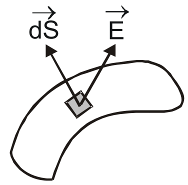
ELECTRIC FLUX

Consider some surface in an electric field $\vec{E}$ Let us select a small area element $\vec{dS}$ on this surface.

The electric flux of the field over the area element is given by $d\phi_E = \vec{E} \cdot \vec{dS}$



Direction of $\vec{dS}$ is normal to the surface. It is along $\hat{n}$

$$d\phi_E = EdS\cos\theta \text{ or } d\phi_E = (E\cos\theta)dS \text{ or } d\phi_E = E_n dS$$

Where E_n is the component of electric field in the direction of $\vec{dS}$.

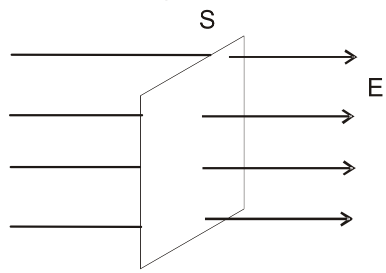
The electric flux over the whole area is given by $\phi_E = \int_S \vec{E} \cdot \vec{dS} = \int_S E_n dS$

If the electric field is uniform over that area then $\phi_E = \vec{E} \cdot \vec{S}$

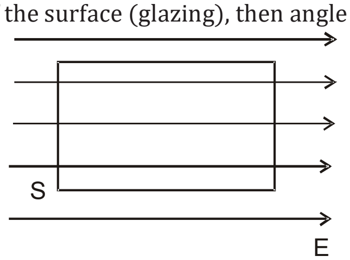
Special Cases :

Case I: If the electric field in normal to the surface, then angle of electric field $\vec{E}$ with normal will be zero

$$\phi = ES\cos 0$$
$$\phi = ES$$



Case II: If electric field is parallel of the surface (glazing), then angle made by $\vec{E}$ with normal=90º



$$\phi = ES\cos 90° = 0$$

Physical Meaning:

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface. It is a property of electric field

Unit

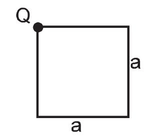
The SI unit of electric flux is $Nm^2\ C^{-1}$ (gauss) or $J\ m\ C^{-1}$.

Electric flux is a scalar quantity. (It can be positive, negative or zero)

Ex. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2m^2$ parallel to the Y–Z plane.

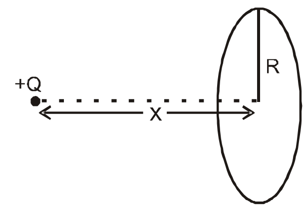
Sol. $\phi_E = \vec{E} \cdot \vec{S} = (\frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{j}) \cdot (0.2\hat{i}) = 240\,\frac{N-m^2}{C}$

Ex. A point charge Q is placed at the corner of a square of side a, then find the flux through the square.



Sol. The electric field due to Q at any point of the square will be along the plane of square and the electric field line are perpendicular to square ; so $\phi = 0$. In other words we can say that no line is crossing the square so flux = 0.

Ex. Find the electric flux due to point charge 'Q' through the circular region of radius R if the charge is placed on the axis of ring at a distance x

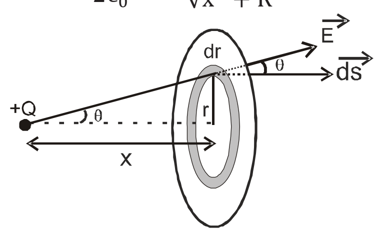


Sol. We can divide the circular region into small rings.
Lets take a ring of radius r and width dr. flux through this small element

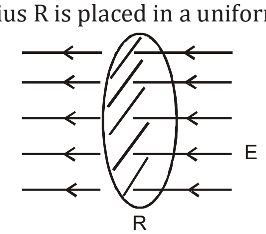
$$d\phi = Eds\cos\theta$$

$$\phi_{net} = \int Eds\cos\theta = \int_{r=0}^{r=R} \frac{KQ}{(x^2+r^2)}(2\pi r dr)\left(\frac{x}{\sqrt{x^2+r^2}}\right)$$

$$\frac{Q}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

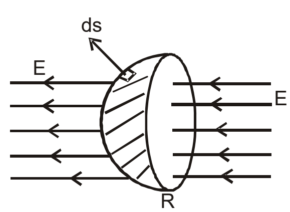


Case-III: Curved surface in uniform electric field.
Suppose a circular surface of radius R is placed in a uniform electric field as shown.



Flux passing through the surface $\phi = E(\pi R^2)$

2. Now suppose, a hemispherical surface is placed in the electric field flux through Hemispherical surface



$$\phi = \int Eds\cos\theta$$

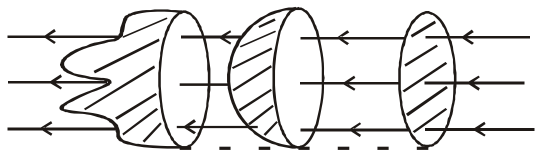
$$\phi = E\int ds\cos\theta$$

Where $\int ds\cos\theta$ is Projection of the spherical surface Area on base.

$$\int ds\cos\theta = \pi R^2$$

So $\phi = E(\pi R^2)$ = same Ans. as in previous case
So we can conclude that

If the number of electric field lines passing through two surfaces are same, then flux passing through these surfaces will also be same, irrespective of the shape of surface



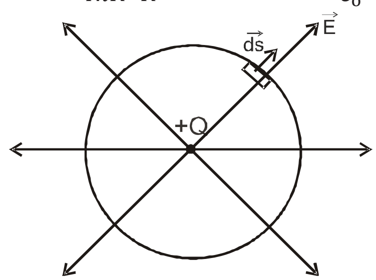
Case IV: Flux through a closed surface

Suppose there is a spherical surface and a charge 'q' is placed at center. flux through the spherical surface

$$\phi = \int \vec{E} \cdot \vec{ds} = \int Eds \qquad \text{as } \vec{E} \text{ is along } \vec{ds} \text{ (normal)}$$

$$\phi = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^2}\int ds \qquad \text{where } \int ds = 4\pi R^2$$

$$\phi = \left(\frac{1}{4\pi R^2}\frac{Q}{R^2}\right)(4\pi R^2) \Rightarrow \phi = \frac{Q}{\varepsilon_o}$$



Now if the charge Q is enclosed by any other closed surface, still same lines of force will pass through the surface.

So here also flux will be $\phi = \frac{Q}{\varepsilon_0}$ that's what Gauss Theorem is

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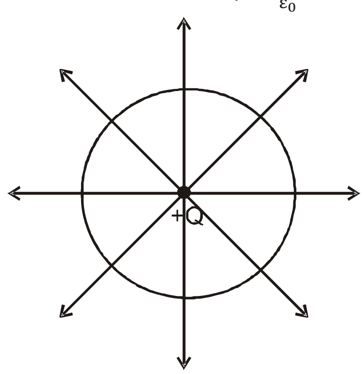
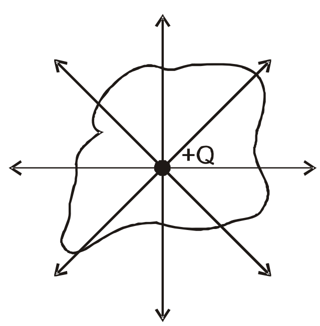
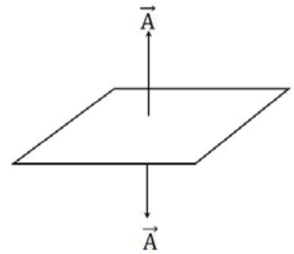


$\phi = \frac{Q}{\varepsilon_0}$



$\phi = \frac{Q}{\varepsilon_0}$

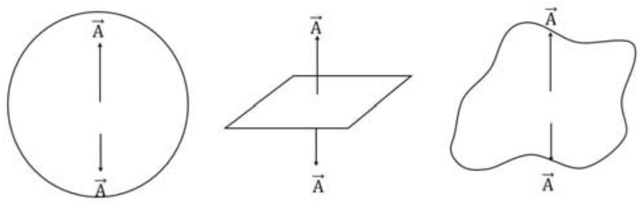
Area Vector



- The orientation of the area vector is consistently perpendicular to the surface.
- Vector Quantity.
- SI Unit: m^2
- Think of one direction of the area vector as positive, while considering the opposite direction as negative.

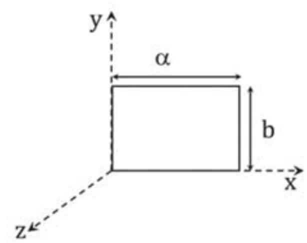
Area vector Open Surfaces

All two-dimensional surfaces are regarded as open surfaces.



For a given problem, only one orientation of the area vector for an open surface is taken into account.

Ex. A rectangle of length a and width b is placed in an $x - y$ plane as shown. Find the area vector of the rectangle

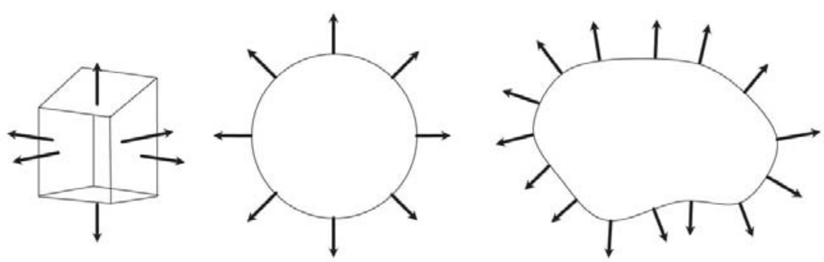


Sol. The magnitude of area of a rectangle of length a and width b is, $A = ab$

Since the rectangle is on xy -plane, the perpendicular vector of the rectangle will be along the z-axis. Thus, the area vector of the rectangle is, $\vec{A} = \pm ab\hat{k}$

Area Vector Closed Surfaces

Three-dimensional surfaces are universally regarded as closed surfaces.



For closed surfaces, the area vector's direction is invariably perpendicular to the surface, with the normal directed outward.