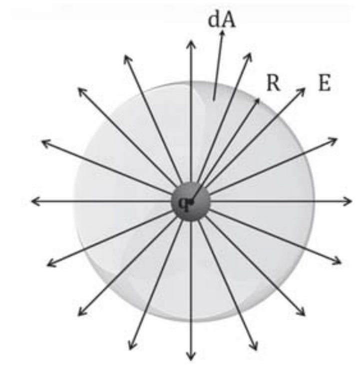
CALCULATION OF E-FIELD DUE CHARGED BODIES USING GAUSS' LAW

Electric Field Due To Point Charge Using Gauss Law

The electric field produced by a point charge at a distance r remains uniform in all directions, akin to how a bulb emits an equal amount of light in every direction at a specific distance from it.

Due to this symmetry (spherical symmetry), we can examine a sphere (Gaussian surface) with a radius r centered around this point charge to determine the electric field at a distance r from the point charge, as illustrated in the diagram.



Total charge enclosed by the Gaussian surface is, $q_{in} = q$

In this case, the electric field $\vec{E}$ and the small area element $d\vec{A}$ are along same direction. Hence, the angle between them is, $\theta = 0°$

Applying Gauss's law, we get,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$EdA\cos \oint 0° = \frac{q}{\varepsilon_0}$$

$$E\oint dA = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

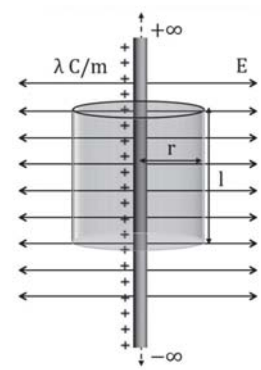
$$E = \frac{Kq}{r^2} \text{ since } k = \frac{1}{4\pi\varepsilon_0}$$

$$|\vec{E}| = \frac{kq}{r^2}$$

Electric Field Due To Infinite charged Wire Using Gauss Law

Let's consider an infinitely charged wire with a uniform line charge density of $+\lambda$. Our objective is to determine the electric field at a distance r from the wire using Gauss's law.

The infinite wire exhibits cylindrical symmetry. Therefore, in order to determine the electric field resulting from the uniformly charged infinite wire at a distance r from the wire, we consider a cylindrical Gaussian surface with a radius r and length l, as depicted in the illustration.



Total charge enclosed by the cylindrical surface, $q_{in} = \lambda l$

Applying Gauss's law, we get,

$$\oint_{Curved} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\int_{Top} \vec{E} \cdot d\vec{A} + \int_{Bottom} \vec{E} \cdot d\vec{A} + \int_{\vec{E}} \vec{A}\,\vec{A} = \frac{\lambda l}{\epsilon_0}$$

As the area vectors of the top and bottom surfaces of the cylindrical Gaussian surface are perpendicular to the electric field, the electric flux through these surfaces is zero. Consequently,

$$\int_{Curved} \vec{E} \cdot d\vec{A} = \frac{\lambda l}{\epsilon_0}$$

$$E2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

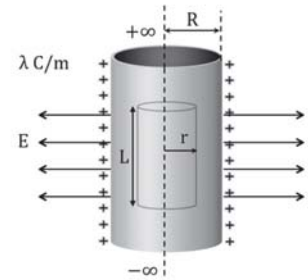
$$E = \frac{\lambda}{2\pi\ _0 r} = \frac{2k\lambda}{\gamma}$$

$$|\vec{E}| = \frac{2k}{r}$$

Gauss's law holds true universally, but it is only in highly symmetric scenarios where one can effectively determine the electric field using Gauss's law.

Electric Field Due To Infinite Hollow Conducting Wire

When $r < R$



Consider an infinitely long hollow conducting wire with uniform charge density λ and radius R. Our aim is to determine the electric field at a distance r where $r < R$ inside the wire using Gauss's law.

In order to calculate the electric field within the wire at a distance r from it, we employ a cylindrical Gaussian surface with a radius r and length l, depicted in the figure.

As all charges reside on the surface of the conducting wire, the total charge encompassed by the Gaussian surface is, $q_{in} = 0$

Applying Gauss's law, we get,

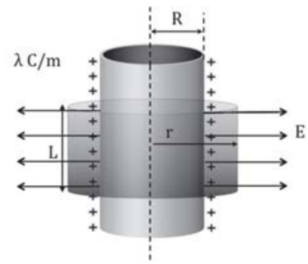
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \int_{Curved} \vec{E} \cdot d\vec{A} + \int_{Top} \vec{E} \cdot d\vec{A} + \int_{Bottom} \vec{E} \cdot d\vec{A} = 0$$

$$\int_{Curved} \vec{E} \cdot d\vec{A} = 0 \Rightarrow E(2\pi rl) = 0 \Rightarrow E = 0$$

$$|\vec{E}| = 0$$

Electric Field Due To Infinite Hollow Conducting Wire

When $r > R$



In order to determine the electric field outside the wire at a distance r from it, we consider a cylindrical Gaussian surface with a radius r and length l, as depicted in the figure.

The net charge enclosed by the Gaussian surface is $q_{in} = \lambda l$

Applying Gauss's law, we get,

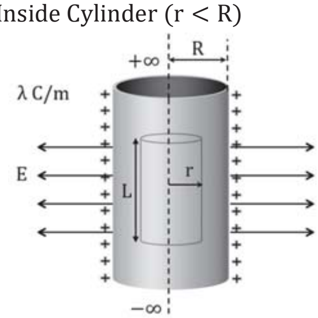
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \int_{Curved} \vec{E} \cdot d\vec{A} + \int_{Top} \vec{E} \cdot d\vec{A} + \int_{Bottom} \vec{E} \cdot d\vec{A} = \frac{\lambda l}{\epsilon_0}$$

$$\int_{Curved} \vec{E} \cdot d\vec{A} = \frac{\lambda l}{\epsilon_0} \Rightarrow E(2\pi rl) = \frac{\lambda l}{\epsilon_0} \Rightarrow E = \frac{2k\lambda}{r}$$

$$|\vec{E}| = \frac{2k\lambda}{r}$$

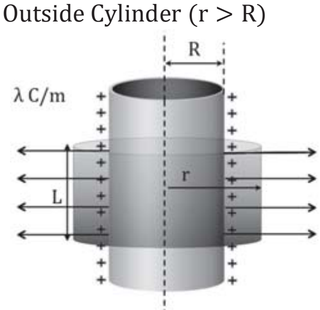
Electric Field Due To Infinite Hollow Conducting Wire

Inside Cylinder ($r < R$)



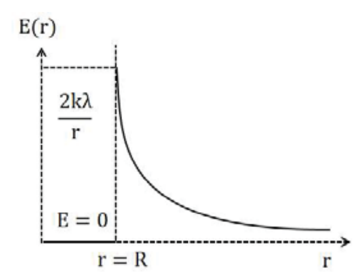
$|\vec{E}| = 0$

Outside Cylinder ($r > R$)



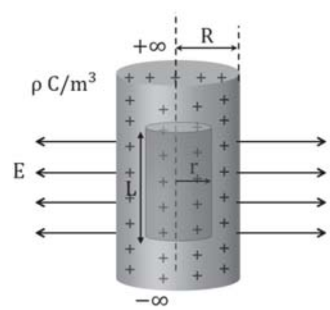
$|\vec{E}| = \frac{2k}{r}$

Variation of the electric field with r



Electric Field Due To Infinite Solid Non-Conducting Wire

When $r < R$



Consider an infinitely long solid non-conducting wire with uniform volume charge density r and radius R. Our objective is to determine the electric field at a distance r where \(r < R\)) inside the wire using Gauss's law.

In order to determine the electric field inside the wire at a distance r from it, we employ a cylindrical Gaussian surface with a radius r and length l as depicted in the figure. Consequently, the volume of the Gaussian surface is given by $V = \pi r^2 l$

The net charge enclosed by the Gaussian surface is, $q_{in} = \rho V = \rho(\pi r^2 l)$

Applying Gauss's law, we get,

$$\oint_{\vec{E}} d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \cdot \int_{Curved} \vec{E} \cdot d\vec{A} + \int_{Top} \vec{E} \cdot d\vec{A} + \int_{Bottom} \vec{E} \cdot d\vec{A} = \frac{\rho(\pi r^2 l)}{\epsilon_0}$$

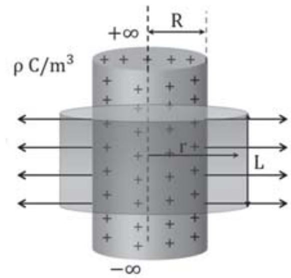
$$\int_{Curved} \vec{E} \cdot d\vec{A} = \frac{\rho(\pi r^2 l)}{\epsilon_0} \Rightarrow E(2\pi rl) = \frac{\rho(\pi r^2 l)}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}$$

As long as $r < R$, the electric field linearly increases with the radial distance r i.e., $E \propto r$ when $r < R$.

$$|\vec{E}| = \frac{\rho r}{2\epsilon_0}$$

Electric Field Due To Infinite Solid Non-Conducting Wire



When $r > R$

Assume that the uniformly charged infinite solid non- conducting wire of radius *R* has volume charge density $+\rho$ and line charge density $+\lambda$.Since the radius of the wire is R, the relation between ρ and λ is $\lambda = \rho(\pi R^2)$

Therefore, the electric field outside the wire will be,

$$E = \frac{2k\lambda}{r} \Rightarrow E = \frac{2k\rho(\pi R^2)}{r} \Rightarrow E = \frac{2\rho(\pi R^2)}{4\pi\varepsilon_0 r} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}$$

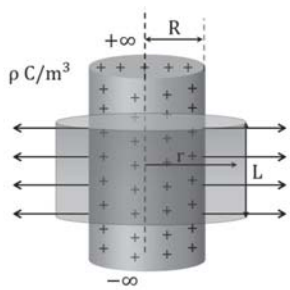
Outside the wire, the electric field E is inversely proportional to r.

At $r = R$, the electric field E will be $\frac{\rho R}{2\epsilon_0}$.

$$|\vec{E}| = \frac{\rho R^2}{2\epsilon_0 r}$$

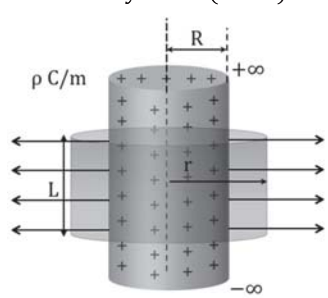
Electric Field Due To Infinite Solid Non-Conducting Wire

Inside Cylinder ($r < R$)



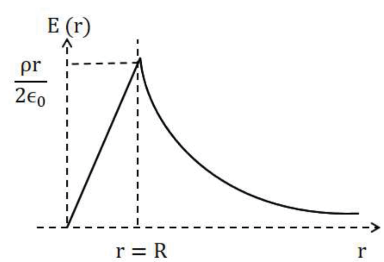
$$|\vec{E}| = \frac{\rho r}{2\epsilon_0}$$

Outside Cylinder ($r > R$)

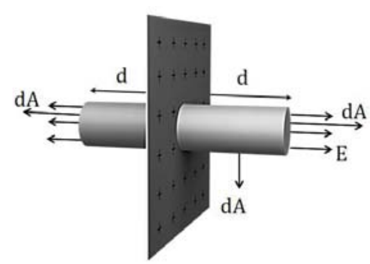


$$|\vec{E}| = \frac{\rho R^2}{2\epsilon_0 r}$$

Variation of the electric field with



Electric Field Due To Thin Infinite Sheet



The uniformly charged thin infinite sheet has surface charge density $+\sigma$. Suppose we want to find the electric field at a distance *d* from the sheet by using Gauss's law.

In order to determine the electric field resulting from the uniformly charged thin infinite sheet at a distance d from it, we consider a cylindrical Gaussian surface with a radius r and length d, as depicted in the figure.

Total charge enclosed by the cylindrical surface $q_{in} = \sigma(\pi r^2)$

Applying Gauss's law, we get,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \int_{Curved} \vec{E} \cdot d\vec{A} + \int_{Left} \vec{E} \cdot d\vec{A} + \int_{Right} \vec{E} \cdot d\vec{A} = \frac{\sigma(\pi r^2)}{\epsilon_0}$$

As the area vector of the curved surface of the cylindrical Gaussian surface is perpendicular to the electric field, the electric flux through this surface is zero. Hence,

$$\int_{Left} \vec{E} \cdot d\vec{A} + \int_{Right} \vec{E} \cdot d\vec{A} = \frac{\sigma(\pi r^2)}{\epsilon_0} \Rightarrow E(\pi r^2) + E(\pi r^2) = \frac{\sigma(\pi r^2)}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

$$|\vec{E}| = \frac{\sigma}{2\epsilon_0}$$

Electric Field Due To Thick Infinite Sheet

When $r < d$

The uniformly charged thick infinite sheet has volume charge density $+\rho$. Suppose we want to find the electric field at a distance r ($r < d$) inside the sheet by using Gauss's law.

In order to determine the electric field inside the sheet at a distance r from it, we consider a cylindrical Gaussian surface with a radius R and length 2r as depicted in the figure. Thus, the volume of the Gaussian surface is. $V = \pi R^2(2r)$

The net charge enclosed by the Gaussian surface is,

$$q_{in} = \rho V = \rho[\pi R^2(2r)]$$

Applying Gauss's law, we get,

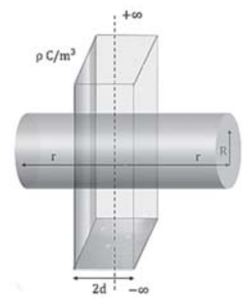
$$\oint_{Left} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \int_{Cuived} \vec{E} \cdot d\vec{A} + \int_{Left} \vec{E} \cdot d\vec{A} + \int_{Right} \vec{E} \cdot d\vec{A} = \frac{\rho[\pi R^2(2r)]}{\epsilon_0}$$

$$\int_{Right} \vec{E} \cdot d\vec{A} + \int_{\vec{A}} \vec{A} = \frac{\rho[\pi R^2(2r)]}{\epsilon_0} \Rightarrow E(\pi R^2) + E(\pi R^2) = \frac{\rho[\pi R^2(2r)]}{\epsilon_0} \Rightarrow E = \frac{\rho r}{\epsilon_0}$$

As long as $r < d$, the electric field linearly increases with the distance r i.e., $E \propto r$ when $r < d$.

$$|\vec{E}| = \frac{\rho r}{\epsilon_0}$$

Electric Field Due To Thick Infinite Sheet



When $r > d$

We want to find the electric field at a distance r ($r > d$) outside the thick sheet by using Gauss's law.

To find the electric field outside the sheet at a distance r from it, we assume a cylindrical Gaussian surface of radius R and length 2r as shown in the figure. Therefore, the volume of the Gaussian surface up to which the charge located is, $V = \pi R^2(2d)$

The net charge enclosed by the Gaussian surface is,

$$q_{in} = \rho V = \rho[\pi R^2(2d)]$$

Applying Gauss's law, we get,

$$\oint_{Cyived} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow \int_{Left} \vec{E} \cdot d\vec{A} + \int_{Right} \vec{E} \cdot d\vec{A} + \int_{Right} \vec{E} \cdot d\vec{A} = \frac{\rho[\pi R^2(2d)]}{\epsilon_0}$$

$$\int_{Left} \vec{E} \cdot d\vec{A} + \int_{E} \vec{E} \cdot d\vec{A} = \frac{\rho[\pi R^2(2d)]}{\epsilon_0} \Rightarrow E(\pi R^2) + E(\pi R^2) = \frac{\rho[\pi R^2(2d)]}{\epsilon_0}$$

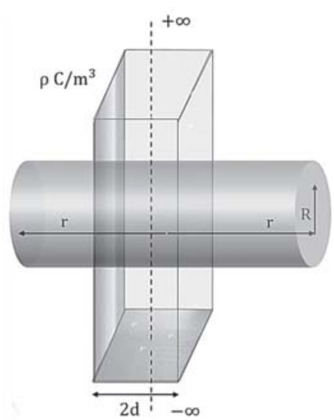
$$E = \frac{\rho d}{\epsilon_0}$$

$$|\vec{E}| = \frac{\rho d}{\epsilon_0}$$

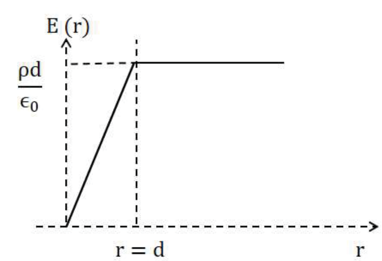
Electric Field Due To Thick Infinite Sheet

Inside the sheet ($r < d$) Outside the sheet ($r < d$) Variation of the electric field with r

$|\vec{E}| = \frac{\rho r}{\epsilon_0}$



$|\vec{E}| = \frac{\rho d}{\epsilon_0}$



Introduction Of Solid Angel

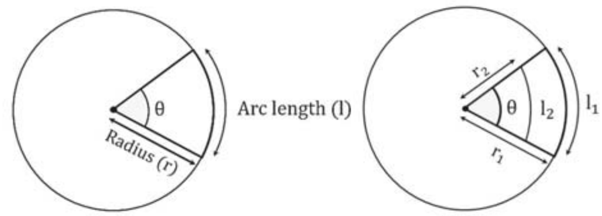
Plane Angle

Angle formed by an arc (segment of a circle) at a specific point...

Mathematical definition: $\theta = \frac{\text{Arc length}}{\text{Radius}} = \frac{l}{r}$

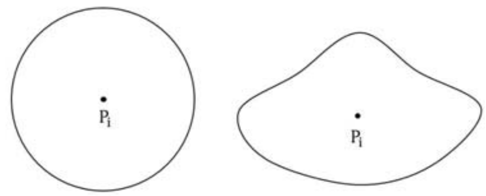
Unit: radian (rad)

180° is equal to π radian



Circle Circle

Angle subtended by a closed arc at any interior point: $\theta = 2\pi$



Circle Random arc

Angle subtended by a closed arc at any exterior point: $\theta = 0$

Solid Angle

Angle formed by a surface at a point.

Assumption: Area of surface (A) is so small that the distance between the point O and any point on the surface is r.

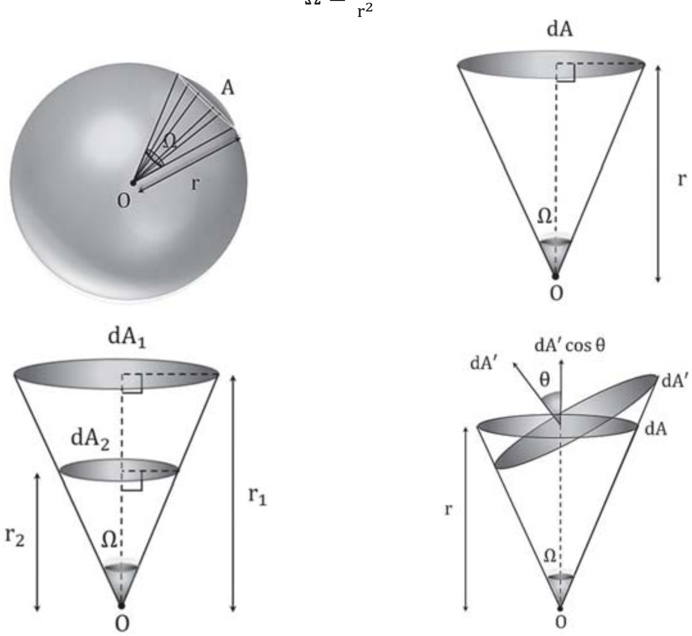
The surface area dA should be perpendicular to r.

Unit of solid angle: steradian (sr)

Solid angle is represented by the symbol 'Ω'

Solid angle for the adjacent figure

$$\Omega = \frac{dA}{r^2}$$

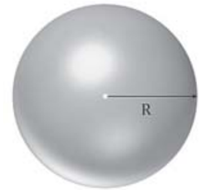


$\Omega = \frac{dA_1}{r_1^2} = \frac{dA_2}{r_2^2}$

Solid angle : $\Omega = \frac{\text{Component of Area whose surface is} \perp \text{to r}}{r^2}$

$\Omega = \frac{dA}{r^2} = \frac{dA' \cos\theta}{r^2}$

Ex. Find out the solid angle subtended by a sphere at its centre

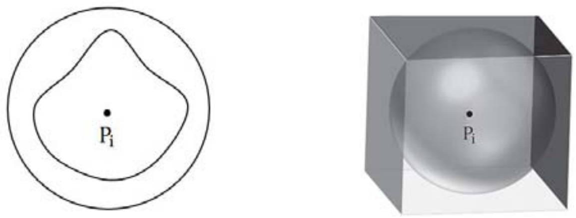


Sol. Total solid angle at the centre of the sphere will be:

$$\Omega = \int \frac{dA}{R^2} = \frac{\text{Area of sphere}}{R^2} = \frac{4\pi R^2}{R^2} = 4\pi$$

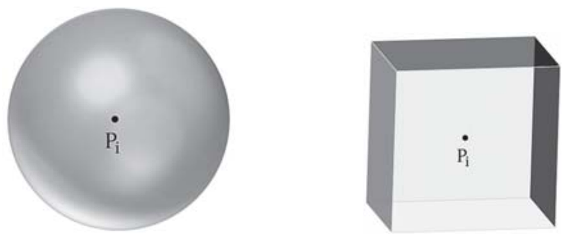
$$\Omega = 4\pi(sr)$$

	Angle subtended at Point P_i
Any closed Arc	2π
Any closed surface	4π



Solid angle subtended by any closed surface at an interior point

$$\Omega = 4\pi sr$$



Position of point	Solid angle subtended by any closed surface (in sr)
At any interior point (P_1)	4π
At any exterior point (P_0)	0