ENERGY DENSITY OF E-FIELD

The energy density of an electric field refers to the amount of energy stored per unit volume in the region occupied by the electric field. It represents the concentration of energy within that space due to the presence of the electric field. Mathematically, the energy density (u_E) of an electric field (E) can be expressed as:

$$U_E = \frac{1}{2} \epsilon_o E^2$$

Where:

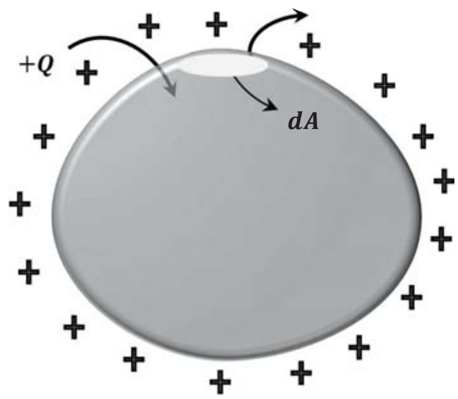
- ➢ uE is the energy density of the electric field,
- ➢ E is the magnitude of the electric field,
- ➢ ϵ0 is the vacuum permittivity constant.

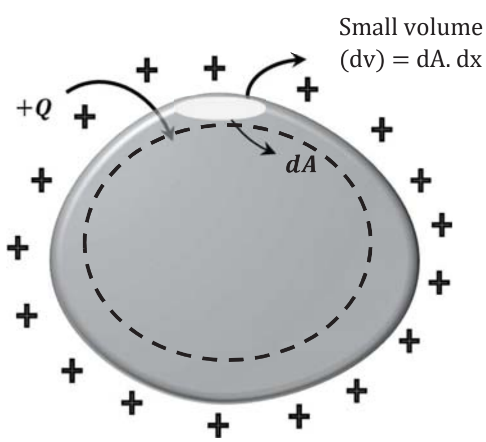
This equation indicates that the energy density of an electric field is directly proportional to the square of the electric field strength. Therefore, regions with higher electric field strengths will have higher energy densities, signifying greater stored energy per unit volume.

Energy Density

The concept of energy density refers to the amount of energy contained within a given volume. Mathematically, it is represented as $\frac{dE}{dV}$, where dE represents the energy contained within a volume element dV.

In the context of electric fields, as electric fields carry energy, we establish the notion of energy density specifically for electric fields. This is expressed in the phrase "Wherever there is an electric field, there is energy," emphasizing the correlation between electric fields and energy.





Let's consider a small area dA depicted in the illustration. The force acting on this area, denoted as dA, is given by the expression $\frac{\sigma 2}{2\epsilon 0} \cdot$ dA.

Now, when this area is expanded through dx, the work done in the expansion is calculated as the product of the force dF and the displacement dx, which yields dW = dF × dx = $\frac{\sigma 2}{2\epsilon 0} \cdot$dA dx.

This work performed during expansion is transformed into stored energy within the small element. This energy stored per unit volume is represented by the expression $\frac{\sigma 2}{2\epsilon 0}$. = dv = dϵ.

In essence, the energy stored per unit volume, known as energy density, is a measure of the amount of energy stored within a given volume.

The concept of energy density, denoted as ϵ·d, signifies the amount of energy stored within a given volume. This energy density can be expressed mathematically as $\frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2}\epsilon_0 E^2$,

where ϵ. $d = \frac{1}{2}\epsilon_0 E^2$.

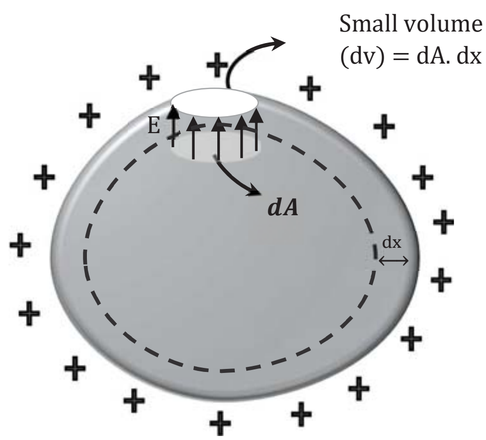
This equation arises from the relationship $\frac{\sigma}{\epsilon 0} = E$,

Where σ is the surface charge density and E represents the electric field intensity.

To determine the energy associated with the expansion through a volume dV, we can calculate the energy using the integral:

$$U_{expansion} = \epsilon \cdot d \times \text{"volume"}$$
$$= \int \epsilon \cdot d \times dv$$

Here, ϵ·d is a variable factor, and dv represents the differential volume. This integral allows us to find the energy associated with the expansion process throughout the volume dV.



The energy that is stored in the form of an electric field within a volume dV can be represented by dU, and it is equivalent to the work done dW during the process. Mathematically,

this is expressed as dU= $dW = \frac{\sigma^2}{2\varepsilon_0} dA \cdot dx$

This expression signifies that the energy stored in the electric field within the volume dV is determined by the product of the force $\frac{\sigma^2}{2r_0}$ acting on the area dA and the displacement dx. In essence, it quantifies the energy stored per unit volume due to the electric field within dV.

The energy density, denoted as ED, is defined as the ratio of the energy stored dU to the volume dV, represented mathematically as ED = $\frac{dU}{dV}$.

For this specific scenario, the energy density is calculated as ED= $\frac{\sigma^2}{2r_0}$, which is equivalent to $\frac{1}{2}\varepsilon_0 E^2$,where E represents the electric field intensity.

This formulation stems from the relationship between the electric field E and the surface charge density σ, given by E = = $\frac{a}{r0}$. Thus, the energy density ED is directly influenced by the square of the electric field intensity E, which is in turn determined by the ratio of the surface charge density σ to the vacuum permittivity ε_0.

Ex. When a cube with side length a is positioned in proximity to an infinite sheet characterized by a charge density of $\sigma\ C/m^2$, the task is to determine the energy stored within the cube due to this configuration.

Sol. The energy stored within the cube is calculated by multiplying the energy density by the volume of the cube. Mathematically, this can be expressed as follows:

$$\text{Energy stored} = \frac{1}{2}\epsilon_0 E^2 \times \text{volume} \qquad (1)$$

The electric field generated by the infinite sheet is given by the equation:

$$E = \frac{\sigma}{2\epsilon 0}$$

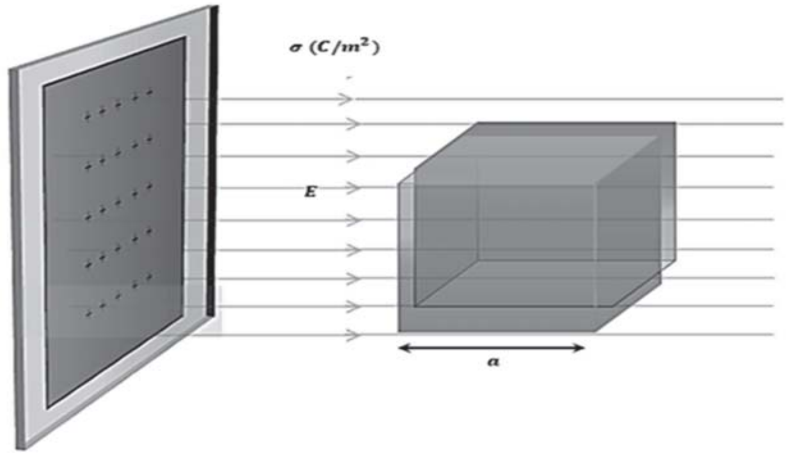
Substituting this expression for electric field into equation (1), we obtain:

$$\text{Energy} = \frac{1}{2}\epsilon_0 \left(\frac{a}{2e_0}\right)^2 \times Q^3$$

$$= \frac{\sigma^2}{Br_0} \times Q^3$$

$$= \frac{\sigma^2 a^3}{8e_0}$$

This formula represents the energy stored within the cube due to the presence of the infinite sheet with charge density σ, where a denotes the side length of the cube.



Energy of HC, SC, HNC and SNC Sphere using energy density

To calculate the energy of a hollow conducting (HC), solid conducting (SC), hollow non-conducting (HNC), and solid non-conducting (SNC) sphere using energy density, we first need to establish the expression for energy density ϵd in terms of the electric field E and other relevant parameters for each type of sphere.

1. **Hollow Conducting Sphere (HC):**
For a hollow conducting sphere, the electric field is zero inside the sphere and $E = \frac{kq}{r^2}$ outside the sphere (where k is a constant, Q is the charge on the sphere, and r is the distance from the center of the sphere). Since there is no electric field inside, the energy density inside the sphere is zero. Outside the sphere, the energy density can be calculated using $\epsilon_d = \frac{1}{2}\epsilon_0 E^2$. We then integrate this energy density over the volume of the sphere to find the total energy.

2. **Solid Conducting Sphere (SC):**
For a solid conducting sphere, the electric field inside is $E = \frac{kq}{\pi} r$ (where R is the radius of the sphere), and zero outside. We calculate the energy density using $\epsilon_d = \frac{1}{2}\epsilon_0 E^2$ and integrate over the volume of the sphere.

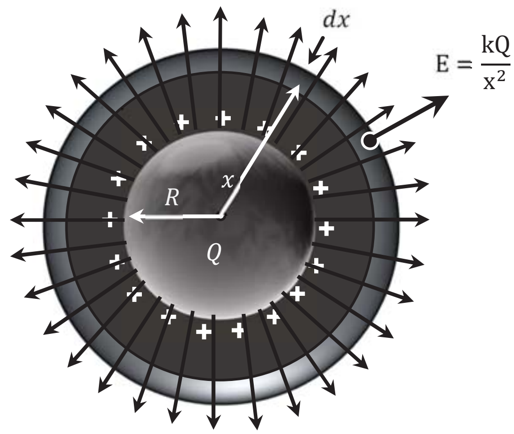
3. **Hollow Non-Conducting Sphere (HNC):**
For a hollow non-conducting sphere, the electric field is zero both inside and outside the sphere. Therefore, the energy density is zero everywhere, and the total energy is zero.

4. **Solid Non-Conducting Sphere (SNC):**
For a solid non-conducting sphere, the electric field inside is $E = \frac{kQ}{k_0 R^2}(3R^2 - r^2)$, and zero outside. We calculate the energy density using $\epsilon_d = \frac{1}{2}\epsilon_0 E$ and integrate over the volume of the sphere.

After obtaining the expression for energy density and integrating over the appropriate volume, we can find the total energy for each type of sphere.

Energy of Hc, Sc, HNC Sphere

Consider a narrow strip with a thickness of dx located at a distance x from the center of a sphere, as illustrated. At this distance, the electric field is described by the equation $\epsilon_f = \frac{K\theta}{x^2}$, where k is a constant and θ represents some parameter associated with the field.



The energy density at this distance x is given by $\varepsilon d = \frac{1}{2}\left(\frac{k^2 e^2}{x^4}\right)$, indicating the energy per unit volume in the vicinity.

The energy contained within the shaded region can be determined by integrating the expression for energy density d∊, yielding $d\epsilon = \frac{1}{2}\left(\frac{ek^2\theta^2}{x^4}\right) \cdot 4\pi x^2 dx$

The total energy is then computed by integrating d∊ over the range R to infinity,

resulting in $\int d\epsilon = (\frac{k\theta^2}{2}) \int_R^\infty \frac{dx}{x^2}$.

Evaluating this integral, we find. $d\varepsilon_I = \frac{K\theta^2}{2}\left[\frac{-1}{x}\right]_R^\infty = \frac{K\theta^2}{2R}$

This expression represents the total energy associated with the given scenario, where R denotes a certain parameter relevant to the problem.

Certainly, here's a rephrased version with detailed explanations:

HC stands for "Hollow Conducting", referring to a sphere that is conductive but contains an empty space within. SC represents "Solid Conducting", indicating a sphere made of a conductive material throughout its volume. HNC denotes "Hollow Non-Conducting", describing a sphere that lacks conductivity and has an empty interior. SNC stands for "Solid Non-Conducting", characterizing a sphere made of a non-conductive material and lacking any internal voids.

Each abbreviation corresponds to a specific type of sphere, distinguished by its material composition and internal structure. Understanding these designations helps clarify the nature of the spheres being discussed, facilitating the analysis of their respective energy characteristics and behaviors in electrostatic scenarios.

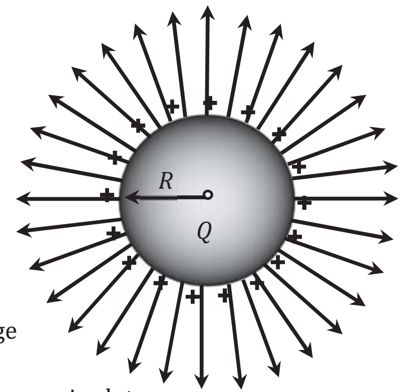
The total energy of the sphere, often referred to as the self-energy of the sphere, is represented by the equation $U_{\text{Total}} = \frac{kQ^2}{2R}$.

This energy is distributed in two distinct regions:

In the region $R < x < \infty$, the energy exists in the form of an electric field.

However, within the region $0 < x < R$, the energy is effectively nonexistent, indicated by $U = 0$.

This delineation signifies the distribution of energy within and around the sphere, with energy concentrated in the outer space beyond the sphere's radius R, while within the sphere itself, no significant energy presence is observed.



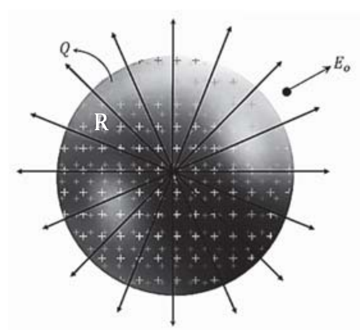
The total energy U_{Total} of the sphere is given by the formula
$U_{Total} = 2R/kQ^2$.
This expression denotes the total energy stored within the sphere, where:

- k represents a constant related to the electrostatic force.
- Q is the charge magnitude of the sphere.
- R signifies the radius of the sphere.

The energy calculation provides valuable insight into the electrostatic properties and behaviors of the sphere, illustrating the energy stored within it due to its charge distribution.

The potential energy of the sphere, along with the work required to assemble the system, corresponds to the energy that can be obtained by dismantling the system. This potential energy signifies the energy stored within the sphere due to its charge configuration and position in an electric field. When assembling the system, work is done against forces, which results in the accumulation of this potential energy within the sphere. Conversely, when disassembling the system, this energy is released or made available, indicating its potential utility or significance in various electrostatic applications or analyses.

Energy of SNC Sphere



Consider a slender strip with a thickness of dx, positioned at a distance x from the center of a sphere, as depicted.
The electric field at this distance x is represented by

$$\epsilon \cdot f = \frac{K\theta x}{R^3}.$$

This equation describes the strength of the electric field, where K is a constant, θ is a parameter associated with the field, and R denotes the radius of the sphere.
The energy density at this distance x is given by

$$\epsilon_d = \frac{1}{2}\epsilon_0\left(\frac{k^2\theta^2x^2}{R^6}\right).$$

This expression quantifies the energy per unit volume in the vicinity.
The energy contained within the shaded region can be calculated using $d\epsilon = \frac{1}{2}\left(\frac{\epsilon_1 K^2\theta^2}{K^6}\right) \cdot 4\pi x^2 \cdot dx$.
This equation accounts for the energy within the strip and integrates over its thickness.
The total energy is then determined by integrating dϵ from 0 to R, resulting in

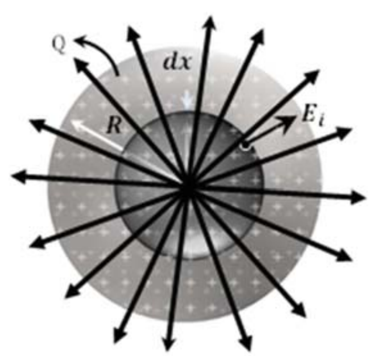
$$\int d\epsilon = \frac{K\theta^2}{2K^5}\int_0^R x^4 dx.$$

Evaluating this integral, we find $\epsilon_T = \frac{K\theta^2}{2K^5} \cdot \frac{R^5}{5} = \frac{K\theta^2}{10}$.

This expression represents the total energy associated with the given scenario, where R denotes the radius of the sphere and S signifies its surface area.

Energy inside $+$ Energy artside $[R - \infty)$
$(0 - R)$ 11
11 Same as S · C, H · C, H · N · C

$$\frac{K\theta^2}{10R} + \frac{K\theta^2}{2R} = \frac{3K\theta^2}{5R}$$

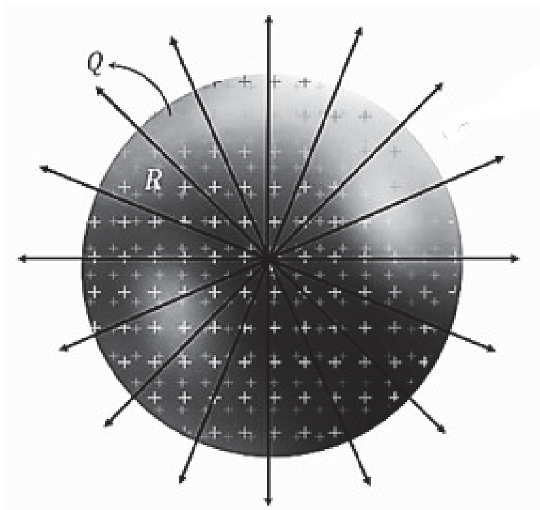


The energy existing beyond the sphere (for R < x < ∞) is given by $U_o = \frac{kQ^2}{2R}$.

This expression quantifies the energy present in the region outside the sphere, extending infinitely away from its surface.

Conversely, within the interior of the sphere ($0 < x < R$), the energy is represented by $U_i = \frac{kQ^2}{10R}$. This equation describes the energy contained within the sphere's confines, specifically within the range from its center to its surface radius.

By delineating the energy within and outside the sphere, these expressions provide a comprehensive understanding of the distribution of energy across different spatial regions relative to the sphere's boundaries.



The total energy (E_T), also known as the self-energy of the sphere, is determined by summing the energies present both inside and outside the sphere.

Mathematically, this is expressed as $U_T=U_i+U_o$,

where U_i represents the energy within the sphere and U_o denotes the energy outside the sphere.

Substituting the expressions for U_i and U_o,

we get, $U_T= \frac{kQ^2}{10R} + \frac{kQ^2}{2R}$.

Simplifying these yields, $UT= \frac{3kQ^2}{5R}$.

This equation represents the total energy of the sphere, incorporating contributions from both its interior and exterior regions, thus providing a comprehensive assessment of its overall energy content.

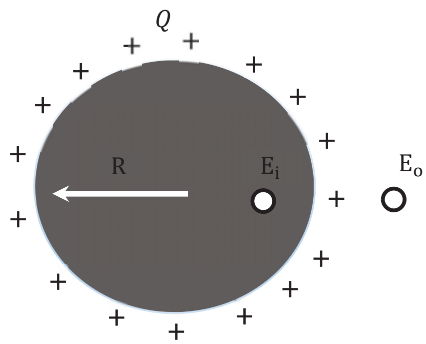
The equation $U_T = \frac{3kQ^2}{5R}$ represents the total potential energy, denoted as U_T, of a sphere within a certain system. This equation is derived from physical principles, where k represents a constant, Q denotes the charge of the sphere, and R signifies its radius.

The potential energy of the sphere within the system can be calculated using this equation. It indicates the amount of energy associated with the configuration and arrangement of the components within the system, specifically in relation to the charge and size of the sphere.

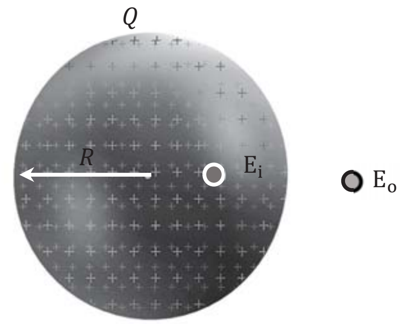
To dismantle the system or alter its configuration, work must be done. This work requirement involves applying force to change the arrangement of components, which consequently changes the potential energy of the system. The amount of work done is proportional to the change in potential energy.

Upon dismantling the system, the energy represented by U_T becomes available. This energy can be harnessed or utilized for various purposes once the system is disassembled. Therefore, the equation provides insight into the energy potential inherent within the system, which can be accessed by dismantling it.

Sphere \ Energy	U_i	U_o	U_{Total}
HC/SC/HNC	0	$\frac{KQ^2}{2R}$	$\frac{KQ^2}{2R}$
SNC	$\frac{KQ^2}{10R}$	$\frac{KQ^2}{2R}$	$\frac{3KQ^2}{5R}$



HC/SC/HNC Sphere



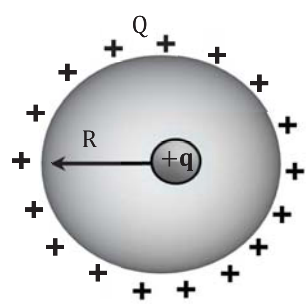
SNC Sphere

Ex. In this scenario, we position a positive charge denoted as +q at the precise center of a hollow, non-conducting sphere with a radius R. This sphere itself bears a charge Q, as illustrated. The objective is to calculate the potential energy associated with this configuration.

Sol. In this context, the overall potential energy of the system is composed of two main components: the self-energy of the sphere and the interaction energy between the sphere and the nearby charge. These components together contribute to the total potential energy of the system.

The self-energy of the sphere accounts for the energy associated with its own charge distribution and configuration. It reflects the energy required to assemble or maintain the sphere's charge arrangement.

On the other hand, the interaction energy between the sphere and the nearby charge refers to the energy associated with the electrostatic interaction between them. This interaction energy arises due to the presence of the charge within the sphere's electric field, or vice versa.



By summing up these two components – the self-energy of the sphere and the interaction energy between the sphere and the charge – we obtain the total potential energy of the system. This summation captures the combined energy effects resulting from the configuration and interactions within the system.

Potential energy:

$$E_T = \frac{K\theta^2}{2R} + \frac{Kq\theta}{R}$$

In this equation, E_T, denoting the total energy, is determined by the sum of two distinct components.

The first component $\frac{K\theta^2}{2R}$. represents the potential energy associated with the system's configuration, where K is a constant, θ signifies a variable parameter, and R represents a characteristic distance.

The second component, $\frac{Kq\theta}{R}$, depicts the interaction energy resulting from the presence of a separate charge, q, within the system. Here, K represents a constant, q denotes the charge magnitude, and θ symbolizes the relevant variable.

When combined, these components provide a comprehensive understanding of the total energy within the system, encompassing both the intrinsic potential energy due to its configuration and the additional energy arising from the interaction with the external charge.

E-Field and Potential Due to Sphere with Cavity Inside

The electric field and potential resulting from a sphere with a cavity inside can be analyzed by considering the charge distribution and geometry of the system.

When there's a cavity within the sphere, the electric field and potential are influenced by the distribution of charge both inside and outside the cavity.

1. **Electric Field (E-field):**

   Inside the cavity: The electric field is zero within the cavity because there are no charges present inside it.

   Outside the cavity (within the material of the sphere): The electric field behaves as if all the charge is concentrated at the center of the sphere. This is a result of Gauss's Law, which states that the electric field due to a uniformly charged sphere is the same as that of a point charge located at its center. The field strength decreases with distance from the center following an inverse square law.

2. **Electric Potential (Voltage):**
   - Inside the cavity: Similar to the electric field, the potential is constant within the cavity and is equal to the potential at the surface of the cavity. This is because there are no charges inside the cavity, so there is no work done in moving a charge within it.
   - Outside the cavity (within the material of the sphere): The potential due to the sphere with the cavity behaves similarly to that of a solid sphere. It decreases with distance from the center, following the inverse relation of $\frac{1}{r}$, where r is the distance from the center of the sphere.

In summary, within the cavity, both the electric field and potential are zero. Outside the cavity but within the material of the sphere, they behave as if all the charge is concentrated at the center of the sphere, resulting in a radial field and potential.