## VELOCITY AND INTENSITY OF SOUND

### Speed of a Sound Wave in a Material Medium

We understand that mechanical waves can propagate through any medium. Of all the wave parameters, the wave's velocity is the sole parameter influenced by the properties of the medium, specifically its elasticity and inertia.

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$$

Here, $B =$ Bulk modulus of the medium, and, $\rho =$ Density of the medium

For fluids, $v = \sqrt{\frac{B}{\rho}}$

For a solid rod, $v = \sqrt{\frac{Y}{\rho}}$

**Note** In the case of fluids, the elastic property is characterized by the bulk modulus, while for solid materials, the elastic property is represented by Young's modulus.

### Newton's Formula for Speed of Sound in Gas

Newton hypothesized that the transmission of sound through a gaseous medium follows an isothermal process.

For isothermal process, $PV = \text{Constant}$

$$PdV + VdP = 0$$
$$PdV = -VdP$$
$$P = -\frac{dP}{\left(\frac{dV}{V}\right)} = B_{isothermal}$$

By substituting this in $v = \sqrt{\frac{B}{\rho}}$, we get the speed of sound.

$$v = \sqrt{\frac{P}{\rho}}$$

For a medium like air at temperature 273 K,

$$P = 1.01325 \times 10^5 \text{Pa}$$
$$\rho = 1.293 \text{ kgm}^{-3}$$
$$v_{sound} = 280 \text{ms}^{-1}$$

It is different from the other experimental results.

### Laplace's Correction

- Laplace determined that the transmission of sound through a gaseous medium adheres to an adiabatic process.
- The rapid occurrence of compressions and rarefactions leaves little time for heat exchange between the medium and its surroundings.

Thus, for the propagation of sound, we have,

$$PV^{\gamma} = \text{Constant}$$

On differentiating both the sides,

$$\left(\frac{dP}{dV}\right)V^{\gamma} + P(\gamma V^{\gamma-1}) = 0$$
$$\left(\frac{dP}{dV}\right)V^{\gamma} = -P(\gamma V^{\gamma-1})$$
$$\left(\frac{dP}{dV}\right)V^{\gamma} = -P\gamma\frac{V^{\gamma}}{V}$$
$$-\frac{(dP)}{\left(\frac{dV}{V}\right)} = \gamma P = B_{adiabatic}$$

Therefore $v = \sqrt{\frac{B_{adiabatic}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

The provided equation provides the speed of sound within the gaseous medium. Derived from the ideal gas equation in relation to density,

$$PM = \rho RT$$
$$\frac{P}{\rho} = \frac{RT}{M}$$

We get the following relation for the speed of sound in a gaseous medium.

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

**Note** For a medium like air at temperature 273 K

$$\gamma = 1.4 = \frac{7}{5}$$
$$P = 1.01325 \times 10^5 \text{Pa}$$
$$\rho = 1.293 \text{ Kgm}^{-3}$$
$$v_{sound} = 332\text{ms}^{-1}$$

This resembles the speed of sound in air.

**Factors Affecting Speed of Sound in Atmosphere**

**Effect of temperature**

The relation between the speed of sound and temperature is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$
$$v \propto \sqrt{T}$$

(With $\gamma$, R, and M constants for a specific gaseous medium) Therefore, as temperature rises, the velocity of sound also increases. This can be deduced from the provided equation.

$$v = \sqrt{\frac{\gamma R}{M}} \times T^{\frac{1}{2}}$$
$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$
$$\Delta v = \frac{v}{2T}\Delta T$$

We have, $\Delta v = \frac{v}{2T} \times \Delta T$

Near the room temperature, $v = 330 \text{ ms}^{-1}$, T = 273 K,

Therefore,

$$\Delta v \simeq (0.6)\Delta T$$

For a slight temperature increase above room temperature, the speed of sound in air exhibits a linear increase of 0.6 m/s for every 1°C or 1 K rise in temperature.

**Effect of pressure**

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

When the temperature remains constant and there is a change in pressure, the variation occurs in a manner such that $\frac{P}{\rho}$ remain constant.

Therefore, as long as the temperature remains constant, the pressure does not influence the velocity of sound.

**Effect of humidity**

As humidity rises, the effective density of air decreases, leading to an increase in the velocity of sound.

**Note** In dry air, the predominant components are nitrogen (78%), oxygen (21%), $CO_2$ (0.03%), and trace amounts of other elements. Introducing water vapor reduces the effective molecular weight of the mixture, as $H_2O$ has a lower molecular weight (18 g $\text{mol}^{-1}$) compared to other air components such as $CO_2$, $O_2$, $N_2$, etc.

Since $M_{\text{wet air}} < M_{\text{dry air}}$, therefore for a fixed amount of volume,$\rho_{\text{wet air}} < \rho_{\text{dry air}}$ [ Since $\rho = \frac{M}{V}$]

Now since, $v \propto \frac{1}{\sqrt{\rho}}$, thus, $v_{\text{wet air}} > v_{\text{dry air}}$

**Ex.** Adiabatic constant for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 470 ms$^{-1}$, then what will be the speed of sound in hydrogen at the same temperature and pressure?

**Sol.** Given $(\gamma)_{H_2} = (\gamma)_{O_2} = 1.4$ and $(v)_{O_2} = 470ms^{-1}$

$$\text{Since, } v = \sqrt{\frac{\gamma RT}{M}}, \text{ therefore,}$$

$$(v)_{O_2} = \sqrt{\frac{\gamma RT}{M_{O_2}}} \text{ and } (v)_{H_2} = \sqrt{\frac{\gamma RT}{M_{H_2}}}$$

$$\frac{(v)_{O_2}}{(v)_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{(v)_{O_2}}{(v)_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$(v)_{H_2} = 4 \times (v)_{O_2}$$

$$(v)_{H_2} = 4 \times 470$$

$$(v)_{H_2} = 1880m^{-1}$$

**Intensity of Sound Wave**

As sound waves propagate through the medium, they transfer both energy and momentum. The equations representing pressure and displacement are as follows:

$$S = S_0 \sin(\omega t - kx)$$

$$p = p_0 \cos(\omega t - kx)$$

[To prevent confusion with power notation, in this context, the pressure is represented by the p.]

The power transmitted (P) by the wave across cross section A is,

$$P = \vec{F} \cdot \vec{v}$$

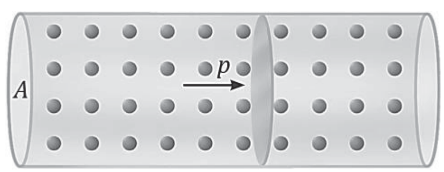
$$P = Fv\cos 0^\circ = Fv$$

$$P = (pA)\frac{\partial S}{\partial t}$$

$$P = A[p_0\cos(\omega t - kx)] \times [S_0\omega\cos(\omega t - kx)]$$

$$P = p_0 S_0 A\omega\cos^2(\omega t - kx)$$

$$P_{avg} = \langle P \rangle = \frac{p_0 S_0 A\omega}{2}$$




(Since the average value of $\cos^2\theta$ is $\frac{1}{2}$ in one complete cycle.)

Intensity is characterized as the power conveyed per unit area. Mathematically, its definition is as follows:

$$I = \frac{P_{avg}}{A} = \frac{p_0 S_0 \omega}{2}$$

We know that, $p_0 = BkS_0$ and $v = \sqrt{\frac{B}{\rho}} \Rightarrow B = v^2\rho$

$$I = \frac{p_0 S_0 \omega}{2} = \frac{S_0\omega}{2}(BkS_0)$$

Therefore $I = \frac{S_0\omega}{2}\left(\frac{BS_0\omega}{v}\right)$[ Since $v = \frac{\omega}{k}$]

$$I = \frac{B}{2v}\omega^2 S_0^2$$

$$I = \frac{(v^2\rho)}{2v}(2\pi f)^2 S_0^2$$

$$I = 2\pi^2 f^2 S_0^2 \rho v$$

This is the expression for the intensity of a wave, illustrating that intensity is dependent on the frequency, medium density, and wave speed.

From the equation, $\omega = 2\pi f$, and $I = \frac{p_0 S_0 \omega}{2}$ we get another form of the equation for the intensity of sound. $I = \frac{p_0^2}{2\rho v}$

**Note** In a specific medium, the sound intensity is directly correlated with the amplitude of excess pressure. $I \propto p_0^2$

**Ex.** The pressure amplitude in a sound wave from a radio receiver is $2.0 \times 10^{-2}$ $Nm^{-2}$ and the intensity at a point is $5.0 \times 10^{-7}$ $Wm^{-2}$. If the pressure amplitude is increased to $2.5 \times 10^{-2}$ Nm–2 by turning the volume knob, then evaluate the intensity.

**Sol.** Given

$$(P_0)_i = 2.0 \times 10^{-2} Nm^{-2}$$
$$I_i = 5.0 \times 10^{-7} Wm^{-2}$$
$$(P_0)_f = 2.5 \times 10^{-2} Nm^{-2}$$
$$I = 2\pi^2 f^2 S_0^2 \rho v = \frac{P_0^2}{2\rho v}$$
$$I \propto P_0^2$$
$$\frac{I_i}{I_f} = \frac{(P_0)_i^2}{(P_0)_f^2}$$
$$I_f = I_i \times \frac{(P_0)_f^2}{(P_0)_i^2}$$
$$I_f = (5 \times 10^{-7}) \times \frac{(2.5\times10^{-2})^2}{(2.0\times10^{-2})^2}$$
$$I_f = 7.8 \times 10^{-7} Wm^{-2}$$

**Appearance of Sound to the Human Ear**

Typically, the perception of sound to the human ear is characterized by three parameters: pitch, loudness, and quality.

**Pitch**

Pitch refers to the sensation or characteristic that corresponds to the predominant frequency of a sound wave. In the real world, waves are typically polychromatic, consisting of various frequencies. The human ear is particularly sensitive to higher frequencies. Consequently, it can be stated that a higher frequency corresponds to a higher pitch, and conversely, a lower frequency results in a lower pitch. For instance, a woman's voice, having a higher frequency than a man's voice, is characterized by a higher pitch.

**Loudness**

Loudness, associated with a sound wave, is linked to its intensity, and our perception of loudness corresponds to the sound level. The sound level is defined as $\beta = 10\log_{10}(\frac{I}{I_0})$

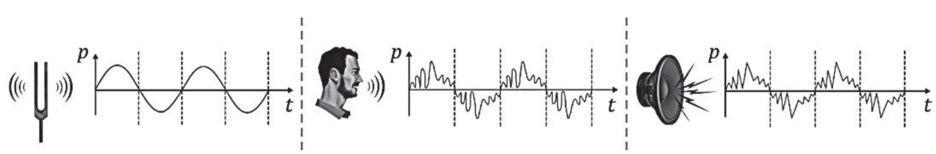
Where I is the intensity of the concerned sound wave and $I_0$ is the minimum audible intensity, which is $10^{-12}$ W $m^{-2}$.

The sound level is measured in decibels (dB). For $I = I_0$, the sound level, $\beta = 0$ dB.

**Quality**

Quality, in addition to the predominant frequency and intensity, is another aspect the human ear is attuned to in a sound wave. It hinges on the waveform of the sound wave and serves to differentiate between two sounds of identical loudness and pitch.

In the figure, three sound waveforms originating from distinct sources are depicted. Despite having identical pressure amplitude and time period, the waveforms differ, leading to distinct qualities in the sound produced by these three sources.




**Ex.** If the intensity is multiplied by a factor of 20, what will be the increase in sound level measured in decibels?

**Sol.** Assume that the initial intensity is I and the minimum audible intensity is $I_0$.

Therefore, the sound level is $\beta = 10\log_{10}(\frac{I}{I_0})$

When the intensity is multiplied by a factor of 20, the resulting sound level is as follows:

$$\beta' = 10\log_{10}(\frac{20I}{I_0})$$

Therefore, $$\beta' - \beta = 10\log_{10}(\frac{20I}{I_0}) - 10\log_{10}(\frac{I}{I_0})$$

$$\beta' - \beta = 10\log_{10}[\frac{(\frac{20I}{I_0})}{(\frac{I}{I_0})}][\text{ Since } \log(\frac{a}{b}) = \log a - \log b]$$

$$\beta' - \beta = 10\log_{10}[\frac{20I}{I_0} \times \frac{I_0}{I}]$$

$$\beta' - \beta = 10\log_{10}(20)$$

$$\beta' - \beta = 10\log_{10}(2 \times 10)$$

$$\beta' - \beta = 10\log_{10} 2 + 10\log_{10} 10$$

$$\beta' - \beta = 10 \times 0.3 + 10[\text{ Since } \log_{10} 2 = 0.3 \text{ and } \log_{10} 10 = 1]$$

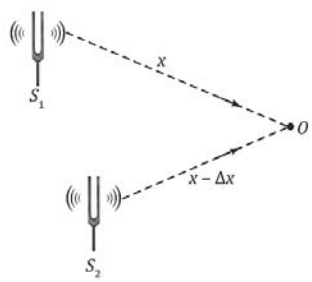
$$\beta' - \beta = 13\text{dB}$$

Therefore, the sound level will increase by 13dB.

**Interference of Sound Waves**

Contemplate two tuning forks, denoted as S1 and S2, serving as sources of sound waves. Assume these two sources to be coherent, meaning there is either no phase difference or a constant phase difference between the waves they emit.

Suppose point O is situated at a distance x from source $S_1$ and x – Δx from source $S_2$. Thus, $S_1O = x$ and $S_2O = x - \Delta x$ and therefore, the path difference between them is $|S_1O - S_2O| = \Delta x$.




Let the equation of the sound waves from $S_1$ and $S_2$ be as follows:

$$p_1 = p_{01}\sin(\omega t - kx) \quad \text{... (1)}$$

$$p_2 = p_{02}\sin(\omega t - k(x - \Delta x)) \quad \text{... (2)}$$

Equation (2) can also be rewritten as follows

$$p_2 = p_{02}\sin(\omega t - kx + k\Delta x)$$

$$p_2 = p_{02}\sin(\omega t - kx + \phi) \quad \text{... (3)}$$

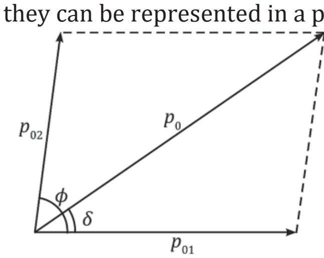
Where ϕ the phase difference between the waves, and this is can be formulated as follows

$$\phi = k\Delta x \quad \text{... (4)}$$

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference ... (5)

Hence, any disparity in the path traveled by the waves results in a corresponding phase difference. Referring to equations (1) and (3), it can be asserted that they represent sinusoidal waves sharing identical frequencies and propagation directions but differing in amplitudes. A parallel situation has been previously discussed in Simple Harmonic Motion (SHM). By recalling this, it can be concluded that the resultant wave will also manifest as a sinusoidal wave, as expressed by: $p = p_0\sin(\omega t - kx + \delta)$ and they can be represented in a phasor diagram as shown.



Where $p_0$ and δ are the amplitude and phase angle of the resultant wave and they are defined as follows:

$$p_0 = \sqrt{p_{01}^2 + p_{02}^2 + 2p_{01}p_{02}\cos\phi} \quad \text{... (6)}$$

$$\delta = \tan^{-1}\left(\frac{p_{02}\sin\phi}{p_{01}+p_{02}\cos\phi}\right) \quad \text{... (7)}$$

Now, in addition to the phase difference arising from the path disparity, if there exists an initial phase difference, denoted as the epoch ($\phi_0$), between the waves, the cumulative phase difference will be: $\Delta\phi = (\frac{2\pi}{\lambda})\Delta x + \phi_0$ Then, equation will reconstruct itself as.

$$p_0 = \sqrt{p_{01}^2 + p_{02}^2 + 2p_{01}p_{02}\cos(\Delta\phi)} \quad \text{... (8)}$$

Therefore the magnitude of the resultant wave depends upon the total phase difference between the waves.

**Constructive interference**

Referring to equation (8), it can be asserted that constructive interference will occur when the specified condition is met.

$$\cos\Delta\phi = 1$$
$$\Delta\phi = 2n\pi$$

Where $n = 0,1,2,3$

Hence, from equation (8), we get $p_0 = p_{01} + p_{02}$

Now, $\Delta\phi = (\frac{2\pi}{\lambda})\Delta x + \phi_0$ and we know that for constructive interference, $\Delta\phi = 2n\pi$

If the epoch becomes zero, then we get $\Delta x = n\lambda$

Therefore, for constructive interference to take place, the necessary and sufficient condition is $\Delta\phi = 2n\pi$, and if the initial phase difference between the interfering waves becomes zero, then $\Delta\phi = 2n\pi$ and $\Delta x = n\lambda$ are both true

**Destructive interference**

Destructive interference will occur when the specified condition is met.

$$\cos\Delta\phi = -1$$
$$\Delta\phi = (2n \pm 1)\pi$$

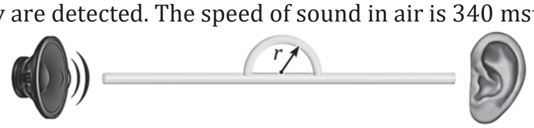
Where $n = 0,1,2,3$

Hence from equation (8), we get, $p_0 = |p_{01} - p_{02}$

Now, $\Delta\phi = (\frac{2\pi}{\lambda})\Delta x + \phi$ . and we know that for destructive interference $\Delta\phi = (2n \pm 1)\pi$ if the epoch become zero the we get $\Delta x = n\lambda \pm \frac{\lambda}{2}$ therefore for destructive interference to take place the necessary and sufficient condition is $\Delta\phi = (2n + 1)\pi$ and if the initial phase difference between the interfering wave become zero then the $\Delta\phi = (2n + 1)\pi$ and $\Delta x = n\lambda \pm \frac{\lambda}{2}$ are both true.

- When dealing with sound waves (longitudinal waves), the phase difference between two waves may arise from two distinct factors: the initial phase difference and the path difference. In contrast, for transverse waves, the phase difference between two waves is solely attributed to the initial phase difference between them.
- Criterion for constructive interference: $\Delta\phi = 2n\pi$ (always) and $\Delta x =$ integer multiple of $\lambda = n\lambda$ (if $\phi_0 = 0$ )
- Condition for destructive interference $\Delta\phi = (2n + 1)\pi$ (always) and $\Delta x = n\lambda \pm \frac{\lambda}{2} =$ half integral multiple of $\lambda$ (if $\phi_0 = 0$ )

**Ex.** The figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 20.0 cm. The frequency of the sound source can be varied electronically between 1,000 Hz and 4,000 Hz. Find the frequencies at which the maxima of intensity are detected. The speed of sound in air is 340 $ms^{-1}$.

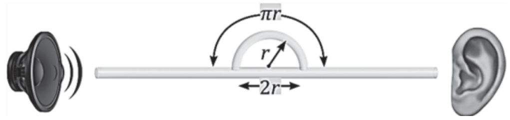


**Sol.** First of all, it should be remembered that the intensity will be the maximum for constructive interference only.

Now given, Radius of the semicircular part,

$r = 20.0\ cm = 0.2\ m$

The required path difference between the two routes is $\Delta x = \pi r - 2r$




Therefore, for constructive interference to take place, the following condition has to be satisfied

$$\Delta x = n\lambda$$

$$\pi r - 2r = n\left(\frac{v}{f}\right) \qquad \begin{bmatrix} \text{Where } v = \text{ velocity of the wave,} \\ f = \text{ Frequency of the wave} \end{bmatrix}$$

$$f = \frac{nv}{(\pi - 2)r}$$

This equation will give the required frequencies for maximum intensity. By putting the known values in the equation, we get,

$$f = n\frac{340}{(\pi - 2)\times 0.2}$$

$$f = n\frac{340}{1.14\times\left(\frac{1}{5}\right)}$$

$$f = n\frac{1,700}{1.14}$$

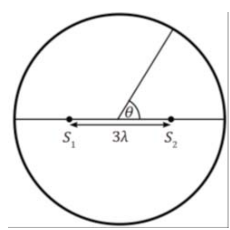
$$f \approx 1,490 \times n$$

By putting $n = 1,2,3,...$, we get,

$$f_1 = 1,490\ Hz, f_2 = 2,980\ Hz, f_3 = 4,470\ Hz$$

Since the source has the frequency range 1,00 Hz to 4000 Hz, the required frequency at which intensity will be maximum are $f_1 = 1,490$ Hz and $f_2 = 2,980$ Hz

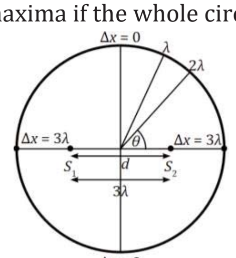
**Ex.** The figure shows two coherent sources, $S_1$ and $S_2$, which emit sound of wavelength λ in phase. The separation between the sources is 3λ. A circular wire of a large radius is placed in such a way that $S_1 S_2$ lies in its plane and the midpoint of $S_1 S_2$ is at the centre of the wire. Find the angular positions θ (in first quadrant) on the wire for which constructive interference takes place.




**Sol.** If we consider the separation between the sources to be d, then along the horizontal diameter for the two points on the circumference of the circle, the path difference between $S_1 S_2$ will be $d = 3\lambda$ . Similarly, for the two points on the circumference of the circle along the vertical diameter, the path difference between $S_1 S_2$ will be zero.

Thus, it is natural to conclude that there will be two positions in the first quadrant where the path difference should be λ and 2λ, and constructive interference will take place at those two positions. The whole scenario is depicted in the figure.

Hence, in total, there will be 12 maxima if the whole circle is considered.



Now, consider any point (P) on the circumference of the circle. The path difference at point P between the sound waves originating from sources $S_1$ and $S_2$ is, $\Delta x = |S_1 P - S_2 P| = S_1 M$ (assuming that $d = 3\lambda$ is very small compared to the radius of the circle). Now, $S_1 M = d \cos\theta$ Therefore, if $\theta_1$ and $\theta_2$ are the two positions of the maxima in the first quadrant,

$$d\cos\theta_1 = \lambda \quad \text{... (1)}$$

$$d\cos\theta_2 = 2\lambda \quad \text{... (2)}$$

From equation (1), we get,

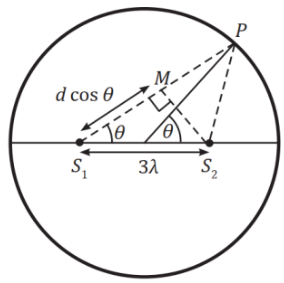
$$\cos\theta_1 = \frac{\lambda}{d} = \frac{\lambda}{3\lambda}$$

$$\theta_1 = \cos^{-1}(\frac{1}{3})$$

From equation (2), we get,

$$\cos\theta_2 = \frac{2\lambda}{d} = \frac{2\lambda}{3\lambda}$$

$$\theta_2 = \cos^{-1}(\frac{2}{3})$$




Therefore the angular position (in the first quadrant) on the wire for which constructive interference takes place are $\cos^{-1}(\frac{1}{3})$ and $\cos^{-1}(\frac{2}{3})$.