ELECTRIC FIELD

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

Intensity of Electric Field

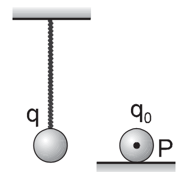
Electric field intensity at a point is equal to the electrostatic for experienced by a unit positive point charge both in magnitude and direction. If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source charges is given by

$$\vec{E} = \frac{\vec{F}}{q_0}$$

If the $\vec{E}$ is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution on the source which is producing the electric field and hence modify the quantity which is measured.

Ex. A positively charged ball hangs from a long silk thread. We wish to measure E at a point P in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0. Will F/q_0 be less than, equal to, or greater than E at the point in question?

Sol. When we try to measure the electric field at point P then after placing the test charge at P it repels the source charge (suspended charge) and the measured value of electric field $E_{measured} = \frac{F}{q_n}$ will be less than the actual value E_{act} that we wanted to measure.

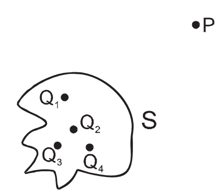


Properties of electric field intensity $\vec{E}$:

1. It is a vector quantity. Its direction is the same as the force experienced by positive charge.
2. Direction of electric field due to positive charge is always away from it while due to negative charge always towards it.
3. Its S.I. unit is Newton/Coulomb.
4. Its dimensional formula is $[MLT^{-3}A^{-1}]$
5. Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is $\vec{E}$ is given by $\vec{F} = q\vec{E}$. Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.
6. It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots.$$
7. It is produced by source charges. The electric field will be a fixed value at a point unless change the distribution of source charges.

Ex. Electrostatic force experienced by – 3μC charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{\mathbf{i}} + 9\hat{\mathbf{j}})\mu N$.

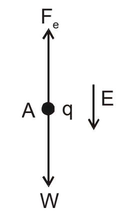
1. Find out electric field intensity at point P due to S.
2. If now 2μC charge is placed and −3μC is removed at point P then force experienced by it will be.



Sol. 1. $\vec{F} = q\vec{E} \Rightarrow (2\hat{i} + 9\hat{j})\mu N = -3\mu C(\vec{E}) \Rightarrow \vec{E} = -7\hat{i} - 3\hat{j}\,\frac{\mu N}{C}$

2. Since the source charges are not disturbed the electric field intensity at 'P' will remain same. $\vec{F}_{2GC} = +2(\vec{E} = 2(-7\hat{i} - 3\hat{j}) = -14\hat{i} - 6\hat{j}\mu N$

Ex. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μc and mass 10 mg. (Take g = 10 ms^2)



Sol. As force on a charge q in an electric field $\vec{E}$ is $\vec{F}_q = q\vec{E}$

So according to given problem

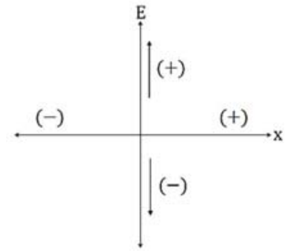
$|\vec{F}_q| = |\vec{W}|$ i.e., $|q|E = mg$

$E = \frac{mg}{|q|} = 10$ N/C., in downward direction.

E – x CURVE

Sign convention

Direction of electric field	Nature
⟶	(+)
⟵	(–)

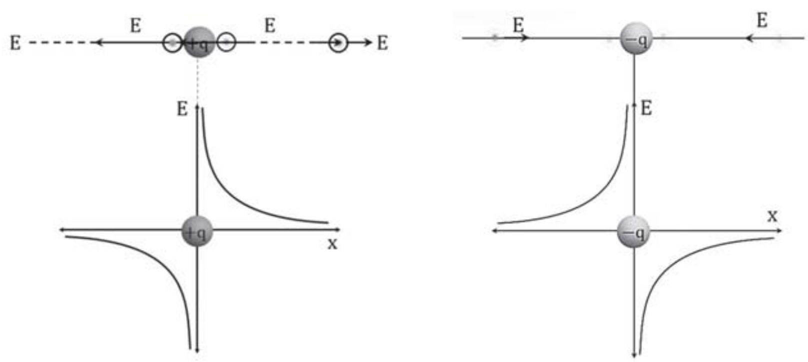


$E(x) \rightarrow x$

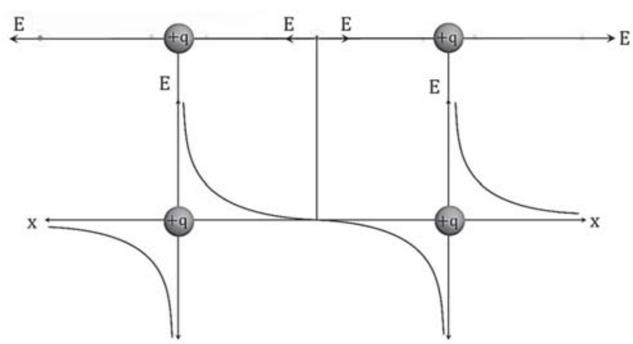
$x = \infty, E = 0$

$x = -\infty, E = 0$

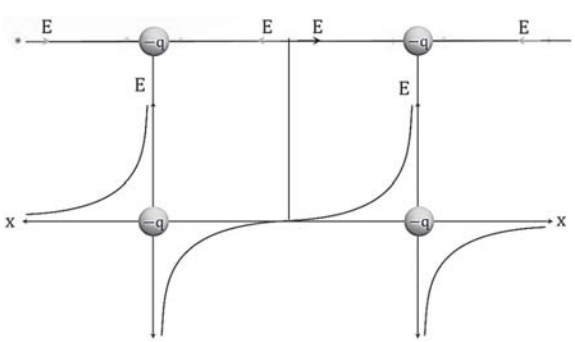
E – x CURVE



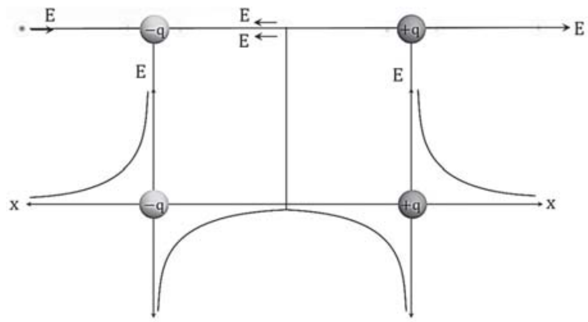
E – x CURVE



E – x CURVE

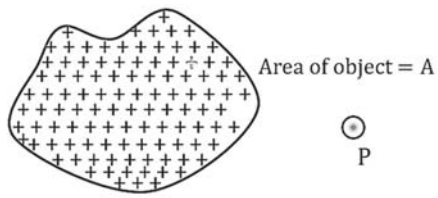


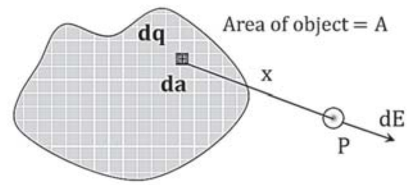
E – x CURVE



Electric Field Due To Continuous Charge Distribution

The charge Q is spread across the surface of the object.

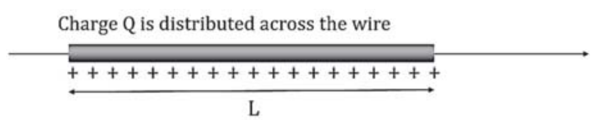




In such scenarios, determine the electric field caused by individual small charge elements, then integrate it across the entire area.

Linear Charge Density (λ)

It represents the charge density per unit length.



$$\lambda = \frac{Q}{L}$$

To determine the electric field along the axis of the rod:

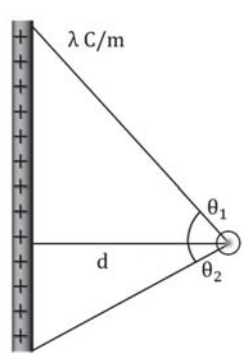
$$dE = \frac{kdq}{x^2} = \frac{k \cdot \lambda d}{x^2}$$

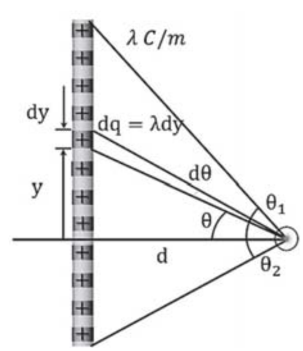
$$E_{net} = k\lambda \int_a^{a+l \cdot} \frac{dx}{x^2}$$

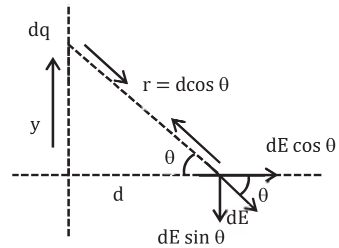
$$k\lambda\left[\frac{1}{a} - \frac{1}{a+l}\right]$$

Because the electric field from each small charge element aligns in the same direction, we can integrate it across the length being considered to obtain the ultimate value.

Electric Field Due To Finitely Charged Rod







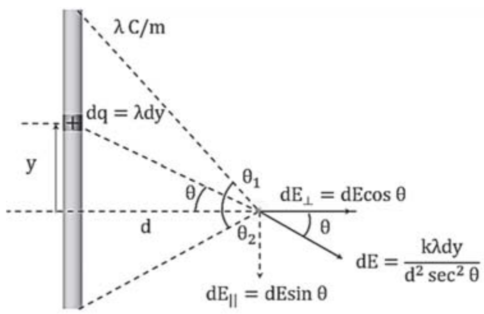
$$\tan\theta = \frac{y}{d}$$
$$y = d\tan\theta.$$

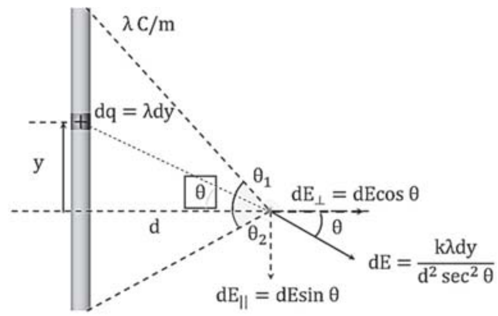
Differentiation w.r.to θ

$$\frac{dy}{d\theta} = d\frac{d\tan\theta}{d\theta}$$
$$dy = d\sec^2\theta d\theta$$
$$dE = \frac{k\cdot dq}{r^2} = \frac{k\cdot\lambda dy}{(d\sec\theta)^2}$$

θ And y both are variable

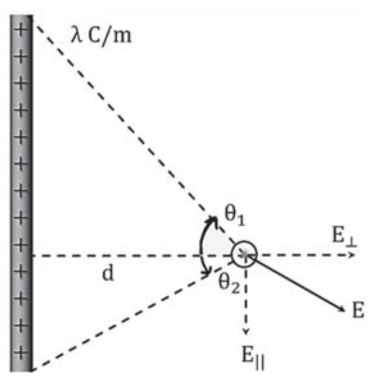
$$\frac{k\cdot\lambda\cdot d\sec^2\theta d\theta}{d^2\cdot\sec^2\theta}$$
$$dE = \frac{k\lambda}{d}d\theta$$

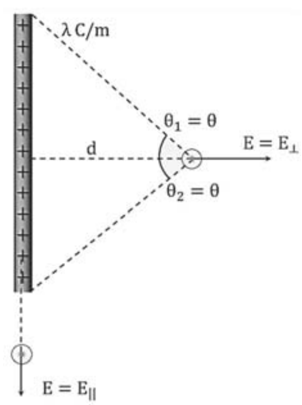




$$E_{11} = dE\sin\int\theta$$
$$E_{11} = \frac{k\lambda}{d}\int_{-\theta_2}^{\theta_1}\sin\theta d\theta$$
$$= \frac{k\lambda}{d}[-\cos\theta]_{-\theta_2}^{\theta_1}$$
$$E_{11} = \frac{k\lambda}{d}[\text{Cos}\theta_2 - \text{Cos}\theta_1]$$

$$E_\perp = \int dE\text{Cos}\theta.$$
$$E_\perp = \frac{k\lambda}{d}\int_{-\theta_2}^{\theta_1}\cos\theta d\theta$$
$$E_\perp = \frac{k\lambda}{d}[\sin\theta]_{-\theta_2}^{\theta_1}$$
$$E_\perp = \frac{k\lambda}{d}[\sin\theta_1 + \sin\theta_2]$$



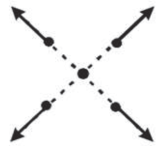
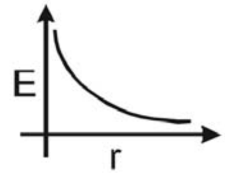
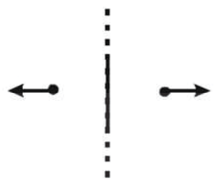
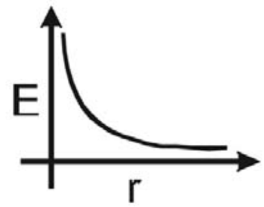
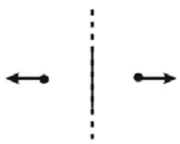
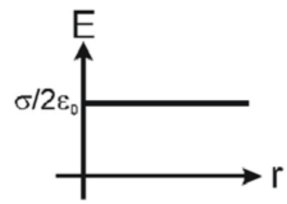
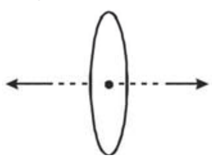
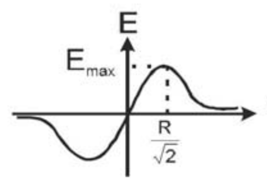
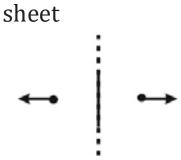
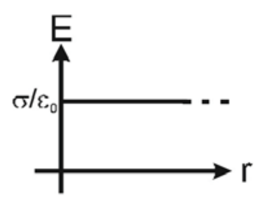
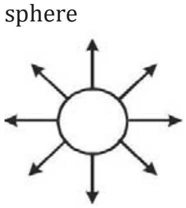
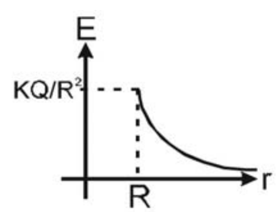


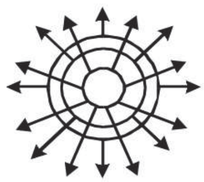
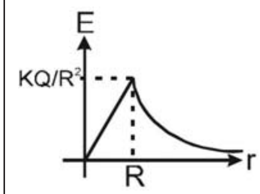
$$E_{||} = \frac{k\lambda}{d}(\cos\theta_2 - \cos\theta_1)$$

$$E_\perp = \frac{k\lambda}{d}(\sin\theta_1 + \sin\theta_2)$$

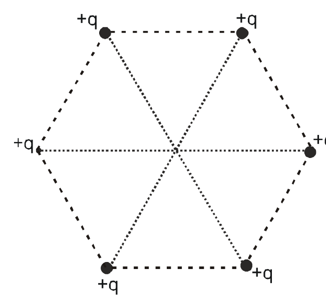
1. d =⊥ Distance of point from wire.
2. θ_1 and θ_2 is taken from⊥.
3. θ_1 and θ_2 are taken in opposite same.

List of formula for Electric Field Intensity due to various types of charge distribution :

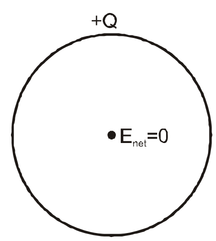
Name/ Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{Kq}{\lvert\vec{r}\rvert^2}\cdot\hat{\mathbf{r}}$	q is source charge. $\bar{r}$is vector drawn from source charge to the test point. outwards duc to +charges & inwards due to charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\hat{\gamma},\hat{r}}{r}$	q is linear charge density (assumed uniform) r is perpendicular distanc of paint from E ne charge. Is radlal unit vector drawn from the charge to test point	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	Is surface charge density. (assumed uniform) Is unit normal vector. x=distance of point on the axis from centre of the ring. electric field is always along the axis,	
Uniformly charged ring 	$E = \frac{KQX}{(R^2+x^2)^{\beta/2}}$ $E_{avs} = 0$	Q is total charge of the ring x-distance of point on the axis from centre of the ring. Electric field is always along the axis.	
Infinitely large charged conducing sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	Is the surface charge. density (assumed uniform) n̂is the unit vector perpendicular is the surface.	
Uniformly charged hollow conducting/ no conducting /solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r < R$ $E = 0$	R is radius of the sphere. $\vec{\mathbf{r}}$ is vector drawn from centre of sphere to the point Sphere acts like a point charge. placed at centre for paints outside the sphere.$\vec{E}$ is always along radial direction. Q is total charge ($= 4R^2$). (=surface charge density)	

Uniformly charged solid nan conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{k\vec{O}}{\lvert\vec{r}\rvert^2} r$ (ii) for $r \leq R$ $\vec{E} = \frac{kQ}{R^3}\vec{r}$	$\vec{\mathbf{r}}$ is vector drawn from centre of sphere to the point *Sphere acts like a point charge placed at the centre for points outside the sphere $\vec{\mathbf{E}}$ is always along radial dir *O is total charge (P = $\frac{4}{3}\pi\mathbf{R}^3$). (p=volume charge density) Inside the sphere E ∝ r.Outside the sphere E = $1/r^2$.	

Ex. Six equal point charges are placed at the corners of a regular hexagon of side 'a'. Calculate electric field intensity at the center of hexagon?



Similarly electric field due to a uniformly charged ring at the centre of ring :



Note 1. Net charge on a conductor remains only on the outer surface of a conductor. This property will be discussed in the article of the conductor.

2. On the surface of isolated spherical conductor charge is uniformly distributed.