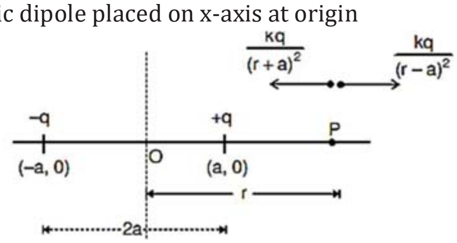
ELECTRIC FIELD AND POTENTIAL DUE TO ELECTRIC DIPLOLE

Electric field and potential due to Electric Dipole

1. **At an axial point**

Figure shows an electric dipole placed on x-axis at origin



Here we wish to find the electric field at point P having coordinates (r, o) (where r >> 2a). Due to positive charge of dipole electric field at P is in outward direction & due to negative charge it is in inward direction.

$$E_{net} \text{ at } P = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}\frac{4\ Kqar}{(r^2-a^2)^2}$$

$$\vec{p} = 2aq$$

$$E_{net} \text{ at } p = \frac{2Kpr}{(r^2-a^2)^2}$$

As r >> 2a

We can neglect a w.r.t. r

$$E_{net} \text{ at } p = \frac{2Kp}{r^3}$$

As we can observe that for axial point direction of field is in direction of dipole moment

$$\text{Vector ally, } \vec{E} = \frac{2k\vec{p}}{r^3}$$

2. **At an equatorial point.**

Again, we consider the dipole placed along the x-axis & we wish to find, electric field at point P which is situated equatorially at a distance r (where r >> 2a) from origin. Vertical component of the electric field vectors cancel out each other.

$$E_{rtt} \text{ at } P = 2E\cos\ \theta[\text{ where} E\ = \frac{Kq}{r^2+a^2}]$$

$$E_{ret} \text{ at } P = \frac{2kq}{r^2+a^2}\cdot\frac{a}{\sqrt{r^2+a^2}}[\cos\ \theta = \frac{a}{\sqrt{r^2+a^2}}]$$

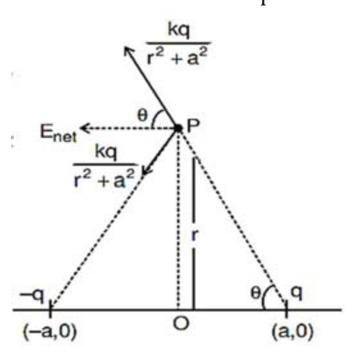
$$E_{rtt} = \frac{2kqa}{(r^2+a^2)^{3/2}} = \frac{kq}{(r^2+a^2)^{3/2}}$$

As we have already stated that r > > 2a

$$E_{net} \text{ at } P = \frac{kp}{r^3}$$

We can observe that the direction of dipole moment & electric field due to dipole at P are in opposite direction.

$$\text{Vector ally } \vec{E} = \frac{-k\vec{P}}{r^3}$$



Electric field at a general Point due to a dipole

Figure shows a electric dipole place on x-axis at origin & we wish to find out the electric field at point P with coordinate (r, θ)

$$E_{net} \text{ at}$$

$$E_{net} = \sqrt{(\frac{2kp\cos\theta}{r^3})^2 + (\frac{kp\sin\theta}{r^3})^2}$$

$$\frac{kp}{r^3}\sqrt{1 + 3\cos^2\theta}$$

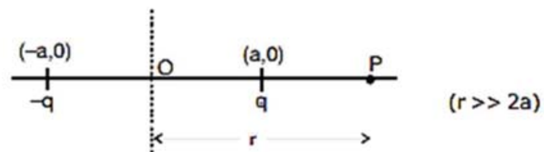
$$\tan\alpha \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}[\frac{\tan\theta}{2}]$$

Electric potential due to a dipole.

1. **At an axial point**



We wish to find out potential at P due to dipole (with p = 2aq)

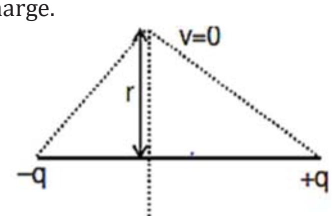
$$V_{met} = \frac{kq}{(r-a)} - \frac{kq}{(r+a)}$$

$$V_{met} = \frac{2kap}{(r^2 - a^2)}$$

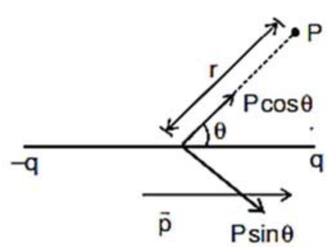
$$V_{met} = \frac{kp}{r^2} \text{ (As P } = 2aq)$$

2. **At a point on perpendicular bisector**

At an equatorial point, electric potential due to dipole is always zero because potential due to +ve charge is cancelled by –ve charge.



3. **Potential due to dipole at a general Point**



Potential at P due to dipole $= \frac{Kp\cos\theta}{r^2}$

Basic torque concept

$$\overline{\tau} = \overline{r} \times \overline{F}$$

- If the net transactional force on the body is zero then the torque of the forces may or may not be zero but net torque of the forces about each point of universe is same
- If we have to prove that a body is in equilibrium then first, we will prove F_{net} is equal to zero & after that we will show τ_{net} about any point is equal to zero.
- If the body is free to rotate then it will rotate about the axis passing through center of mass & parallel to torque vector direction & of the body is hinged then it will rotate about hinged axis.

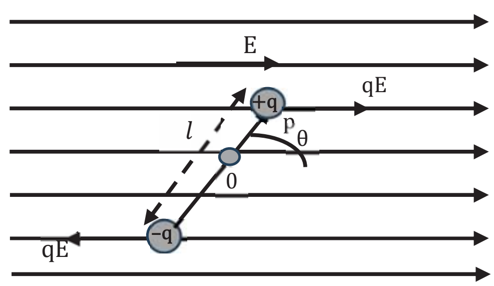
Electric dipole in Uniform electric field:

Imagine a dipole positioned within a uniform electric field, as depicted.

The force acting on the two charges is identical in magnitude but opposite in direction. Consequently, the resultant force on the dipole is zero ($F_{net} = 0$).

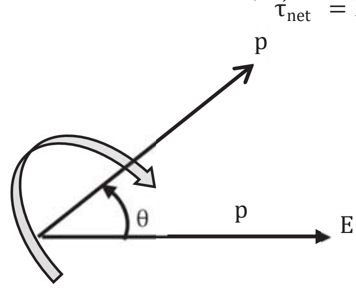
Now, consider the net torque acting on the dipole.

$$\vec{\tau}_{net} = \vec{\tau}_{(+q)} + \vec{\tau}_{(-q)}$$



$$C_A = qEl\sin\theta\,(\times) + qEl\sin 0\,(\times)$$
$$= 2qEl\sin\theta\,(\times)$$
$$= PE\,\text{Sin}\,\theta\,(\times)$$
$$(\because p = 2gl)$$
$$|\vec{\tau}_{net}| = |\vec{p}||\vec{E}|\sin\theta$$

$$|\vec{\tau}_{net}| = |\vec{p}||\vec{E}|\sin\theta \quad \begin{cases} \vec{\tau}_{net} = \vec{p} \times \vec{E} \;\checkmark \\ \vec{\tau}_{net} = \vec{E} \times \vec{p} \;\times \end{cases}$$



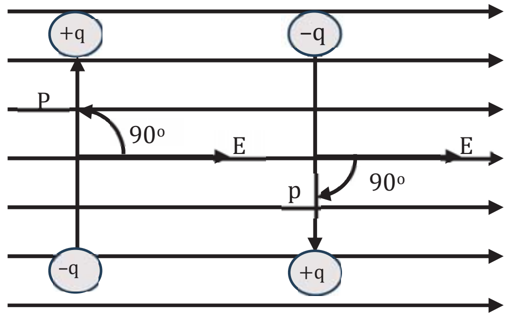
Torque operates with the intention of orienting the dipole in alignment with the electric field's direction.

Torque on dipole:

$$|\vec{\tau}_{net}| = |\vec{p}||\vec{E}|\sin\theta$$
$$\vec{\tau}_{net} = \vec{p} \times \vec{E}$$

For, $|\vec{\tau}_{net}|_{max}$.

To achieve maximum torque, it is essential for the sine of the angle ($\sin\theta$) to be at its maximum, indicating that the angle (θ) should ideally be set at 90 degrees.



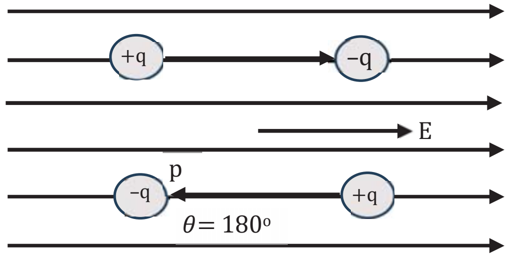
The magnitude of the torque reaches its peak when the dipole is positioned perpendicular to the direction of the electric field.

Torque on dipole:

$$|\vec{\tau}_{net}| = |\vec{p}||\vec{E}| \sin\theta$$

$$\vec{\tau}_{net} = \vec{p} \times \vec{E}$$

For, $|\vec{\tau}_{net}|_{max}$.

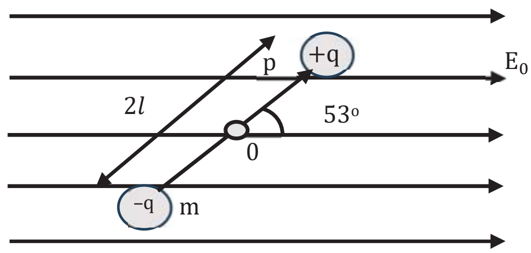


To achieve the lowest possible torque, it is imperative that the sine of the angle ($\sin\theta$) be minimized. This indicates that the angle (θ) should ideally be set to either 0° or 180°.

When the dipole aligns precisely with the direction of the electric field, the magnitude of torque diminishes to either zero or its minimum value.

Ex. Determine the angular acceleration (α) for the depicted mass system.

Sol. Torque on dipole:



$$|\vec{\tau}_{net}| = |\vec{p}||\vec{E}| \sin\theta$$

$$\tau = q \cdot 21 \cdot E_0 \cdot 4/s = \frac{4PE_0}{5} \text{ .... (1)}$$

Moment of inertia of dipole:

$$I = ml^2 + ml^2$$

$$= 2ml^2 \text{ ...... (2)}$$

We have, τ = I alpha

From equation (1) and (2)

$$\frac{(4PE_0)}{5} = \text{Rm } l^2 \text{ alpha}$$

Alpha = $\frac{(2PE_0)}{5\ ml^2}$ ⟺ (P = 2gℓ; Sin 53° = $\frac{4}{5}$) $\alpha = \frac{qE_0}{ml} \sin 5\,3°$