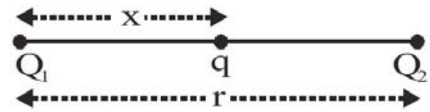
ELECTROSTATIC EQUILIBRIUM

Equilibrium of charged particles

In a state of equilibrium, the net electric force acting on each charged particle is zero. The stability of a charged particle under the influence of Colombian forces alone can never be achieved in equilibrium.



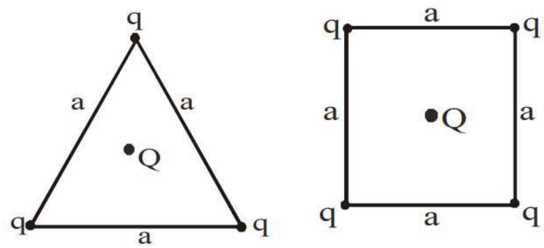
Equilibrium of three point charge

1. Two charge must be like nature as $F_q = \frac{KQ_1q}{x^2} + \frac{KQ_2q}{(r-x)^2} = 0$
2. Third charged should be of unlike nature as $F_{Q_1} = \frac{KQ_1Q_2}{r^2} + \frac{KQ_1q}{x^2} = 0$

   Therefore $x = \frac{\sqrt{Q_1}}{\sqrt{Q_1}+\sqrt{Q_2}} r$ and $q = \frac{-Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$

Equilibrium of symmetric geometrical point charged system

Value of Q at centre for which system to be in state of equilibrium



1. For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$
2. For square $Q = \frac{-q(2\sqrt{2}+1)}{4}$

Equilibrium of suspended point charge system

For equilibrium position

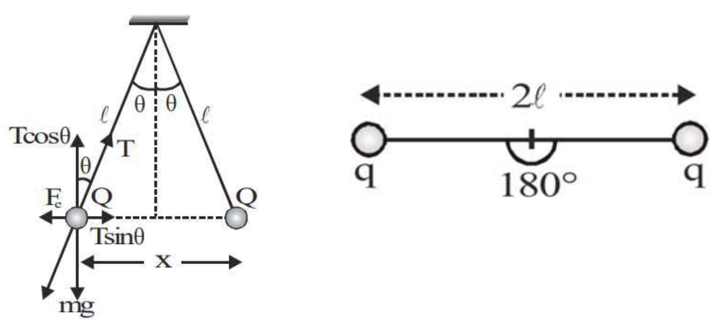
$$T\cos\theta = mg \text{ and} T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2mg}$$

If θ is small then tan

$$\theta \approx \sin\theta = \frac{x}{2\ell} \Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = [\frac{Q^2\ell}{2\pi\epsilon_0 mg}]^{\frac{1}{3}}$$

If whole set up is taken into an artificial satellite$(g_{eff} \simeq 0)$ then

$$T = F_c = \frac{kq^2}{4\ell^2}$$

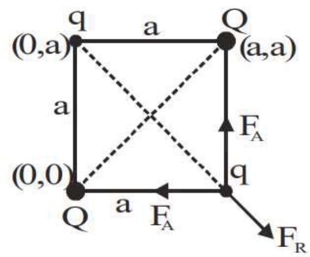


Ex. For the system shown in figure find Q for which resultant force on q is zero.

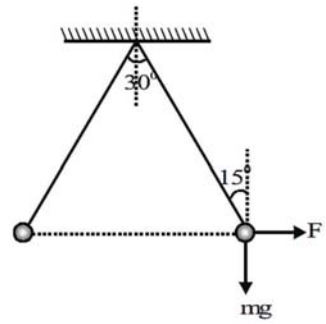
Sol. For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

$$\sqrt{2}F_A = F_R \Rightarrow \sqrt{2}\frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2} \text{ Hence } q = -2\sqrt{2}Q$$



Ex. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 300 with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.



Sol. When set up shown in figure is in air, we have $\tan 15° = \frac{F}{mg}$ When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\frac{F}{\varepsilon_r}$and the effective gravitational force will become $mg(1-\frac{\rho_\ell}{\rho_s})$ Thus we have

$$\tan 15^0 = \frac{F}{mg\epsilon_r(1-\frac{\rho_\ell}{\rho_s})} = \frac{F}{mg} \Rightarrow \epsilon_r = \frac{1}{1-\frac{\rho_\ell}{\rho_s}} = 2$$

Ex. Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law.

$$\vec{F} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

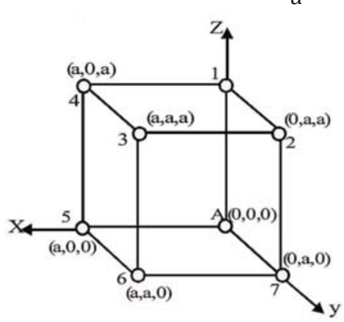
From the figure the different forces acting on A are given as $\vec{F}_{A_1} = \frac{kq^2(-a\hat{k})}{a^3}$

$$\vec{F}_{A_2} = \frac{kq^2(-a\hat{j} - a\hat{k})}{(\sqrt{2}a)^3}, \vec{F}_{A_3} = \frac{kq^2(-a\hat{i} - a\hat{j} - a\hat{k})}{(\sqrt{3}a)^3}; \vec{F}_{A_4} = \frac{kq^2(-a\hat{i} - a\hat{k})}{(\sqrt{2}a)^3}$$

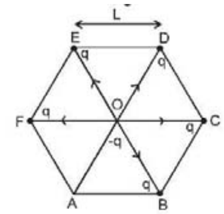
$$\vec{F}_{A_5} = \frac{kq^2(-a\hat{i})}{a^3}, \vec{F}_{A_6} = \frac{kq^2(-a\hat{i} - a\hat{j})}{(\sqrt{2}a)^3}, \vec{F}_{A_7} = \frac{kq^2(-a\hat{j})}{a^3}$$

The net force experienced by A can be given as

$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_5} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_8} + \vec{F}_{A_7} = \frac{-kq^2}{a^2}[(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1)(\hat{i} + \hat{j} + \hat{k})]$$



Ex. Five point charges, each of value +q are placed on five vertices of a regular hexagon of side Lm. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?



Sol. If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O will be zero (as the forces due to individual charges will balance each other).

Now if $\vec{f}$ is the force due to sixth charge and

$\vec{F}$ due to remaining five charges.

$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$