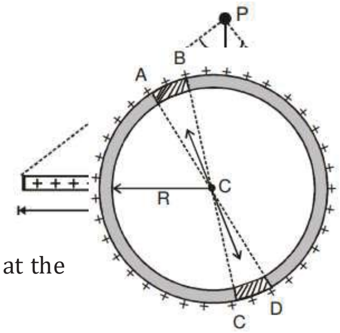
ELECTRIC FIELD DUE TO UNIFORMLY CHARGED BODIES

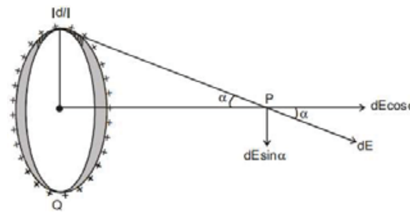
Electric Field on the Axis of Uniformly Charged Ring

Case - I: At its Centre



Here by symmetry we can say that electric field strength at Centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. As shown in figure. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is zero

Case II: At a Point on the Axis of Ring



For this look at the figure. There we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length dl on ring as shown. The charge on this elemental section is

$$dq = \frac{Q}{2\pi} d\ell$$

[Q = total charge of ring]

Due to the element dq, electric field strength dE at point P can be given as

$$dE = \frac{Kdq}{(R^2+x^2)}$$

The component of this field strength dE sin α which is normal to the axis of ring will be cancelled out due to the ring section opposite to dl. The component of electric field strength along the axis of ring dE cosα due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \int dE\cos\alpha$$

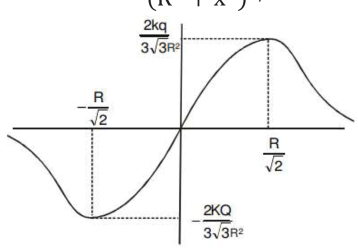
$$\int_0^{2\pi R} \frac{Kdq}{(R^2+x^2)} \times \frac{x}{\sqrt{R^2+x^2}}$$

$$E_p = \int_0^{2\pi R} \frac{KQx}{2\pi R(R^2+x^2)^{3/2}}\ dl$$

$$\frac{KQx}{2\pi R(R^2+x^2)^{3/2}} \int_0^{2\pi R} d\ell$$

$$\frac{KQx}{2\pi R(R^2+x^2)^{3/2}} [2\pi R]$$

$$E_p = \frac{KQx}{(R^2+x^2)^{3/2}}$$



Ex. A thin wire ring of radius r carries a charge q. Find the magnitude of the electric field strength on the axis of the ring as function of distance l from centre. Investigate the obtained function at l >> r. Find the maximum strength magnitude and the corresponding distance.

Sol. See figure (Modify for maximum E) we know due to ring electric field strength at adistance from its centre on its axis can be given as

$$E = \frac{Kq\ell}{(\ell^2+r^2)^{3/2}} \quad \text{... (1)}$$

For $>> r$, we have $E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{\ell^2}$

Thus the ring behaves like a point charge.

For $E_{max}\ \frac{dE}{d\ell} = 0$. From equation we get

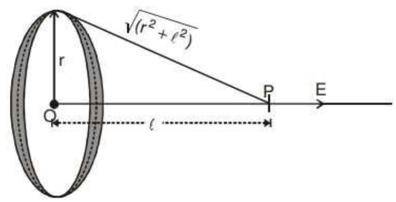
$$\frac{dE}{d\ell} = \frac{q}{4\pi\epsilon_0}\left[\frac{(r^2+\ell^2)^{3/2}\cdot 1 - \frac{3\ell}{2}(r^2+\ell^2)^{1/2}\times 2\ell}{(r^2+\ell^2)^3}\right] = 0$$

$$(r^2+\ell^2)^{3/2} = \frac{3}{2}(r^2+\ell^2)^{1/2}\times 2\ell^2$$

Solving we get, $\ell = \frac{r}{\sqrt{2}}$ ... (2)

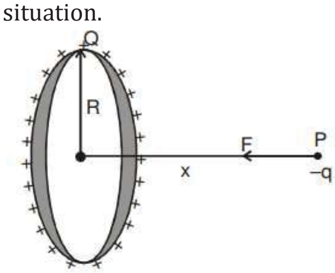
Substituting the value of in equation (1) we get

$$E = \frac{kq(r/\sqrt{2})}{(r^2+r^2/2)^{3/2}} = \frac{2kq}{3\sqrt{3}r^2}$$



Ex. A thin fixed ring of radius 1 meter has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0.9 gm and having a negative charge of 1×10^{-6} coulomb is placed on the axis at distance of 1 cm from the centre of the ring. Shown that the motion of the negatively charged particle as approximately simple harmonic. Calculate the time period of oscillations.

Sol. Let us first find the force on a – q charge placed at a distance x from centre of ring along its axis. Figure shows the respective situation.



In this case force on particle P is

$$F_p = -qE = -q\cdot\frac{KQx}{(x^2+R^2)^{3/2}}$$

For small x, x << R, we can neglect x, compared to R, we have

$$F = -\frac{KqQx}{R^3}$$

Acceleration of particle is $a = -\frac{KqQ}{mR^3}x$

[Here we have x = 1 cm and R = 1 m hence x << R can be used]

This shows that particle P excutes SHM, now comparing this acceleration with

$$a = -\omega^2 x$$

We get $\omega = \sqrt{\frac{KqQ}{mR^3}}$

Thus time period of SHM is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mR^3}{KqQ}} = 2\pi\sqrt{\frac{0.9\times10^{-3}\times(1)^3}{9\times10^9\times10^{-5}\times10^{-6}}} = \frac{\pi}{5}$ seconds

Ex. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

Sol. Force df on the wire = dq $\vec{E}$

$$\frac{Kqx}{(x^2+R^2)^{3/2}}\cdot\lambda dx$$

$$F = Kq\lambda\int_0^\infty \frac{xdx}{(R^2+x^2)^{3/2}}$$

$$F = \frac{\lambda q}{4\pi\epsilon_0 R}$$

Alternate:

Due to wire electric field on the points of ring in y-direction is

$$E_y = \frac{K\lambda}{R}$$

Thus force on ring due to wire is

$$q\frac{K\lambda}{R} = \frac{Kq\lambda}{R} = \frac{\lambda q}{4\pi \in_0 R}$$

And E_X =0 [As cancelled out]

(Here x components of forces on small elements of rings are cancelled by the x component of diametrically opposite elements.)

Electric field Strength at a General Point due to a Uniformly Charged Rod:

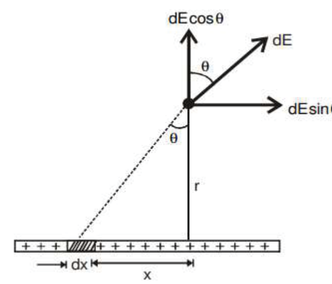
As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, again we consider an element on rod of length dx at a distance x from point O as shown in figure.

Now if dE be the electric field at P due to the element, then it can be given as

$$dE = \frac{Kdq}{(x^2+r^2)}$$

$$dq = \frac{Q}{l}dx$$

Now we resolve electric field in components. Electric field strength in x-direction due to dq at P is $dE_x = dE\sin\theta$



$$dE_x = \frac{Kdq}{(x^2+r^2)}\sin\theta$$

$$= \frac{KQ\sin\theta}{L(x^2+r^2)}dx$$

Here we have $x = r\tan\theta$ and

$$dx = r\sec^2\theta d\theta$$

Thus we have $$dE_x = \frac{KQ}{L}\frac{r\sec^2\theta d\theta}{r^2\sec^2\theta}\sin\theta$$

Strength= $\frac{KQ}{Lr}\sin\theta d\theta$

Net electric field strength due to dq at point P in x-direction is

$$E_x = \int dE_x = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\sin\theta d\theta$$

$$E_x = \frac{KQ}{Lr}[-\cos\theta]_{-\theta_2}^{+\theta_1}$$

$$E_x = \frac{KQ}{Ir}[\cos\theta_2 - \cos\theta_1]$$

Similarly, electric field strength at point P due to dq in y-direction is

$$dE_y = dE\cos\theta$$

$$dE_y = \frac{KQdx}{L(r^2+x^2)}\times\cos\theta$$

Again we have $x = r\tan\theta$

$$dx = r\sec^2\theta d\theta$$

Thus we have $$dE_y = \frac{KQ}{L}\cos\theta\times\frac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \frac{KQ}{Lr}\cos\theta d\theta$$

Net electric field strength at P due to dq in y-direction is

$$E_y = \int dE_y = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta d\theta$$

$$E_y = \frac{KQ}{Lr}[+\sin\theta]_{-\theta_2}^{+\theta_1}$$

$$E_y = \frac{KQ}{Lr}[\sin\theta_1 + \sin\theta_2]$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

In ||-direction $E_x = \frac{KQ}{Lr}(\cos\theta_2 - \cos\theta_1) = \frac{k\lambda}{r}(\cos\theta_2 - \cos\theta_1)$

In ⊥ -direction $E_y = \frac{KQ}{Lr}(\sin\theta_1 + \sin\theta_2) = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2)$

r is the perpendicular distance of the point from the wire

θ_1 and θ_2 should be taken in opposite sense

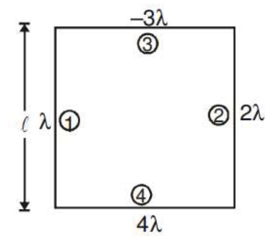
Ex. In the given arrangement of a charged square frame find field at centre. The linear charged density is as shown in figure.

Sol. E.F. due to $1 = \frac{2K\lambda}{f}(\sin 45^\circ + \sin 45^\circ)\hat{i} = \frac{2\sqrt{2}K\lambda}{\ell}\hat{i}$

E.F. due to $2 = -\frac{4\sqrt{2}K\lambda}{\ell}\hat{i}$

E.F. due to $3 = \frac{6\sqrt{2}K\lambda}{\ell}\hat{j}$

E.F. due to $4 = \frac{8\sqrt{2}K\lambda}{\ell}\hat{j}$

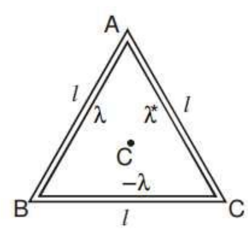


$$\vec{E}_{net} = \vec{E}_{due\ to\ 1} + \vec{E}_{due\ to\ 2} + \vec{E}_{due\ to\ 3} + \vec{E}_{due\ to\ 4}$$

$$\left(\frac{2\sqrt{2}K\lambda}{\ell} - \frac{4\sqrt{2}K\lambda}{\ell}\right)\hat{i} + \left(\frac{6\sqrt{2}K\lambda}{\ell} + \frac{8\sqrt{2}K\lambda}{\ell}\right)\hat{j}$$

$$\frac{-2\sqrt{2}K\lambda}{\ell}\hat{i} + \frac{14\sqrt{2}K\lambda}{\ell}\hat{j}$$

Ex. Given an equilateral triangle with side. Find E at the centroid. The linear charge density is as shown in figure.



Sol. The electric field strength due to the three rods AB, BC and CA are as shown in figure.

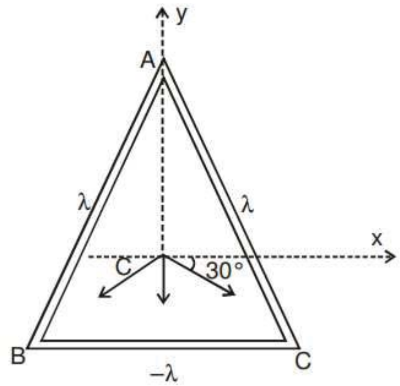
$$\vec{E}_{AC} = \frac{-2K\lambda}{\ell/\sqrt{3}}(2\sin 30^\circ)(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\vec{E}_{AB} = \frac{2\,K\lambda}{\ell/\sqrt{3}}(2\sin 30^\circ)(\cos\theta\hat{i} - \sin\theta\hat{j})$$

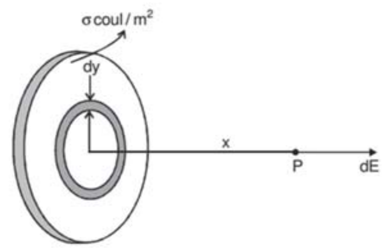
$$\vec{E}_{BC} = \frac{2\,K\lambda}{\ell/\sqrt{3}}(2\sin 30^\circ)\hat{j}$$

$$\vec{E}_{net} = \vec{E}_{AC} + \vec{E}_{AB} + \vec{E}_{BC}$$

$$\vec{E}_{net} = \frac{-\lambda}{2\pi\in_0\ell}\hat{j}$$



Electric field Strength due to a Uniformly Surface Charged Disc:



If there is a disc of radius R, charged on its surface with surface charge density σ coul/m^2, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.

To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. Now the charge on this elemental ring dq can be given as

$$dq = \sigma 2\pi y dy \qquad \text{[Area of elemental ring } ds = 2\pi y\, dy]$$

Now we know that electric field strength due to a ring of radius R. Charge Q at a distance x from its centre on its axis can be given as

$$E = \frac{KQx}{(x^2+R^2)^{3/2}} \qquad \text{[As done earlier]}$$

Here due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{Kdqx}{(x^2+y^2)^{\frac{3}{2}}} = \frac{K\sigma 2\pi y dy}{(x^2+y^2)^{\frac{3}{2}}}$$

Net electric field at point P due to this disc is given by integrating above expression from O to R as

$$E = \int dE = \int_0^R \frac{K\sigma 2\pi x y dy}{(x^2+y^2)^{3/2}}$$

$$K\sigma\pi x \int_0^R \frac{2ydy}{(x^2+y^2)^{3/2}} = 2K\sigma\pi x\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^R$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

Case: (i) If x >> R

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1+\frac{R^2}{x^2}\right)^{-1/2}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} + \text{ higher order terms }\right]$$

$$\frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\ _0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e. behavior of the disc is like a point charge.

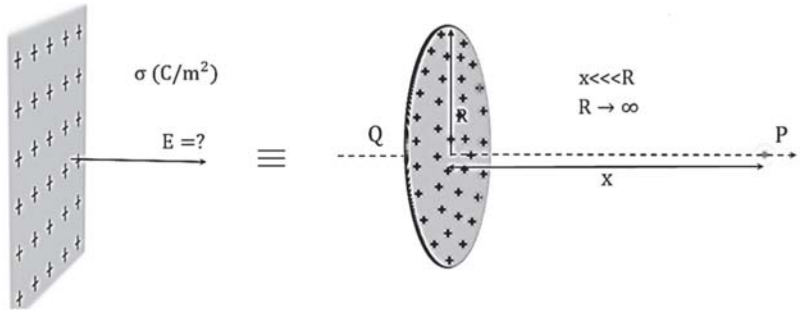
Case: (ii) If x << R

$$E = \frac{\sigma}{2\varepsilon_0}[1-0] = \frac{\sigma}{2\varepsilon_0}$$

I.e. behavior of the disc is like infinite sheet.

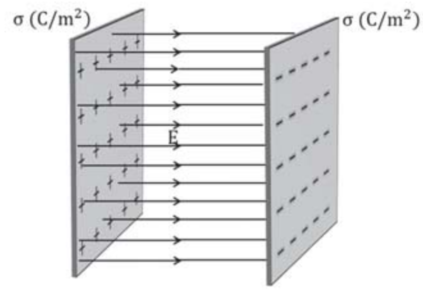
Electric Due To Infinite Charged Sheet

For a disc if R → ∞, It behaves like an infinite sheet. Therefore, for an infinite sheet, we can apply the formula for a disc.



Electric field at paraxial point of disc: $E = \frac{\sigma}{2\varepsilon_0}[1 - \frac{x}{\sqrt{R^2+x^2}}]$

For infinite sheet or disc, $x <<$ R Thus $E = \frac{\sigma}{2\varepsilon_0}$



Electric field resulting from two infinite parallel plates: $E = \frac{\sigma}{\varepsilon_0}$

Electric Field Strength due to a uniformly charged Hollow Hemispherical Cup:

Figure shows a hollow hemisphere, uniformly charged with surface charge density σ coul/m². To find electric field strength at its centre C, we consider an elemental ring on its surface of angular width dθ at an angle θ from its axis as shown. The surface area of this ring will be

$$ds = 2\pi R\sin\theta \times Rd\theta$$

Charge on this elemental ring is

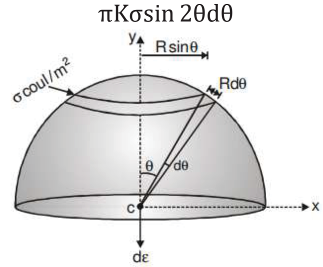
$$dq = \sigma ds = \sigma \cdot 2\pi R^2 \sin\theta d\theta$$

Now due to this ring electric field strength at centre C can be given as

$$dE = \frac{Kdq(R\cos\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$\frac{K\sigma \cdot 2\pi R^2 \sin\theta d\theta \cdot R\cos\theta}{R^3}$$

$$\pi K\sigma \sin 2\theta d\theta$$



Net electric field at centre can be obtained by integrating this expression between limits 0 to $\frac{\pi}{2}$ as

$$E_0 = \int dE = \pi K\sigma \int_0^{\pi/2} \sin 2\theta d\theta = \frac{\sigma}{4\epsilon_0}[-\frac{\cos 2\theta}{2}]_0^{\pi/2} = \frac{\sigma}{4\epsilon_0}[\frac{1}{2} + \frac{1}{2}] = \frac{\sigma}{4\epsilon_0}$$