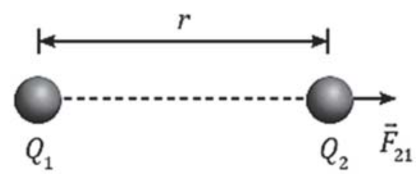
COULOMB'S LAW IN VECTOR FORM

Vector form of coulomb's law:

Let us consider that two like charges, Q_1 and Q_2, are placed somewhere in space. According to coulomb's law, the electrostatic forces act along the direction of the line joining the two charges. Since we have taken the charges of the same nature, there must be a repulsive force acting between them.



The force on charge Q_1 due to charge Q_2 is $\vec{F}_{12}$. The position vector of charge Q_1 as seen from charge Q_2 is $\vec{r}_{12} = r\hat{r}_{12}$, where r is the separation between the charges.

By applying coulomb's law, we get the following:

$$F = \frac{kQ_1Q_2}{r^2}$$

Also, the position vector is given by,

$$\vec{r}_{12} = r\hat{r}_{12}$$

Therefore, force acting on charge due to charge is given by,

$$\overrightarrow{F_{12}} = \frac{kQ_1Q_2}{r^2}\hat{r}_{12} \quad \text{... (1)}$$

The unit position vector is given by,

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$$

By substituting $\hat{r}_{12}$ in equation (1), we get,

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2} \times \frac{\vec{r}_{12}}{r}$$

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2}\vec{r}_{12} \quad \text{... (2)}$$

Similarly, the force on charge Q_2 due to charge Q_1 is $\vec{F}_{21}$. The position vector of charge Q_2 as seen from charge Q_1 is, $\vec{r}_{21} = r\hat{r}_{21}$

Therefore, the force acting on charge due to charge is given by,

$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^2}\hat{r}_{21} \quad \text{... (3)}$$

The unit position vector is given by,

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$$

By substituting in $\hat{r}_{12}$ equation (3), we get,

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2} \times \frac{\vec{r}_{12}}{r}$$

$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^3}\vec{r}_{21} \quad \text{... (4)}$$

For unlike charges, the forces become attractive in nature and hence, their directions get changed. Therefore, the force on Q_1 due to Q_2 is expressed as, $\vec{F}_{12} = \frac{kQ_1Q_2}{r^3}\vec{r}_{21}$ and the force on Q_2 due to Q_1 is expressed as, $\vec{F}_{21} = \frac{kQ_1Q_2}{r^3}\vec{r}_{12}$

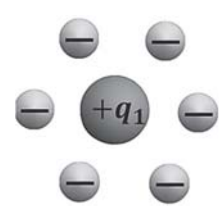
Only the formulas for the like charges are enough to remember because to obtain the formulas for the unlike charges, we just need to put the value of charges with sign.

Since $\vec{r}_{12} = -\vec{r}_{21}$,what we get from equations (1) and (2) is, $\vec{F}_{12} = -\vec{F}_{21}$. Thus, coulomb's law agrees with newton's third law of motion.

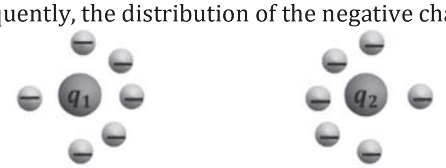
Force On Charge In A Medium

The force experienced by two charged particles when positioned in vacuum (or in air) varies from the force acting on the same pair of charged particles when situated within a medium.

When a + q_1 charge is introduced into a neutral medium (where opposite charges are balanced), the negative charges within the medium are drawn towards + q_1, resulting in a symmetric distribution of negative charges around + q_1. These negative charges are termed as 'induced charges'.



When a + q_2 charge is positioned near + q_1, the induced negative charges experience an attractive force from + q_2. Consequently, the distribution of the negative charges becomes asymmetric.

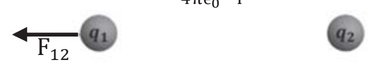


Net force on charge $+q_1$ is: $\vec{F}_{\text{net }1} = \vec{F}_{12} + \vec{F}_{\text{induced}}$

Net force on charge + q_1 will be lesser than $\vec{F}_{12}$ which is the force on + q_1 due to + q_2 only.

Case 1: When tasked with computing the force exerted solely by q_2 on q_1 in a vacuum, then

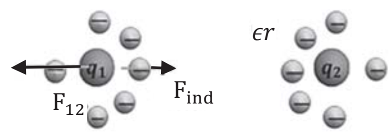
$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$



Case 2: When tasked with determining the total force acting on q_1 due to q_2 in a medium with permittivity ϵ_r, then

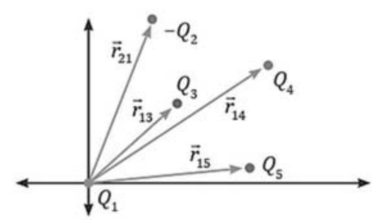
$$\vec{F}_{\text{net }1} = \vec{F}_{12} + \vec{F}_{\text{induced}} = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2} = \frac{F_{12}}{\epsilon_r}$$

Where ϵ_r represents the relative permittivity or dielectric constant of the medium.



Superposition of coulomb's force:

The resultant electrostatic force on a point will be the vector sum of electrostatic forces due to individual point charges. For finding the net force on any charge, we have to find the forces by each charge present in the vicinity of it.

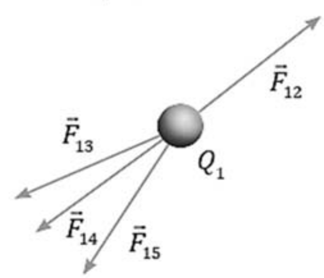


Let us consider charge Q_1 and analyses all the forces acting on it. The forces acting on Q_1 are shown in the Figure.

The net force acting on Q_1 is the vector sum of all the electrostatic force acting on it, which is given by,

$$\vec{F}_{\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$

The force applied by one charge does not affect the force by other charges. They have their individual effects, but the net force acting on the charge changes.



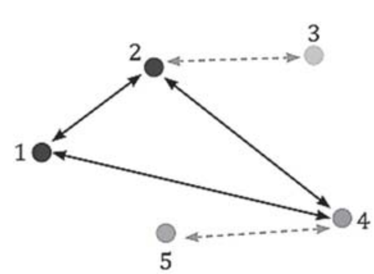
Ex. Five balls numbered 1 to 5 are suspended using separate threads. Pairs (1, 2), (2, 4), and (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion. What should ball 1 therefore be?

(A) Positively charged **(B)** Negatively charged
(C) Neutral **(D)** made of metal

Sol. Consider five balls as shown in the figure.

The dotted arrow represents the repulsive force between two balls and the solid arrow represents the attractive force between two balls.

It is given that the pairs (1, 2), (2, 4), and (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion.



To show electrostatic repulsion, two charges must be like Charges. Suppose that there is a repulsive force between balls 2 and ball 3, assuming that they have charge A.
Now, for electrostatic attraction to take place between two charges, it is not necessary that both the charges should have opposite charges. The electrostatic attraction can also happen between a charge particle and a neutral particle recall the example of the glass rod attracting dry papers). Now, since ball 4and ball 5 arer repelled to each other, they should have the same charge. Therefore, Ball 4 cannot be neutral. On top of this, ball 2 and ball 4 are attracted to each other and since ball 4 cannot be neutral, it should have the opposite charge of type a. Let ball 4 have a charge of type B. Therefore, ball 1 gets attracted by ball 2 having charge of type A and also gets attracted by ball 4 having charge of type B. Hence, it is possible only if ball 1 is neutral.
Therefore, option (c) is the correct answer.

Ex. Two identical charges in vacuum are separated by a distance of r. The electrostatic force between them is given by F. If 75 % of the charge is taken from one of the charges and given to the other, then the new force becomes F'. Find the ratio.$\frac{F}{F\prime}$.

(A) 1 (B) $\frac{16}{9}$ (C) $\frac{16}{7}$ (D) $\frac{7}{16}$

Sol. Initially, let there be two identical charges of magnitude Q separated by a distance of r. Therefore, according to coulomb's law, the magnitude of the electrostatic force is,

$$F = \frac{kQ^2}{r^2} \qquad \dots (1)$$

Now, if 75 % (or $\frac{3}{4}$) of one charge is given to the other, then those two charges become, $Q_1 = \frac{Q}{4}$ and $Q^2 = \left(Q + \frac{3Q}{4}\right) = \frac{7Q}{4}$

However, the separation between them remains the same.
Therefore, the electrostatic force in between is,

$$F' = \frac{kQ_1Q_2}{r^2}$$

$$F' = \frac{K\left(\frac{Q}{4}\right)\left(\frac{7Q}{4}\right)}{R^2}$$

$$F' = \frac{7}{16}\frac{KQ^2}{R^3}$$

$$F' = \frac{7}{16}F$$

$$\frac{F}{F'} = \frac{16}{7}$$

Thus, option (C) is the correct answer.

Ex. Two charge particles, each having charge q and mass m, are apart with a distance of d from each other. If two particles are in equilibrium under the gravitational and electrostatic force, then find the ratio $\frac{q}{m}$.

(A) 10–8 (B) 10–10 (C) 108 (D) None of these

Sol. According to the problem, both the particles are in equilibrium under the gravitational and electrostatic force. So, it can be said that the electrostatic force is balanced by the gravitational force. Therefore,

$$F_{electrostatic} = F_{gravitational}$$

$$\frac{kq^2}{d^2} = \frac{Gm^2}{d^2}$$

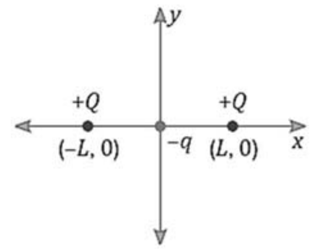
$$\left(\frac{q}{m}\right)^2 = \frac{G}{k}$$

$$\frac{q}{m} = \sqrt{\frac{G}{k}} \approx \sqrt{\frac{10^{-1}}{10^9}}$$

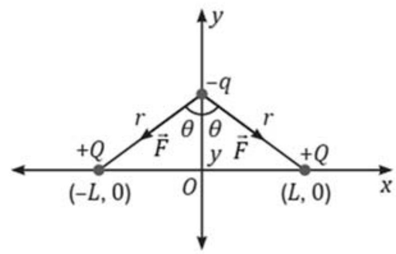
$$\frac{q}{m} \approx \sqrt{10^{-2}} \approx 10^{-1}$$

Thus, option (B) is the correct answer

Ex. Two identical point charges +Q are fixed in a gravity-free space at points (L, 0) and (–L, 0). Another particle with mass m and charge –q is placed at the origin. Now, this particle is displaced by a distance of y along the y-axis and then released. Show that this particle will execute oscillatory motion.



Sol. The only way to prove the particle executes oscillatory motion is to prove that the particle executes SHM because oscillatory motion is a consequence of SHM. Now, if we are able to prove that the force on the particle of charge –q is restoring and is proportional to the displacement, then it will be enough to conclude that the charged particle executes SHM.



Suppose the particle of charge –q is displaced along the y-axis by a distance of y as shown in the figure.

If the force on –q by +Q is given by F, then the net force on the charge –q by both the charges +Q is given by,

$$F_{net} = 2F\cos\theta$$

Also, this force is pointing towards the equilibrium position (O) of the charge –q. Therefore, the net force on the charge –q is restoring in nature.

Now,

$$F_{net} = 2F\cos\theta$$

$$F_{NET} = 2\left(\frac{KQQ}{R^2}\right)\left(\frac{Y}{R}\right)$$

$$F_{NET} = 2\left(\frac{KQQ}{R^3}\right)Y$$

From the figure, it is seen that $r = \sqrt{L^2 + y^2}$

$$F_{net} = 2\left(\frac{kQq}{(L^2+y^2)^{\frac{3}{2}}}\right)y$$

Since the charge is displaced slightly along the y-axis, y << L. Hence,

$$F_{net} = 2\left(\frac{kQq}{L^3\left(1+\frac{y^2}{L^2}\right)^{\frac{3}{2}}}\right)y$$

$$F_{net} = 2\left(\frac{kQq}{L^3}\right)y \qquad \text{[Since } y << L]$$

Therefore, the net force on the charge –q is proportional to the displacement of the charge from its equilibrium position and hence, the charge will execute SHM with time period,

$$T = 2\pi\sqrt{\frac{m}{\left(2\left(\frac{kQq}{L^3}\right)\right)}}$$

$$T = 2\pi\sqrt{\frac{mL^3}{2kQq}}$$

Forces Between Multiple Charges

The mutual electric force between two charges is given by Coulomb's law. How to calculate the force on a charge where there are not one but several charges around? Consider a system of n stationary charges q_1, q_2, and q_3... q_n in vacuum. What is the force on q_1 due to q_2, q_3... q_n? Coulomb's law is not enough to answer this question. Recall that forces of mechanical origin add according to the parallelogram law of addition. Is the same true for forces of electrostatic origin?

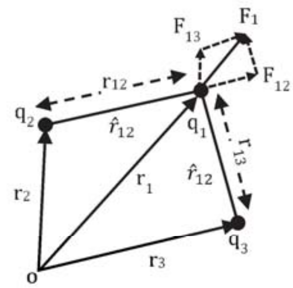
Experimentally, it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition.

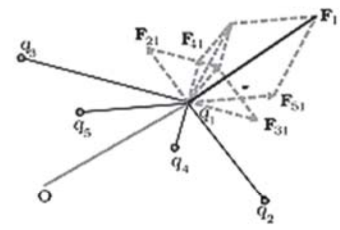
To better understand the concept, consider a system of three charges q_1, q_2 and q_3 , as. The force on one charge, say q_1 , due to two other charges q_2 , q_3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges. Thus, if the force on q_1 due to q_2 is denoted by F_{12}, F_{12} Even though other charges are present. Thus,

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

In the same way, the force on q_1 due to q_3 , denoted by F_{13} is given by

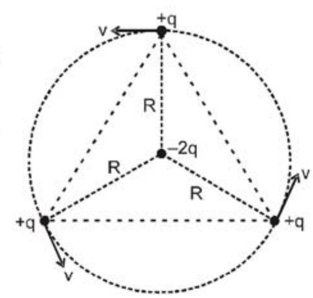
$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{13}$$





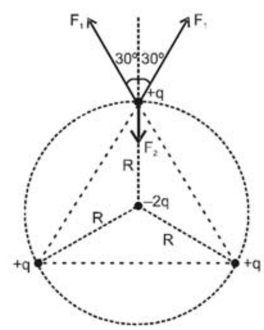
(b)

Ex. Three equal point charges of charge +q are moving along a circle of radius R and a point charge –2q is also placed at the center of circle as (figure), if charges are revolving with constant and same speed then calculate speed



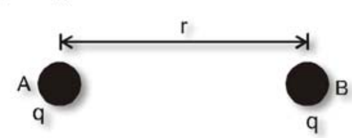
Sol. $F_2 - 2F_1 \cos 30 = \frac{mv^2}{R} \Rightarrow \frac{K(q)(2q)}{R^2} - \frac{2(Kq^2)}{(\sqrt{3}R)^2}\cos 30 = \frac{mv^2}{R}$

$v = \sqrt{\frac{kq^2}{Rm}[2 - \frac{1}{\sqrt{3}}]}$



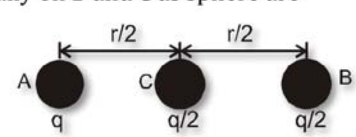
Ex. Two equally charged identical small metallic spheres A and B repel each other with a force 2×10^{-5}N when placed in air (neglect gravitation attraction). Another identical uncharged sphere C is touched to B and then placed at the midpoint of line joining A and B. What is the net electrostatic force on C?

Sol. Let initially the charge on each sphere be q and separation between their centers be r; then according to given problem.



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{r^2} = 2 \times 10^{-5}\text{N}$$

When sphere C touches B, the charge of B, q will distribute equally on B and C as sphere are identical conductors, i.e., now charges on spheres;



$$q_B = q_c = (\frac{q}{2})$$

So sphere C will experience a force

$F_{CA} = \frac{1}{4\pi\varepsilon_0}\frac{q(q/2)}{(r/2)^2} = 2$ F along $\overrightarrow{AB}$ due to charge on A and

$F_{CB} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(q/2)}{(r/2)^2} =$ F along $\overrightarrow{BA}$ due to charge on B

So the net force F_C on C due to charges on A and B,

$$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5}\text{N along } \overrightarrow{AB}.$$

Ex. Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value – q coulomb placed at the centre of the hexagon?

Sol. **Method-I :**

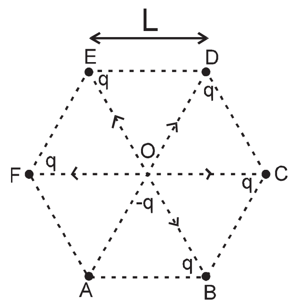
If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O would be zero (as the forces due to individual charges will balance each other), i.e. $\vec{F}_R = 0$

Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$$\vec{F} + \vec{f} = 0 \text{ i.e. } \vec{F} = -\vec{f}$$

$$|F| = |f| = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$$

$$\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2} \text{ along OD}$$



Method: II

In the diagram we can see that force due to charge A and D are opposite to each other

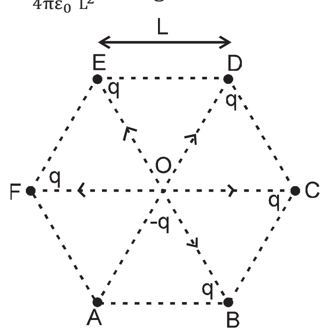
$$\vec{F}_{DO} + \vec{F}_{AO} = 0$$

$$\vec{F}_{DO} + \vec{F}_{AO} = 0 \quad \text{... (1)}$$

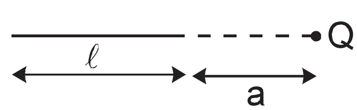
$$\vec{F}_{BO} + \vec{F}_{EO} = 0 \quad \text{... (2)}$$

$$\vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{CO} + \vec{F}_{DO} + \vec{F}_{EO} = \vec{F}_{Net}$$

Using (1) and (2) $\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$ along OD



Ex. A thin straight rod of length l carrying a uniformly distributed change q is located vacuum. Find the magnitude of the electric force on a point charge 'Q' kept as shown the figure.



Sol. As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part called element of length dy can be considered as point charge.

Charge on element $dq = \lambda dy = \frac{q}{\ell} dy$

Electric force on 'Q' due to element $= \frac{K.dq.Q}{y^2} = \frac{K.Q.q.dy}{y^2 \cdot \ell}$

All forces are along the same direction

$F = \sum dF$ This sum can be calculated using integration,

$$F = \int_{y=a}^{a+\ell} \frac{KQqdy}{y^2 \ell} = \frac{KqQ}{\ell}\left[-\frac{1}{y}\right]_a^{a+\ell} = \frac{KQ \cdot q}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right] = \frac{KQq}{a(a+\ell)}$$

Note:

1. The total charge of the rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.
2. If $a >> 1$ then $F = \frac{KQq}{a^2}$ behavior of the rod is just like a point charge.