POINT CHARGE IN CONSTANT ELECTRIC FIELD

Electric Field:

Electric field is a region around a charged particle or object within which a force would be exerted on other charged particles or objects.

The force that is exerted by the source charge on the test charge is a two-step process:

1. At first, the source charge creates its own field, which means that it creates a region up to which it will be able to exert force on any other charged particle or object.
2. Whenever the test charge comes in that region, it feels the force due to the source charge.

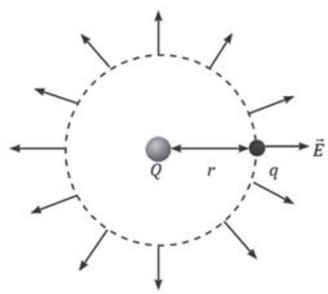
The term 'field' signifies how some distributed quantity (scalar or vector) varies with position

Electric field strength($\vec{E}$):

The electric field strength (often simply called electric field) at a point is defined as the electrostatic force F_e per unit positive charge at that point.

Electric field strength due to a point charge at a distance:

Consider a source charge Q (it creates the field) and a test charge q (it feels the force) as shown in the figure. Let the separation between them be r.



The magnitude of electrostatic force on the test charge q is, $F_e = \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2}$. The magnitude of this electrostatic force will remain unchanged as long as the test charge is at the surface of a sphere of radius r. The field from the source charge can be visualised as light spreading out from a bulb in all directions.

The electric field strength or the electric field of a charge Q at a distance r is given by,

$$E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

SI unit: The SI unit of the electric field is NC^{-1}.

1. The nature of the electric field produced by a point charge is non-uniform because at every point in space, even though the magnitude is the same, the direction of the electric field is different.
2. The electric field of a charge Q at a distance r is given by,

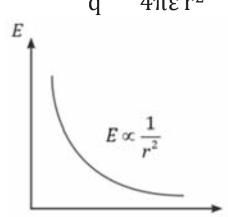
$$E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

3. The direction of the electric field is radially outwards for a positive charge and radially inwards for a negative charge.

Graphical plot of electric field strength variation with distance from a point charge:

The electric field of a point charge Q at a distance r is given by,

$$E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$



Hence, it can be said that the electric field is inversely proportional to the square of the distance from the charge itself. Therefore, the E-r graph will be parabolic in nature as shown in the figure.

Ex. Calculate the electric field strength at a point 1 cm away from a point charge of magnitude, 10 μC. (Assume no other electric charge to be present)

Sol. Given,
The charge is, $Q = 10\ \mu C$.
The distance from the charge where the electric field strength is to be measured is,
$r = 1\ cm = 0.01\ m$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E = (9 \times 10^9)\left[\frac{10 \times 10^{-6}}{(0.01)^2}\right]$$

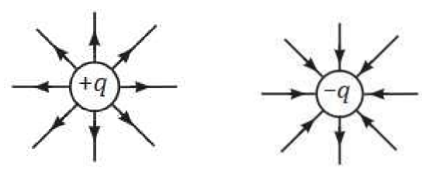
$$E = (9 \times 10^9)\left[\frac{10 \times 10^{-6}}{10^{-4}}\right]$$

$$E = 9 \times 10^8 NC^{-1}$$

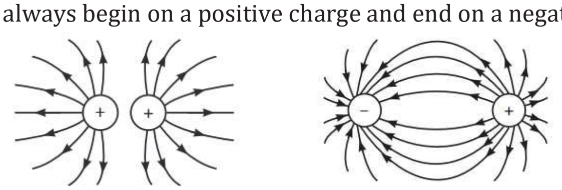
The direction of the electric field strength will be radially outward.

Properties of Electric Field Lines:

1. For a positive charge, the field lines will be radially outwards, and for a negative charge, the field lines will be radially inwards.

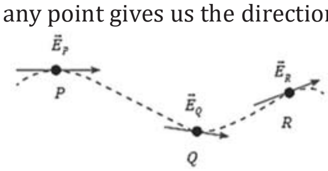


2. Electric field lines always begin on a positive charge and end on a negative charge.



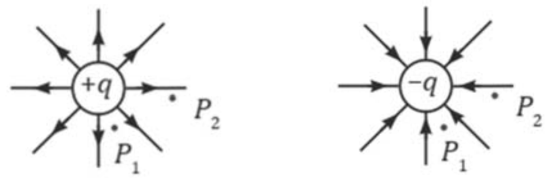
3. The number of lines leaving a positive charge or ending at a negative charge is proportional to the magnitude of the charge. The greater the magnitude of the charge, the more dense the field lines will be at the location of the charge.
It should be noted that if 1 C charge and 3 C charge are considered, and 4 lines are used to represent the field lines for the 1 C charge, then 12 lines should be used to represent the field lines for the 3 C charge.

4. The tangent to a field line at any point gives us the direction of electric field at that point.



5. Two electric field lines can never intersect because if it happens, then there will be two different directions for a single value of electric field at the point of intersection of those two field lines, which is impossible.

6. The field lines never form closed loops, as a line can never start and end on the same charge.
7. In a region of a uniform electric field, the field lines are straight, parallel, and uniformly spaced.
8. Let us take a case of a region of non-uniform electric field.



	AT P_1	AT P_2
Electric Field strength	High	Low

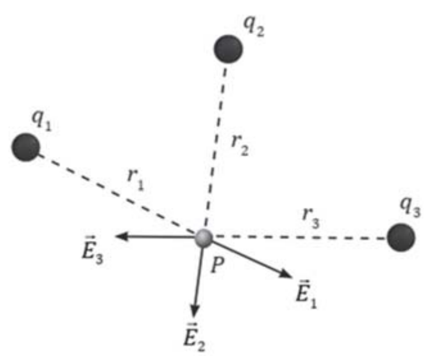
The strength of the electric field is greater where the density of the field lines is larger.

Principle of Superposition:

The resultant electric field at a point will be the vector sum of the electric fields due to all individual point charges.

Consider three point charges q_1, q_2, and q_3, as shown in the figure. If $\vec{E}_1, \vec{E}_2, \vec{E}_3$ are the electric fields due to q_1, q_2, and q_3 respectively, then the net electric field at point P is,

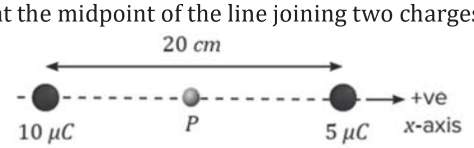
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



Therefore, if n number of charges are present in the space, then the electric field at point P will be,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots. + \vec{E}_n = \sum_{i=0}^{n} \vec{E}_i$$

Ex. Find the electric field at the midpoint of the line joining two charges separated by 20 cm.



Sol. Let $q_1 = 10\mu C = 10 \times 10^{-6} C$ and $q_2 = 5\mu C = 5 \times 10^{-6} C$

The distance of point P from both the charges is, r = 10 cm = 0.1 m.

Therefore, at point P,

The electric field due to q_1 is, $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{i}$.

The electric field due to q_2 is, $\vec{E}_2 = \frac{1}{4\pi\ _0}\frac{q_2}{r^2}(-\hat{i})$

Hence, the net electric field at point P is,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_0}[\frac{q_1}{r^2} - \frac{q_2}{r^2}]\hat{i}$$

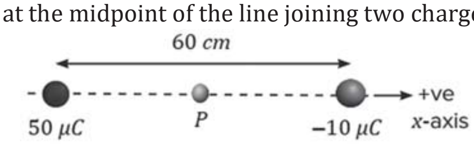
$$\vec{E}_{net} = (9 \times 10^9)[\frac{10 \times 10^{-6}}{(0.1)^2} - \frac{5 \times 10^{-6}}{(0.1)^2}]\hat{i}$$

$$\vec{E}_{net} = (9 \times 10^9)[10 - 5] \times \frac{10^{-6}}{(0.1)^2}\hat{i}$$

$$\vec{E}_{net} = (45 \times 10^9) \times \frac{10^{-6}}{10^{-2}}\hat{i}$$

$$\vec{E}_{net} = 45 \times 10^5\hat{i}NC^{-1}$$

Ex. Find the electric field at the midpoint of the line joining two charges separated by 60 cm.



Sol. Let $q_1 = 50\mu C = 50 \times 10^{-6}C$ and $q_2 = -10\mu C = -10 \times 10^{-6}C$

The distance of point P from both the charges is, r = 30 cm = 0.3 m.

Therefore, at point P,

The electric field due to q_1 is, $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{i}$

The electric field due to q_2 is, $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\hat{i}$

In this case, the directions of the electric field at point P due to both the charges are along the positive x-axis. Hence, the net electric field at point P is,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_0}[\frac{q_1}{r^2} + \frac{q_2}{r^2}]\hat{i}$$

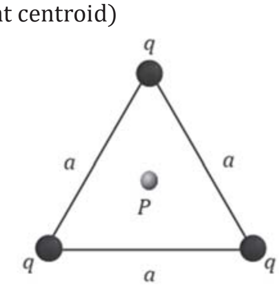
$$\vec{E}_{net} = (9 \times 10^9)[\frac{50 \times 10^{-6}}{(0.3)^2} + \frac{10 \times 10^{-6}}{(0.3)^2}]\hat{i}$$

$$\vec{E}_{net} = (9 \times 10^9)[50 + 10] \times \frac{10^{-6}}{(0.3)^2}\hat{i}$$

$$\vec{E}_{net} = (9 \times 60 \times 10^9) \times \frac{10^{-6}}{9 \times 10^{-2}}\hat{i}$$

$$\vec{E}_{net} = 6 \times 10^6\hat{i}NC^{-1}$$

Ex. Find the net electric field at P (at centroid)

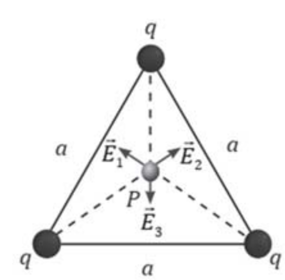


Sol. The given triangle is an equilateral triangle of side length a, and we know that the centroid divides the altitude in the ratio of 2: 1. Hence, the distance of the centroid from any vertex of the triangle is $\frac{a}{\sqrt{3}}$.

The magnitude of $\vec{E}_1, \vec{E}_2, \vec{E}_3$, will be,

$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = \frac{1}{4\pi\ _0}\frac{q}{(\frac{a}{\sqrt{3}})^2}$

By symmetry, the net electric field at point P will be zero.



Short trick:

By polygon law, if we have any number of vectors forming the polygon, and they obey cyclic symmetry, the resultant of that vectors will always be zero.

In this case, $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ can also form a triangle and they have a cyclic symmetry since the angle between each of them is 120°.

Conservative Force

The work performed by or against a conservative force relies solely on the initial and final positions of the object.

Ex. Gravitational force, elastic force, electrostatic force, and so on.

Work done by conservative forces

1st format:

When a constant force is applied.

$$\vec{F} = 4\hat{\imath} + 3\hat{\jmath} \longrightarrow \text{ constant force}$$

If the displacement is specified $\vec{s}$ then work done will be $W = \vec{F} \cdot \vec{s}$

2nd format:

When $\vec{F}$ is given as a function of x, y, z.

$$\vec{F} = f(x)\hat{\imath} + f(y)\hat{\jmath} + f(z)\hat{k}$$

If the displacement is described as $\vec{s}$ then the work done will be.

$$W = \int \vec{F} \cdot d\vec{r}$$

$$W = \int (F(x)\hat{\imath} + F(y)\hat{\jmath} + F(z)\hat{k}) \cdot (dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k})$$

$$W = \int_{x_i}^{x_f} F(x)dx + \int_{y_i}^{y_f} F(y)dy + \int_{z_i}^{z_f} F(z)dz$$

3rd format:

When $\vec{F}$ is given as a perfect differential.

$$\vec{F} = y\hat{\imath} + x\hat{\jmath}$$

In this arrangement, the x-component of the force comprises any function of y, while the y-component of the force comprises any function of x.

Steps to identify a force to be perfect differential

1. Suppose the force is, $\vec{F} = F(x, y)\hat{\imath} + F(x', y')\hat{\jmath}$
2. Integrate $F(x, y)$ w.r.t x i.e., find $\int F(x, y)dx$ keeping y to be constant.
3. Integrate $F(x', y')$ w.r.t y i.e., find $\int F(x', y')dy$ keeping x to be constant.
4. If $\int F(x, y)dx = \int F(x', y')dy$, then the force will be perfect differential.

Ex. $\vec{F} = 2xy^3\hat{\imath} + 3x^2y^2\hat{\jmath}$

Here
$$F(x, y) = 2xy^3$$
$$F(x', y') = 3x^2y^2$$
$$\int F(x, y)dx = \int 2xy^3dx = 2y^3 \int xdx = x^2y^3$$
$$\int F(x', y')dy = \int 3x^2y^2dy = 3x^2 \int y^2dy = x^2y^3$$

Since $\int F(x, y)dx = \int F(x', y')dy$, the force is perfect differential. Work done:

$$W = \int F(x, y)dx + \int F(x', y')dy$$
$$W = \int d(x^2y^3)$$

Ex. Find work done for the force $\vec{F} = x^2\hat{\imath} + y\hat{\jmath}$ from (1,1) to (2,3).

Sol. We have
$$WD = \int_1^2 F(x)dx + F(y)dy + F(z)dz$$
$$= \int_1^2 x^2 \cdot dx + \int_1^3 ydy.$$
$$= [\tfrac{x^3}{3}]_1^2 + [\tfrac{y^2}{2}]_1^3$$
$$= \frac{7}{3} + 4$$
$$= \frac{19}{3} \text{ joule}$$