Chapter 2

Electrostatic Potential and Capacitance

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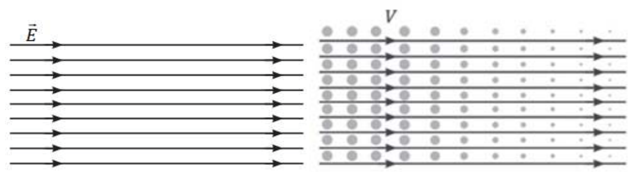
INTRODUCTION

Electric Potential due to point charge, Rod, charged circular ring and disc:

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. Pictorially, the uniform electric field can be described as equispaced, parallel electric lines of force, and the direction of the electric field is denoted by the arrowheads as shown in the figure.

The same field can also be described by assigning a scalar quantity V at each point known as electric potential.



Assume that a particle of charge +q is moving from point A to point B in a non-uniform electric field and the potential energy at point A and point B is U_A and U_B, respectively, with respect to the reference position fixed at any point in space. If V_A and V_B are the electric potentials at points A and B, respectively, then the change in the electric potential is defined as,

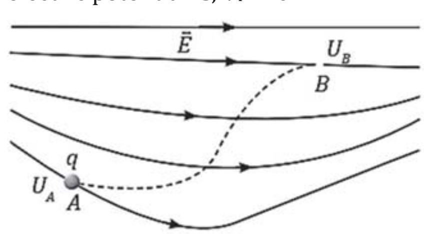
$$V_B - V_A = \frac{U_B - U_A}{q}$$

Therefore, the change in potential is defined as the change in potential energy per unit charge.

Hence, the change in potential energy is,

$$U_B - U_A = q(V_B - V_A) = -W_{el}$$

Now, suppose that we choose some point (P) at infinity where the electric potential is, $V_P = 0$



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  - Introduction to electric current
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- Work done by battery, voltage and potential energy of parallel plate capacitor
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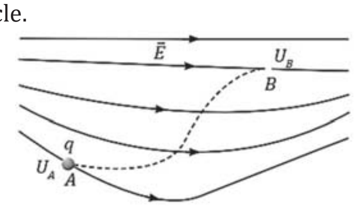
It means that we chose the datum of the electric field at infinity. However, there are no restrictions to choose the datum of the electric potential at infinity. Rather, it can be chosen at any point in space, depending on the given scenario. This is also true for the electric potential energy. If a particle of charge +q is brought from point P (at infinity) to point A, then the change in electric potential is,

$$V_A - V_P = V_A = \frac{U_A - U_P}{q}$$

1. If a particle of charge +q is moving from point A to point B in an electric field and the potential energy at point A and point B is U_A and U_B, respectively, then the change in potential energy is, $U_B - U_A = q(VB - VA) = -W_{el}$.
2. If a particle of charge +q is moving from point A to point B in an electric field under the action of an external force and the potential energy at point A and point B is U_A and U_B, respectively, then the change in potential energy is $U_B - U_A = q(VB - VA) = W_{ext}$, provided that Δ(K.E.) = 0

Ex. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V To a point at potential 200 V. Find the charge of the Particle.



Sol. It is given that the kinetic energy decreases by 10 J as the charged particle moves from point B to point A.
Therefore, Δ (K.E.) = $K_f - K_i = -10$
Since the potential energy is a scalar quantity, path-independent, and a state function, it depends only on the initial and final positions. Similarly, electric potential is a scalar quantity and path-dependent.
Given,
The electric potential at point A is, $V_A = 100$ V.
The electric potential at point B is, $V_B = 200$ V.
By applying the work-energy theorem, we get,

$$W_{ext} + W_{el} = \Delta(K.E.)$$

$0 + (-\Delta U) = -10$ {Since there is no external agent, the external work done is zero.}

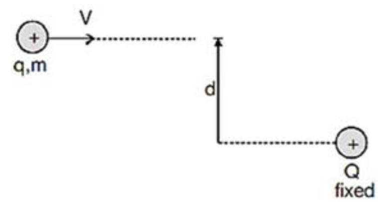
$$\Delta U = 10$$
$$U_f - U_i = 10$$
$$q(V_f - V_i) = 10$$
$$q(V_A - V_B) = 10$$
$$q(200 - 100) = 10$$
$$q = +0.1C$$

Therefore, the charge of the particle is +0.1 C.

Motion of a Charge Particle and Angular Momentum Conservation:

We know that a system of particles when no external torque acts, the total angular momentum of system remains conserved. Consider following examples which explains the concept for moving charged particles.

Ex. Figure shows a charge +Q fixed at a position in space. From a large distance another charge particle of charge +q and mass m is thrown toward +Q with an impact parameter d as shown with speed v. find the distance of closest approach of the two particles.



Here we can see that as +q moves toward +Q, a repulsive force acts on –q radially outward +Q. Here as the line of action of force passes through the fix charge, no torque act on +q relative to the fix point charge +Q, thus here we can say that with respect to +Q, the angular momentum of +q must remain constant. Here we can say that +q will be closest to +Q when it is moving perpendicularly to the line joining the two charges as shown.

If the closest separation in the two charges is r_{min}, from conservation of angular momentum we can write

$$mvd = mv_0 r_{min}$$

Now from energy conservation, we have

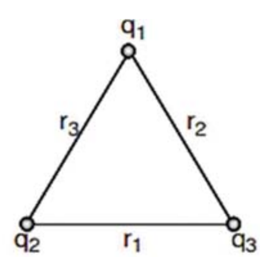
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{KqQ}{r_{min}}$$

Here we use from equation (1) $v_0 = \frac{vd}{r_{min}}$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2\frac{d^2}{r_{min}^2} + \frac{kqQ}{r_{min}}$$

Solving equation (2) we'll get the value of r_{min}.

Potential Energy for a System of charged Particles:



When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles.

For example, if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure.

The total interaction energy of this system can be given as

$$U = \frac{Kq_1q_2}{r_3} += \frac{Kq_1q_2}{r_3} += \frac{Kq_1q_2}{r_3}$$

Derivation for a system of point charges:

1. Keep all the charges at infinity. Now bring the charges one by one to its Corresponding position and find work required. PE of the system is algebraic sum of all the works.
   Let
   W_1 = work done in bringing first charge
   W_2 = work done in bringing second charge against force due to 1st charge
   W_3 = work done in bringing third charge against force due to 1st and 2nd charge.
   $$P_E = W_1 + W_2 + W_3 + \cdots$$
   $$\text{(This will contain } \frac{n(n-1)}{2} = {}^nc_2 \text{ terms)}$$
2. Method of calculation (to be used in problems) U = sum of the Interaction energies of the charges.
   $(U_{12} + U_{13} + \dots\dots + U_{1n}) + (U_{23} + U_{24} + \dots\dots\dots + U_{2n}) + (U_{34} + U_{35} + \dots\dots U_{3n})\dots\dots$
3. Method of calculation useful for symmetrical point charge systems. Find PE of each charge due to rest of the charges.
   If U_1 = PE of first charge due to all other charges.
   $$(U_{12} + U_{13} + \cdots \dots \dots + U_{1n})$$
   U_2 = PE of second charges due to all other charges.
   $$(U_{21} + U_{23} + \cdots \dots \dots + U_{2n})$$
   U = PE of the system $= \frac{U_1 + U_2 + \cdots\dots}{2}$

Electrostatic Potential:

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, electric potential is defined as the interaction energy of a unit positive charge.

If at a point in electric field a charge q0 has potential energy U, then electric potential at that point can be given as

$$V = \frac{U}{q_a} \text{ Joule/coulomb}$$

As potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly, we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces."

Properties:

1. Potential is a scalar quantity, its value may be positive, negative or zero.
2. S.I. Unit of potential is volt = joule/ coulomb and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.
3. Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.
4. Electric potential due to a positive charge is always positive and due to negative charge, it is always negative except at infinity. (Taking $V\infty = 0$)
5. Potential decreases in the direction of electric field.

Electric Potential due to a Point Charge in its Surrounding:

We know the region surrounding a charge is electric field. Thus, we can also define electric potential in the surrounding of a point charge.

The potential at a point P at a distance x from the charge q can be given as

$$V_\rho = \frac{U}{q_0}$$

Where U is the potential energy of charge q_0, if placed at point P, which can be given as

$$U = \frac{Kqq_0}{x}$$

Thus, potential at point P is

$$V_P = \frac{Kq}{X}$$

The above result is valid only for electric potential in the surrounding of a point charge. If we wish to find electric potential in the surrounding of a charged extended body, we first find the potential due to an elemental charge dq on body by using the above result and then integrate the expression for the whole body.

Charge	Electric field at a distance r	Electric potential at a distance r
Point charge +q	$\frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$	$\frac{q}{4\pi\varepsilon_0 r}$
Point charge –q	$\frac{(-q)}{4\pi\varepsilon_0 r^2}\hat{r}$	$\frac{(-q)}{4\pi\varepsilon_0 r}$

Potential due to point charge:

The electrostatic potential due to a point charge Q at a distance r from the charge can be calculated using Coulomb's law. Coulomb's law states that the electrostatic force between two-point charges is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance between them.

Potential Due to A System of Charges:

Consider a system of charges q_1, q_2,.... q_n with position vectors r_1, r_2 ,..., r_n relative to some origin. The potential V1 at P due to the charge q_1 is

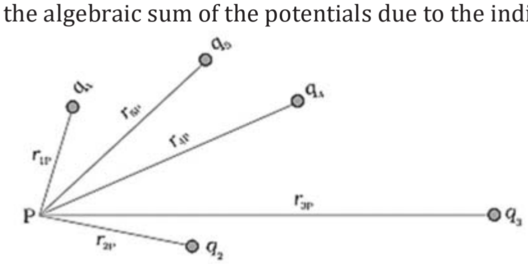
$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

Where r_{1P} is the distance between q_1 and P.

Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}, V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

Where r_{2P} and r_{3P} are the distances of P from charges q2 and q3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges



$$V = V_1 + V_2 + \cdots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \cdots \ldots + \frac{q_n}{r_{np}}\right)$$

If we have a continuous charge distribution characterized by a charge density ρ (r), we divide it, as before, into small volume elements each of size Δυ and carrying a charge ρΔυ. We then determine the potential due to each volume element and sum (strictly speaking, integrate) over all such contributions, and thus determine the potential due to the entire distribution.

A uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by

$$V = \frac{1}{4\pi_{\ 0}}\frac{q}{R} \qquad (r \geq R)$$

Where q is the total charge on the shell and R its radius. The electric field inside the shell is zero. This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell), and, therefore, equals its value at the surface, which is

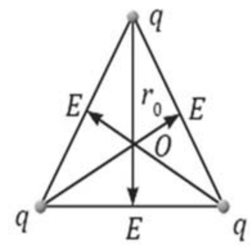
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Ex. If the electric field and the electric potential at a point are E and V, respectively, then which of the following statements is/are incorrect?

(A) If $E = 0$, V must be zero. (B) If $V = 0$, E must be zero.

(C) If $E \neq 0$, V cannot be zero. (D) If $V \neq 0$, E cannot be zero.

Sol. Consider an equilateral triangle with three positive charges at the vertices as shown in the figure.

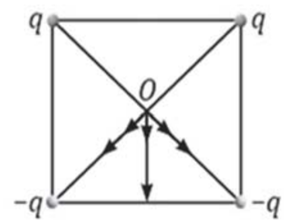


It is easily seen that the magnitude of all the three electric fields at point O is equal, and the resultant of any two electric field vectors perfectly balances the third electric vector. Therefore, the net electric field is zero at point O However, if we assume that the distance from the vertices to point O is r0 and try to calculate the electric potential at point O, then that would be,

$$V_o = \frac{3q}{4\pi\varepsilon_0 r_0}$$

Therefore, the statement 'If $E = 0$, V must be zero' is false.

Thus, option (A) is incorrect.



Now, consider a square with two equal positive charges and two equal negative charges as shown in the figure. In this case, the net electric potential at the centre of the square is zero, but the net electric field is non-zero and its direction is vertically downwards as shown in the figure. Therefore, the statement 'If $V = 0$, E must be zero' is false.

Thus, option (B) is incorrect.

Consider the statement given in option (C): if $E \neq 0$, V cannot be zero.

From the case of the square, we get,

$$E \neq 0, V = 0$$

Therefore, this statement is also false.

Thus, option (C) is also incorrect.

The discussion about the equilateral triangle also implies that the statement in option (D), 'If $V \neq 0$, E cannot be zero' is also incorrect.

Note: We know that$dV = -\vec{E}\cdot d\vec{r}$. So, we might assume that if $E = 0$, $V = 0$. We should be careful. In the expression, $dV = -\vec{E}\cdot d\vec{r}$.if we put $E = 0$, then we get $dV = 0$, which means that the change in potential is zero and this in turn implies that the potential is constant.

The significance of the negative sign in the expression $dV = -\vec{E}\cdot d\vec{r}$ can be understood in the following two ways:

1. Along the direction of the electric field, the value of the potential decreases.

2. The direction of the electric field is from a high potential to a low potential.

For $dV = 0$, the electric field E need not always be zero. This is because if $\vec{E} \perp d\vec{r}$, then $dV = 0$ again. Hence, V becomes constant. This is the concept of equipotential surfaces.

Electrostatic Potential Energy of a System of Charges

(This concept is useful when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without accelerating it.

Types of system of charge

1. Point charge system 2. Continuous charge system.

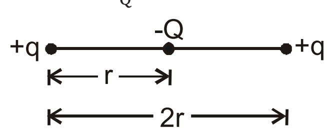
Derivation for a system of point charges:

1. Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebraic sum of all the works. Let
W_1 = work done in bringing first charge
W_2 = work done in bringing second charge against force due to 1st charge.
W_3 = work done in bringing third charge against force due to 1st and 2nd charge.
$$PE = W_1 + W_2 + W_3 + \cdots \ldots. \text{(This will contain } \frac{n(n-1)}{2} = {}^{n}C_2 \text{ terms)}$$
2. Method of calculation (to be used in problems)
U = sum of the interaction energies of the charges.
$= (U_{12} + U_{13} + \ldots\ldots.. + U_{1n}) + (U_{23} + U_{24} + \ldots\ldots.. + U_{2n}) + (U_{34} + U_{35} + \ldots\ldots.. + U_{3n}) \ldots$
3. Method of calculation useful for symmetrical point charge systems.
Find PE of each charge due to rest of the charges.
If U_1 = PE of first charge due to all other charges.
$= (U_{12} + U_{13} + \ldots\ldots.. + U_{1n})$
U_2 = PE of second charge due to all other charges.
$= (U_{21} + U_{23} + \ldots\ldots.. + U_{2n})$ then U = PE of the system $= \frac{U_1 + U_2 + \cdots U_n}{2}$

Ex. Find out potential energy of the two-point charge system having q_1 and q_2 charges separated by distance r.

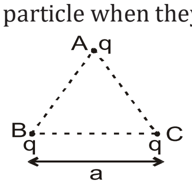
Sol. Let both the charges be placed at a very large separation initially.
Let
W_1 = work done in bringing charge q_1 in absence of $q_2 = q(V_f - V_i) = 0$
W_2 = work done in bringing charge q_2 in presence of $q_1 = q(V_f - V_i) =$
$q_1(Kq_2/r - 0)$
$$PE = W_1 + W_2 = 0 + Kq_1q_2 / r = Kq_1q_2 /$$

Ex. Figure shows an arrangement of three-point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$..



Sol. $U_{SVS} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r}\right] = 0 \Rightarrow -Q + \frac{q}{2} - Q = 0 \text{ or } 2Q = \frac{q}{2} \text{ or } \frac{q}{Q} = \frac{4}{1}.$

Ex. Three equal charges q are placed at the corners of an equilateral triangle of side a.
1. Find out potential energy of charge system.
2. Calculate work required to decrease the side of triangle to a/2.
3. If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.



Sol. 1. **Method I** (Derivation)
Assume all the charges are at infinity initially.
work done in putting charge q at corner A
$W_1 = q\ (v_f - v_i) = q\ (0 - 0)$
Since potential at A is zero in absence of charges, work done in putting q at corner B in

presence of charge at A:

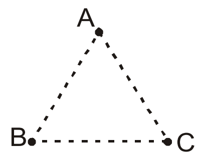
$$W_2 = \left(\frac{Kq}{a} - 0\right) = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$$W_3 = q(v_f - v_i) = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right]$$

So net potential energy PE = $W_1 + W_2 + W_3$

$$0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a}$$



Method II (using direct formula)

$$U = U_{12} + U_{13} + U_{23} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

2. Work required to decrease the sides $W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$

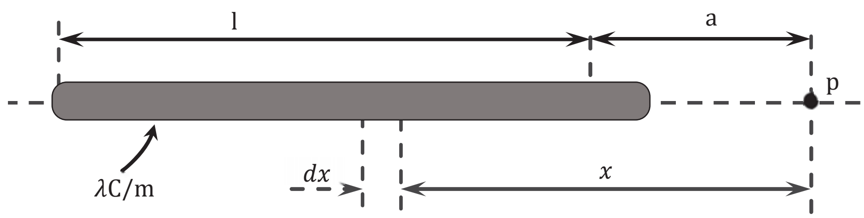
3. Work done by electrostatic forces = change is kinetic energy of particles.

$$U_i - U_f = K_f - K_i \Rightarrow \frac{3Kq^2}{a} - \frac{3K\ ^2}{2a} = 3(\frac{1}{2}mv^2) - 0 \Rightarrow v = \sqrt{\frac{Kq^2}{am}}$$

Electrostatic potential due to charged rod

The electrostatic potential due to a charged rod can be calculated using Coulomb's law and integration techniques. Let's consider a uniformly charged rod of length L and total charge Q with the charge uniformly distributed along its length.

To find the electrostatic potential at a point P at a distance z from the center of the rod, we'll integrate the contribution of each infinitesimal charge element along the length of the rod.



$$V = \frac{kq}{r}$$

Where:

- V is the electrostatic potential at a point due to the point charge.
- k is Coulomb's constant, approximately 8.9875×10^9 N·m²/C².
- q is the magnitude of the point charge.
- r is the distance between the point charge and the point where the potential is being measured.

$$V = k\lambda \ln\left(\frac{a+l}{a}\right)$$

Where:

- V is the electrostatic potential at a point due to the charged rod.
- k is Coulomb's constant.
- λ is the charge density (charge per unit length) of the rod.
- a is the distance from the point to the nearest end of the rod.
- l is the length of the rod.

These formulas are useful in calculating the electrostatic potential at a point in space due to either a point charge or a charged rod

Electrostatic potential due to charged circular ring

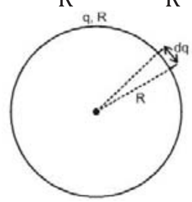
Potential due to a ring

1. Potential at the centre of uniformly charged ring:
Potential arising from the infinitesimal element dq

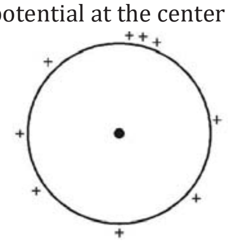
$$dV = \frac{Kdq}{R}$$

$$\text{Net potential: } V = \int \frac{Kdq}{R}$$

$$V = \frac{K}{R}\int dq = \frac{Kq}{R}$$

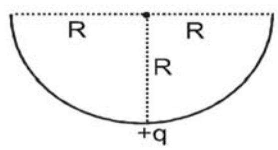


2. For non-uniformly charged ring potential at the center is

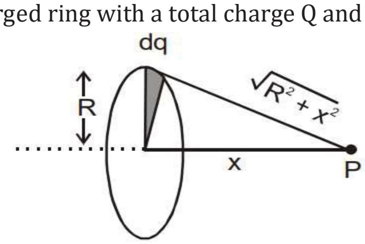


3. Potential due to half ring at center is :

$$V = \frac{Kq}{R}$$



4. Potential at the axis of a ring:
Determining the potential at a point along the axis, located at a distance x from the center, in the vicinity of a uniformly charged ring with a total charge Q and a radius R.

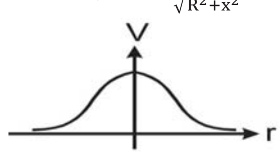


Consider an element of charge dq on the ring. Potential at point P due to charge dq will be

$$dV = \frac{K(dq)}{\sqrt{R^2+x^2}}$$

Net potential at point P due to all such element will be:

$$V = \int dv = \frac{KQ}{\sqrt{R^2+x^2}}$$



Ex. A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R. If the point charge is slightly displaced with negligible force along axis of the ring, then find out its speed when it reaches a large distance.

Sol. Only electric force is acting on q_0

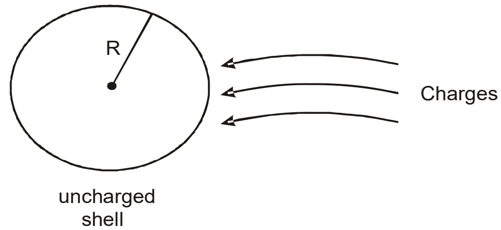
$$W_{cl} = \Delta K = \frac{1}{2}mv^2 - 0 \Rightarrow \text{ Now } W_{el})_{c\to\infty} = q_0V_c = q_0 \cdot \frac{KQ}{R}$$

$$\frac{Kq_0Q}{R} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2Kq_0Q}{mR}}$$

Derivation of electric potential energy for continues charge system:

This energy is also known as self-energy.

1. **Finding P.E. (Self-Energy) of a uniformly Charged spherical shell:**
For this, let's use method 1. Take an uncharged shell Now bring charges one by one from infinite to the surface of the shell. The work required in this process will be stored as potential Energy.



Suppose we have given q charge to the sphere and now we are giving extra dq charge to it.
Work required to bring dq charge from infinite to them shell is

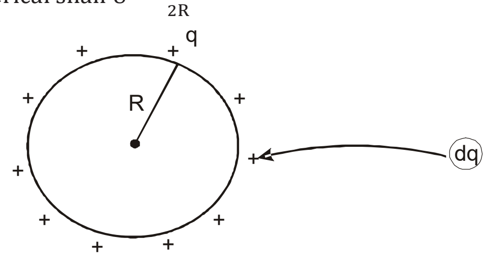
$$dw = (dq)(V_f - V_i)$$

$$dW = (dq)(\frac{Kq}{R} - 0) = \frac{Kq}{R}dq$$

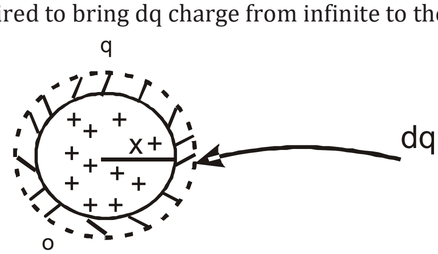
Total work required to give Q charge is $W = \int_{q=0}^{q=Q} \frac{Kq}{R}dq = \frac{KQ^2}{2R}$

This work will stored as a form of P.E. (self-energy)

So P.E. of a charged spherical shall $U = \frac{KQ^2}{2R}$



2. **Self-energy of uniformly charged solid sphere:**
In this case we have to assemble a solid charged sphere. So as we bring the charges one-by-one from infinite to the sphere, the size of me sphere will increase. Suppose we have given q charge to the sphere, and its radius becomes 'x'. Now we are giving extra dq charge to it, which will increase its radius by 'dx' work required to bring dq charge from infinite to the sphere



$$dq(V_f - V_i) = (dq)(\frac{Kq}{x} - 0) = \frac{Kqdq}{x}$$

Total work required to give Q charge: $W = \int \frac{Kqdq}{x} q = \rho(\frac{4}{3}\pi x^3)$

$$dq = \rho(4\pi x^2 dx) \Rightarrow W = \int_{x=0}^{x=R} K\frac{\rho\left(\frac{4}{3}\pi x^3\right)\rho(4\pi x^2 dx)}{x}$$

Solving well get $W = \frac{3}{5}\frac{KQ^2}{R} = U_{self}$ for a solid sphere

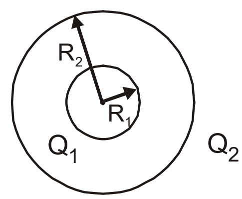
Ex. A spherical shell of radius R with uniform charge q is expanded to a radius 2R. Find the work performed by the electric forces and external agent against electric forces in this process (slow process).

Sol. $W_{ext} = U_f - U_i = \frac{q^2}{16\pi\varepsilon_0 R} - \frac{q^2}{8\pi\varepsilon_0 R} = -\frac{q^2}{16\pi\varepsilon_0 R}$

$$W_{elec} = U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{16\pi\varepsilon_0 R} = \frac{q^2}{16\pi\varepsilon_0 R}$$

Ex. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.

Sol. $U_{total} = U_{self\,1} + U_{self\,2} + U_{Interaction} = \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1 Q_2}{4\pi\ _0 R_2}$



Energy density:

Definition

Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following: Energy density $= \frac{1}{2}\varepsilon E^2$

Where E = electric field intensity at that point ; $\varepsilon = \varepsilon_0\varepsilon_r$electric permittivity of medium

Ex. Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large non-conducting sheet of uniform charge density σ.

Sol. Energy stored $U = \int \frac{1}{2}\varepsilon_0 E^2 dV$where dV is small volume

$$= \frac{1}{2}\varepsilon_0 E^2 \int dV \Theta E \text{ is constant } = \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2}\cdot a^3 = \frac{\sigma^2 a^3}{8\varepsilon_0}$$

Potential Energy in An External Field

Potential energy of a single charge

The source of the electric field was specified – the charges and their locations - and the potential energy of the system of those charges was determined.

In this section, we ask a related but a distinct question. What is the potential energy of a charge q in a given field?

This question was, in fact, the starting point that led us to the notion of the electrostatic potential but here we address this question again to clarify in what way it is different from the discussion in The main difference is that we are now concerned with the potential energy of a charge (or charges) in an external field. The external field E is not produced by the given charge(s) whose potential energy we wish to calculate. E is produced by sources external to the given charge(s). The external sources may be known, but often they are unknown or unspecified; what is specified is the electric field E or the electrostatic potential V due to the external sources. We assume that the charge q does not significantly affect the sources producing the external field.

This is true if q is very small, or the external sources are held fixed by other unspecified forces. Even if q is finite, its influence on the external sources may still be ignored in the situation when very strong sources far away at infinity produce a finite field E in the region of interest. Note again that we are interested in determining the potential energy of a given charge q (and later, a system of charges) in the external field; we are not interested in the potential energy of the sources producing the external electric field.

The external electric field E and the corresponding external potential V may vary from point to point. By definition, V at a point P is the work done in bringing a unit positive charge from infinity to the point P.

(We continue to take potential at infinity to be zero.) Thus, work done in bringing a charge q from infinity to the point P in the external field is qV. This work is stored in the form of potential energy of q. If the point P has position vector r relative to some origin, we can write:

Potential energy of q at r in an external field = qV(r)

where V(r) is the external potential at the point r.

Thus, if an electron with charge $q = e = 1.6\times10^{-19}$ C is accelerated by a potential difference of $\Delta V = 1$ volt, it would gain energy of $q\Delta V = 1.6 \times 10^{-19}$J. This unit of energy is defined as 1 electron volt or 1eV, i.e., 1 eV=1.6×10^{-19J}.

The units based on eV are most commonly used in atomic, nuclear and particle physics,

(1 keV = 103 eV = 1.6×10^{-16J}, 1 MeV = 106 eV = 1.6×10^{-13J}, 1 GeV = 109 eV = 1.6×10^{-10}J and 1 TeV = 1012eV = 1.6×10^{-7}J).

Potential energy of a system of two charges in an external field

Next, we ask: what is the potential energy of a system of two charges q_1 and q_2 located at r_1 and r_2, respectively, in an external field? First, we calculate the work done in bringing the charge q_1 from infinity to r_1. Work done in this step is q_1 V (r_1),

Next, we consider the work done in bringing q_2 to r_2. In this step, work is done not only against the external field E but also against the field due to q_1.

Work done on q_2 against the external field = q_2 V (r_2)

Work done on q_2 against the field due to $q_1 = \frac{q_1q_2}{4\pi\ _0r_{12}}$

Where r_{12} is the distance between q_1 and q_2). By the superposition principle for fields, we add up the work done on q_2 against the two fields (E and that due to q_1):

Work done in bringing q_2 to r_2

$$= q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

Thus, Potential energy of the system

The total work done in assembling the configuration

$$q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

Ex.

1. Determine the electrostatic potential energy of a system consisting of two charges 7 μC and –2 μC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.
2. How much work is required to separate the two charges infinitely away from each other?
3. Suppose that the same system of charges is now placed in an external electric field E = A ($1/r^2$); A = 9×10^5 NC^{-1} m^2. What would the electrostatic energy of the configuration be?

Sol.

1. $U = \frac{1}{4\pi\varepsilon}\frac{qq_2}{r} = 9 \times 10^9 \times \frac{7\times(-2)\times10^{-1}}{0.18} = -0.7$J.
2. $W = U_2 - U_1 = 0 - U = 0 - (-0.7) = 0.7$ J
3. The mutual interaction energy of the two charges remains unchanged. In addition, there is the energy of interaction of the two charges with the external electric field. We find,

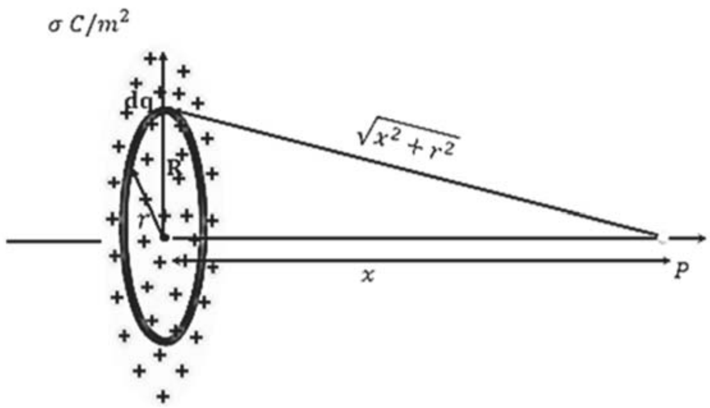
$$q_1V(r_1) + q_2V(r_2) = A\frac{7\mu C}{009m} + A\frac{-2\mu C}{009m}$$

And the net electrostatic energy is

$$q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}} = A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m} - 0.7J$$
$$= 70 - 20 - 0.7 = 49.3\ J$$

Electric Potential at the point on the axis of a Disc

To find the electric potential at a point on the axis of a disc, we can use the formula for the electric potential due to a uniformly charged disc.



$$\sigma = \frac{Q}{\pi R^2}$$

Where,

- Q is the total charge on the disc and
- R is the radius of the disc.

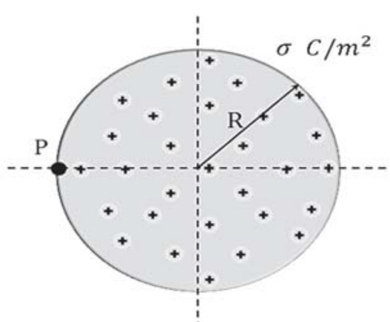
Substituting this expression into the formula for electric potential V at a point on the axis of the disc, we get:

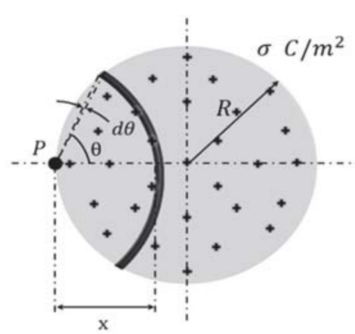
$$V = \frac{\sigma}{2\varepsilon_o}(\sqrt{R^2 + x^2} - x)$$

Where:

- V is the electric potential at the point.
- σ is the surface charge density (charge per unit area) of the disc.
- 0ϵ0 is the permittivity of free space.
- R is the radius of the disc.
- x is the distance from the center of the disc to the point where the potential is being measured along the axis.

This formula gives the electric potential at a point on the axis of the disc in terms of the surface charge density, the radius of the disc, and the distance from the center of the disc along the axis.





The electric potential (VP) at point P is determined by the formula:

$$V_P = \frac{\sigma}{2\varepsilon_o}[\sin\theta - \theta\cos\theta]_{\frac{\pi}{2}}^{0}$$
$$V_P = \frac{\sigma}{2\varepsilon_o}$$

Where:

- σ is the surface charge density in coulombs per square meter (C/m^2).
- 0ϵ0 is the permittivity of free space, approximately

8.854×10−12 C2/N·m28.854×10−12C2/N·m2.

This formula provides a simple expression for the electric potential at point P, which depends solely on the surface charge density and the permittivity of free space.