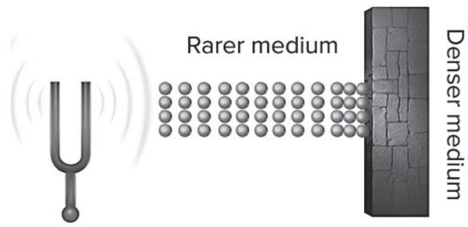
STANDING WAVE IN ORGAN PIPE AND BEATS

Reflection of Sound Waves

Reflection from denser medium

Consider a scenario where a sound wave traverses through a medium and undergoes reflection from a rigid wall, as depicted in the illustration. Immediately preceding the reflection, the particles of the medium adjacent to the wall become compressed, resembling a compression wave in terms of propagation. Following the reflection from the wall particles, the medium's particles rebound in a manner akin to their state just before the reflection. In simpler terms, after reflecting at a fixed end (displacement node), a compression pulse returns as a compression pulse, and a rarefaction pulse returns as a rarefaction pulse. Consequently, the denser medium, representing the fixed end, behaves like a displacement node.

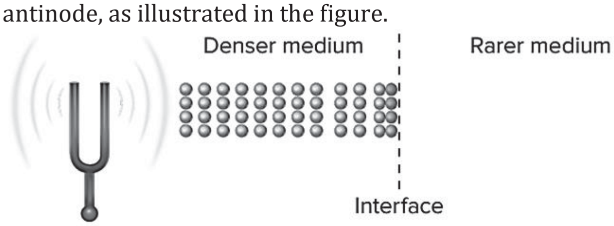


As the wave pulse retains its nature unchanged upon reflection from the denser medium, there is no introduction of any phase difference in the sound wave pulse after this reflection

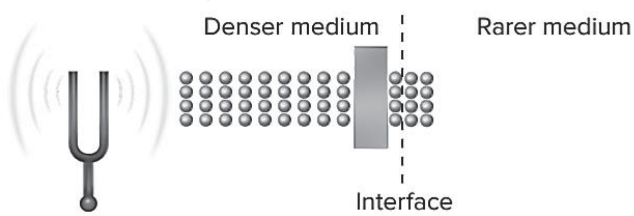
Reflection from rarer medium

Examine a sound wave advancing through a denser medium and undergoing reflection from a rarer medium. In this scenario, as the disturbance progresses through the medium, it reaches the interface precisely in the form of a compression at the moment of reflection, as depicted in the figure.

In this context, the interface acts akin to a free end. When the compression encounters the interface, the particles of the medium displace into the rarer medium, reflecting back into the initial medium as a rarefaction. Consequently, the rarer medium (representing the free end) behaves like a displacement antinode, as illustrated in the figure.



In summary, when a sound wave reflects from the rarer medium, the initial compression transforms into a rarefaction. Therefore, it can be concluded that the sound wave pulse undergoes a phase change of π after reflecting from the rarer medium.



Note **For the reflection of sound waves:**

1. In the denser medium:
- **(a)** A displacement node or pressure antinode will emerge at the interface.
- **(b)** A compression pulse will revert as a compression pulse, and a rarefaction pulse will revert as a rarefaction pulse.
- **(c)** The reflected pressure wave will undergo no phase change, indicating no inversion.

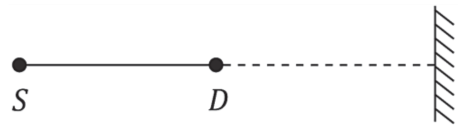
2. In the rarer medium:
- **(a)** A displacement antinode or pressure node will manifest at the interface.

(b) A compression pulse will return as a rarefaction pulse, and a rarefaction pulse will return as a compression pulse.

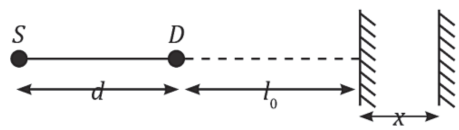
(c) The reflected pressure wave will undergo a phase change of π, signifying wave inversion.

3. The total phase difference between sound waves $\Delta\phi = \Delta\phi_x + \phi_0 + \Delta\phi_r$, where $\Delta\phi$ is the phase difference due to the path difference ϕ_0 0 is the initial phase difference or epoch, and $\Delta\phi_r$ is the phase difference due to the reflection of the sound waves.

Ex. A source of sound S and a detector D are placed at some distance from one another. A big cardboard is placed near the detector and perpendicular to the line SD as shown in the figure. It is gradually moved away, and it is found that the intensity changes from the maximum to the minimum as the board is moved through a distance of 20 cm. Find the frequency of the sound emitted. The velocity of sound in air is 336 ms^{-1}.



Sol. It's important to highlight that due to the interference between the sound wave originating from the source and the reflected sound wave from the cardboard, the detector registers both maximum and minimum intensities of sound waves as the cardboard progressively moves away.



In addressing this issue, considering that the sound wave reflects off the cardboard, it is reasonable to infer that the sound wave undergoes reflection from a denser medium, thus $\Delta\phi_r = 0$.
As the incident and reflected waves stem from the identical source, the initial phase (epoch) $\phi_0 = 0$
Hence, the overall phase difference is, $\Delta\phi = \Delta\phi_x$ Therefore, the phase difference arises exclusively due to the disparity in the path traveled.
Consider the distance between the source and the detector as d, and the initial separation between the detector and the cardboard as l_0. At this cardboard position, let the path difference between the incident and reflected waves be denoted as Δx_i. It is specified that due to this path difference, the detector registers the maximum intensity.
Now, let's assume the cardboard is displaced to a distance x from its original position. Consequently, for this cardboard location, the path difference between the incident wave and the reflected wave becomes: $\Delta x_f = \Delta x_i + 2x$ and due to this variation in path length, the detector registers the minimum intensity.
The intensity at the detector transitions from maximum to minimum due to the cumulative path difference. $(\Delta x_f - \Delta x_i)$, this path difference will be equal to $\frac{\lambda}{2}$.

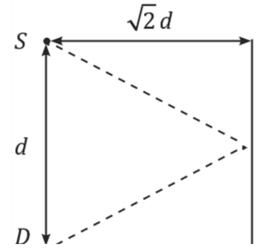
Therefore

$$\Delta x_f - \Delta x_i = \frac{\lambda}{2}$$
$$2x = \frac{\lambda}{2}$$
$$\lambda = 4x$$
$$\lambda = 4 \times 20 \text{ cm}$$
$$\lambda = 80 \text{ cm}$$
$$\lambda = \frac{4}{5} \text{ m}$$

Therefore, the frequency of the sound

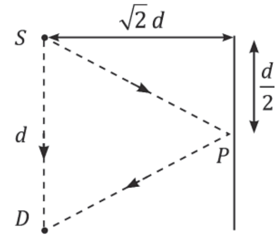
$$f = \frac{v}{\lambda} = \frac{336}{\left(\frac{4}{5}\right)} \text{Hz} \Rightarrow f = 84 \times 5 = 420\text{Hz}$$

Ex. A source S and a detector D are placed at a distance d apart. A big cardboard is placed at a distance $\sqrt{2d}$ from the source and the detector as shown in the figure. The source emits a wave of wavelength $\frac{d}{2}$ which is received by the detector after the reflection from the cardboard. It is found to be in phase with the direct wave received from the source. By what minimum distance should the cardboard be shifted away so that the reflected wave becomes out of phase with the direct wave?



Sol. There are two waves received by the detector.

1. One wave directly goes to the detector.
2. The other one goes to the detector after getting reflected at the cardboard as shown in the figure.

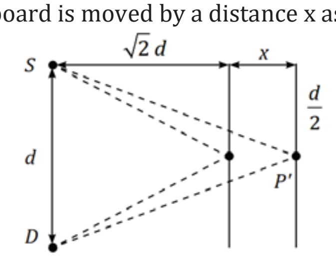


The path difference between these two waves is,

$$\Delta x_i = (SP + PD) - SD$$
$$\Delta x_i = 2(SP) - SD \text{ Since } SP = PD]$$
$$\Delta x_i = 2\sqrt{(\sqrt{2}d)^2 + (\frac{d}{2})^2} - d$$
$$\Delta x_i = 2\sqrt{2d^2 + \frac{d^2}{4}} - d$$
$$\Delta x_i = 2\sqrt{\frac{9d^2}{4}} - d$$
$$\Delta x_i = 2 \times \frac{3d}{2} - d$$
$$\Delta x_i = 2d$$

Given that, in this scenario, they are in phase upon superposition at the detector, the phase difference between them is zero for this specific path difference. Consequently, the detector will register the maximum intensity.

Now, suppose that the cardboard is moved by a distance x as shown in the figure.



For this case, the path difference between these two waves is,

$$\Delta x_f = (SP' + PD') - SD$$
$$\Delta x_f = 2(SP') - SD \text{ [Since } SP' = PD' \text{]}$$
$$\Delta x_f = 2\sqrt{(\sqrt{2}d + x)^2 + (\frac{d}{2})^2} - d$$
$$\Delta x_f = 2\sqrt{(\sqrt{2}d + x)^2 + \frac{d^2}{4}} - d$$

Since for this position of the cardboard, it is given that the waves will be out of phase when superposed at the detector, the phase difference between them will be π for this path difference. Hence, the minimum intensity will be detected at the detector. Therefore,

$$\Delta x_f - \Delta x_i = \frac{\lambda}{2}$$

Given that the wavelength of the sound wave is, $\lambda = \frac{d}{2}$ Thus,

$$\Delta x_f - \Delta x_i = \frac{d}{4}$$

By putting the values of Δx_f and Δx_i, we get,

$$[2\sqrt{(\sqrt{2}d + x)^2 + \frac{d^2}{4}} - d] - 2d = \frac{d}{4}$$
$$2\sqrt{(\sqrt{2}d + x)^2 + \frac{d^2}{4}} = \frac{d}{4} + 3d$$
$$2\sqrt{(\sqrt{2}d + x)^2 + \frac{d^2}{4}} = \frac{13d}{4}$$
$$\sqrt{(\sqrt{2}d + x)^2 + \frac{d^2}{4}} = \frac{13d}{8}$$

Squaring both the sides of the equation, we get,

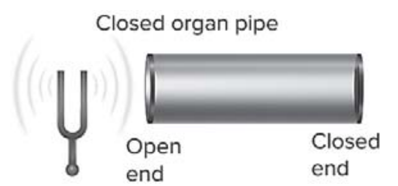
$$(\sqrt{2}d + x)^2 + \frac{d^2}{4} = (\frac{13d}{8})^2$$
$$(\sqrt{2}d + x)^2 = \frac{169d^2}{64} - \frac{d^2}{4}$$
$$(\sqrt{2}d + x)^2 = (\frac{169-16}{64})d^2$$
$$(\sqrt{2}d + x)^2 = \frac{153}{64}d^2$$
$$\sqrt{2}d + x = \sqrt{\frac{153}{64}d^2}$$
$$1.414d + x = 1.546d$$
$$x = 1.546d - 1.414d$$
$$x = 0.132d$$

Therefore, the cardboard should be shifted away by a distance 0.132d for which the reflected wave becomes out of phase with the direct wave.

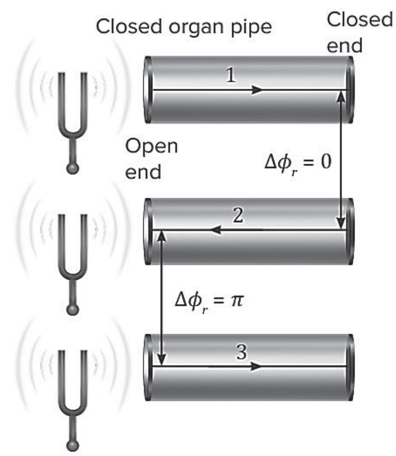
Reflection of Sound Waves

Closed organ pipe

Contemplate a closed organ pipe with one end sealed and the other end open. A source of sound wave is positioned at the open end, as illustrated in the figure.



At the closed end of the organ pipe, the wave exhibits a displacement node, resulting in a pressure antinode. As established, no phase change accompanies the reflected wave when a displacement node or pressure antinode occurs at the point of reflection. Conversely, it's crucial to note that a phase change of π is associated with the reflected wave when a displacement antinode or pressure node occurs at the point of reflection. Consequently, for the incident wav e reflecting from the closed end, the phase difference due to reflection is $(\Delta\phi_r)_1 = 0$, and for the reflec ted wave that again gets reflected from the open end, the phase change due to reflection is, $(\Delta\phi_r)_2 = \pi$



Hence, the cumulative phase difference following two reflections of the wave is $\Delta\phi_r = (\Delta\phi_r)_1 + (\Delta\phi_r)_2 = 0 + \pi$

For an organ pipe with a length of L, the path difference between the incident wave and the reflected wave at the open end is 2L.

We are aware that the overall phase difference for a sound wave is:

$\Delta\phi = \Delta\phi_x + \phi_0 + \Delta\phi_r$

Here, the epoch, ϕ_0, is zero since the source is same for all the waves.

The phase difference originated due to path difference is $\Delta\phi_x = \frac{2\pi}{\lambda}(2L)$

Now, let's contemplate two waves: one emitted directly from the source and the other having undergone double reflection, traversing a path length of 2L. If they undergo constructive interference, then:

$$\Delta\phi = 2n\pi$$
$$\frac{2\pi}{\lambda}(2L) + \pi = 2n\pi$$
$$\frac{4L}{\lambda} + 1 = 2n$$

$$\frac{4L}{\lambda} = 2n - 1$$

$$L = (2n - 1)\frac{\lambda}{4} \quad \text{... (1)}$$

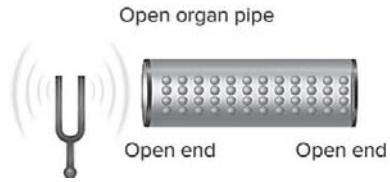
Since we know that $\lambda = \frac{v}{f}$, from the above equation, we get,

$$L = (2n - 1)\frac{v}{4f}$$

$$f = (2n - 1)\frac{v}{4L} \quad \text{... (2)}$$

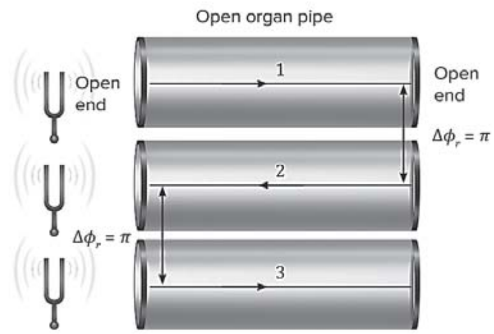
Open organ pipe

Examine an open organ pipe with both ends exposed. A source of sound wave is positioned at one of the open ends, as depicted in the figure.



In this instance, both ends are open. Consequently, the wave emitted from the source travels to the opposite end and reflects from the open end there. The reflected wave then moves back toward the source end and reflects once more from the open end. This leads to a compression pulse transforming into a rarefaction pulse, and vice versa, with each reflection incurring a phase difference of π.

The cumulative phase difference resulting from the wave reflecting as it travels from one open end to the other and returns to the initial open end is: $\Delta\phi_r = 2\pi$ This is equivalent to a full cycle, signifying that there is no phase alteration.



Given the length of the organ pipe as L, the path difference between the incident wave and the reflected wave at the initial open end is 2L, and the associated phase difference due to this path difference is: $\Delta\phi = \frac{2\pi}{\lambda}(2L)$

Now, let's examine two waves: one emitted directly from the source and the other having undergone double reflection, traversing a path length of 2L. If they undergo constructive interference, then:

$$\Delta\phi = 2n\pi$$

$$\frac{2\pi}{\lambda}(2L) = 2n\pi$$

$$\frac{2L}{\lambda} = n$$

$$L = \frac{n\lambda}{2} \quad \text{... (1)}$$

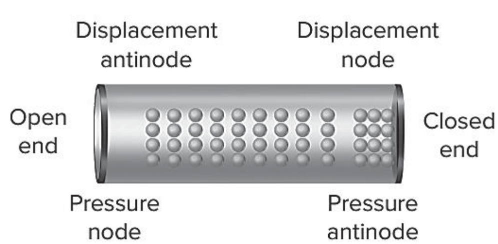
Since we know that $\lambda = \frac{v}{f}$, from the above equation, we get,

$$L = \frac{nv}{2f}$$

$$f = n\left(\frac{v}{2L}\right) \quad \text{... (2)}$$

Modes of Vibration of a Closed Organ Pipe

Following the principles of string waves, when a string is anchored at both ends, a standing wave arises between the two nodes. Conversely, when the string is fastened at one end, a standing wave occurs between a node and an antinode. Analogously, in a closed organ pipe, a standing wave is formed between one node and one antinode, while in an open organ pipe, a standing wave is established between two nodes (pressure nodes).

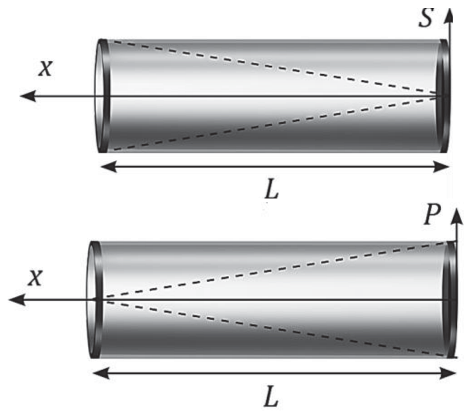


In the scenario of a closed organ pipe with a length of L, one end will feature a displacement antinode (or pressure node), while the closed end will exhibit a displacement node (or pressure antinode), as illustrated in the figure. The criteria for achieving a perfect standing wave between one node and one antinode, as derived in the closed organ pipe section, are $L = (2n-1)\frac{\lambda}{4}$ and $f = (2n-1)\frac{v}{4L}$ which in general can be expressed as,

$$L = \frac{n\lambda}{2} \pm \frac{\lambda}{4} \quad \dots (1)$$

$$f = (2n \pm 1)\frac{v}{4L} \quad \dots (2)$$

Fundamental mode: By substituting $n = 0$ in equation (1) and equation (2), we get $L = \frac{\lambda}{4}$ and $f = \frac{v}{4L}$.

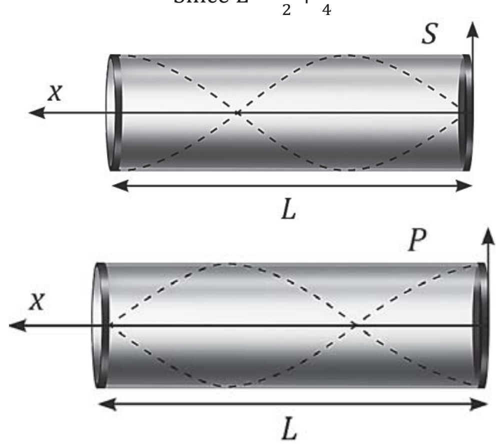


Since $L = \frac{\lambda}{4}$, there will be a half of one loop in between the open end and the closed end as shown in the figure Therefore, the fundamental frequency is, $f_0 = \frac{v}{4L}$

First overtone or third harmonic: By substituting $n = 1$ in equation (1) and equation (2), we get,

$$L = \frac{\lambda}{2} + \frac{\lambda}{4} \text{ and } f = 3\left(\frac{v}{4L}\right)$$

Since $L = \frac{\lambda}{2} + \frac{\lambda}{4}$



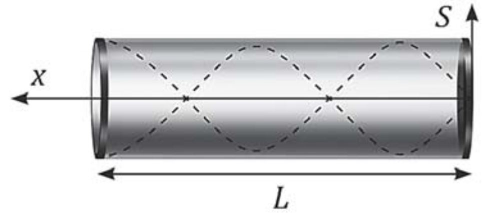
There will be one loop and a half of a loop in between the open end and the closed end as shown in the figure.

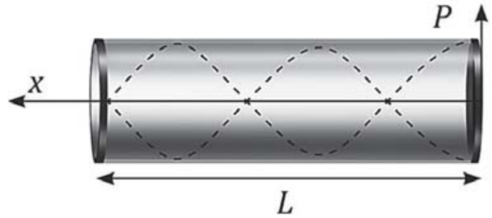
The frequency for this case is represented as, $f_1 = 3\left(\frac{v}{4L}\right) = 3f_0$

Second overtone or fifth harmonic: By substituting $n = 2$ in equation (1) and equation (2), we get,

$$L = 2\left(\frac{\lambda}{2}\right) + \frac{\lambda}{4} \text{ and } f = 5\left(\frac{v}{4L}\right)$$

Since $L = 2\left(\frac{\lambda}{2}\right) + \frac{\lambda}{4}$,





There will be two loops and a half of one loop in between the open end and the closed end as shown in the figure.

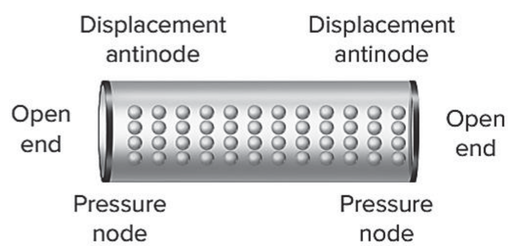
The frequency for this case is represented as, $f_2 = 5(\frac{v}{4L}) = 5f_0$

n^{th} overtone or $(2n+1)^{th}$ harmonic: Here, $L = \frac{n\lambda}{2} + \frac{\lambda}{4}$ And hence the frequency of this cases is

$$f_n = (2n+1)(\frac{v}{4L}) = (2n+1)f_0 \text{ where } n = 0,1,2,3,\ldots$$

Modes of Vibration of an Open Organ Pipe

In the context of an open organ pipe with a length of L, displacement antinodes (or pressure nodes) will be present at both ends, as depicted in the figure.



The requirements for the formation of an ideal standing wave between two nodes with a length of L are as follows:

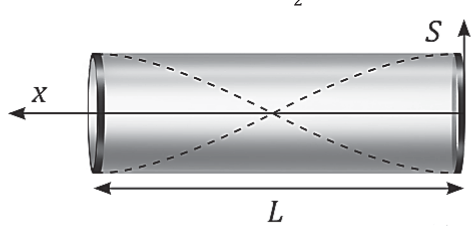
$$L = \frac{n\lambda}{2} \quad \ldots (1)$$

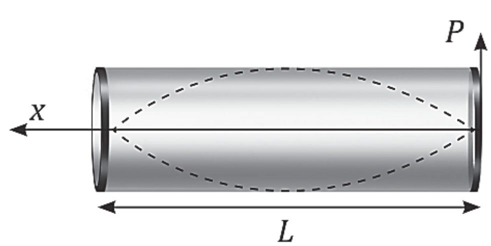
$$f = n(\frac{v}{2L}) \quad \ldots (2)$$

Fundamental mode or first harmonic: By substituting n = 1 in equation (1) and equation (2), we get,

$$L = \frac{\lambda}{2} \text{ and } f = \frac{v}{2L}.$$

Since $L = \frac{\lambda}{2}$,



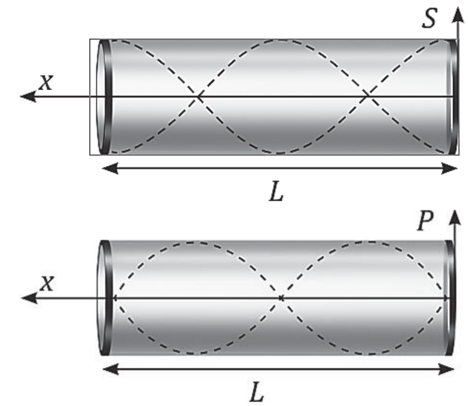


In terms of pressure, there will be a single loop between the open ends, while in terms of displacement, there will be two half loops between the open ends, as depicted in the figure.
Hence, the frequency of the fundamental mode is, $f_0 = \frac{v}{2L}$

Second harmonic or first overtone: By substituting n = 2 in equation (1) and equation (2), we get,

$$L = 2\left(\frac{\lambda}{2}\right) \text{ and } f = 2\left(\frac{v}{2L}\right).$$

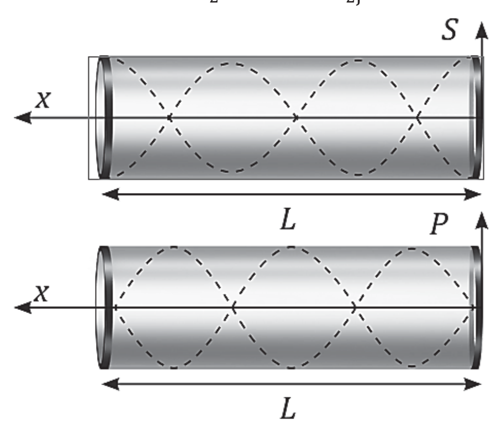
$$\text{Since } L = 2\left(\frac{\lambda}{2}\right)$$



From the perspective of pressure, there will be two loops between the open ends, resulting in one node between them.
In terms of displacement, there will be one loop and two half loops between the fixed ends. Consequently, from the displacement standpoint, there will be two nodes between the open ends, as illustrated in the figure.
The frequency for this case is represented as, $f_1 = 2(\frac{v}{2L}) = 2f_0$

Third harmonic or second overtone: By putting n = 3 in equation (3) and equation (4), we get,

$$L = 3\left(\frac{\lambda}{2}\right) \text{ and } f = 3\left(\frac{v}{2J}\right)$$



In this context, in terms of pressure, there are three loops between the open ends, resulting in two nodes between them. From the displacement perspective, there are two loops and two half loops between the fixed ends, leading to three nodes between the fixed ends, as depicted in the figure.
The frequency for this scenario is denoted as: $f_2 = 3(\frac{v}{2L}) = 3f_0$

$(n+1)^{th}$ harmonic or n^{th} overtone: Here, $L = (n+1)(\frac{\lambda}{2})$ and hence, the frequency of this case is, $f_n = (n+1)(\frac{v}{2L}) = (n+1)f_0$

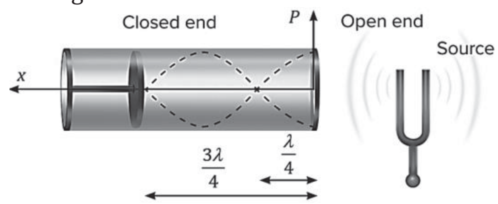
Note In summary of the entire discussion, the following conclusions can be drawn:

1. A closed organ pipe (open at one end) corresponds to the scenario of a string fixed at one end.

2. An open organ pipe (open at both ends) corresponds to the scenario of a string fixed at both ends.

Ex. An air column is constructed by fitting a movable piston in a long cylindrical tube. Longitudinal waves are sent in the tube by a tuning fork of frequency 416 Hz. How far from the open end should the piston be so that the air column in the tube vibrate in its first overtone? The speed of sound in air is 333 ms^{-1}.

Sol. Since an air column is constructed by fitting a movable piston in a long cylindrical tube, this is nothing but the case of a closed organ pipe. Therefore, the first overtone will actually be the third harmonic as shown in the figure.



For the third harmonic, the length should be $L = 3(\frac{\lambda}{4})$ and the wavelength of the sound wave is, $\lambda = \frac{v}{f_1}$

Therefore $$L = \frac{3\lambda}{4}$$

$$L = \frac{3}{4}(\frac{v}{f_1})$$

Given, $$v = 333ms^{-1} \text{ and } f_1 = 416 \text{ Hz}$$

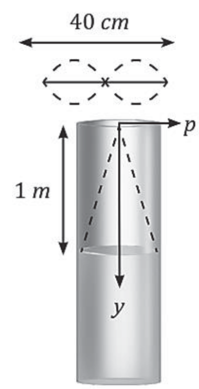
$$L = \frac{3}{4} \times \frac{333}{416}$$

$$L \approx 0.6m$$

$$L \approx 60 \text{ cm}$$

Therefore, the piston should be 60 cm away from the open end so that the air column in the tube may vibrate in its first overtone.

Ex. Consider the situation shown in the figure. The wire, which is fixed at both the ends, has a mass of 4 g, oscillates in its second harmonic, and sets the air column in the tube into vibrations in its fundamental mode. Assuming that the speed of sound in air is 340 ms^{-1}, find the tension in the wire.



Sol. Given,

The length of the wire is, L = 40 cm = 0.4 m

The mass of the wire is, m = 4 g = 4×10^{-3} kg

The speed of sound in air is, $v_{air} = 340$ ms^{-1}

Therefore, the linear mass density of the wire is given by,

$$\mu = \frac{m}{L}$$

$$\mu = \frac{4\times10^{-3}}{0.4}$$

$$\mu = 10^{-2}Kgm^{-1}$$

If the tension in the string is F, then the velocity of the string will be,

$$v = \sqrt{\frac{F}{\mu}}$$

$$v = \sqrt{\frac{F}{10^{-2}}}$$

$$v = 10\sqrt{F}$$

Now, it is given that the wire fixed at both the ends oscillates in its second harmonic. Therefore, if f_0 is the fundamental frequency of the wire fixed at both the ends, then according to the question, the frequency of the wire is, $f = 2f_0$

Therefore, $$f = 2(\frac{v}{2L})$$

[Since $f_0 = \frac{v}{2L}$ for a string fixed at both the ends]

$$f = \frac{v}{L}$$

$$f = \frac{10\sqrt{F}}{0.4}$$

$$f = 25\sqrt{F}$$

This frequency behaves as the source for the air column in the half-filled cylinder. Since the cylinder is half-filled, it can be assumed as a closed organ pipe. It is also given that the air column vibrates in its fundamental mode.

Therefore, the frequency of the air column is, $(f_0)_{air} = \frac{air}{4L'}$

Given, The length of the air column is, L' = 1 m

Therefore,
$$f = 25\sqrt{F} = \frac{V_{air}}{4L'}$$
$$25\sqrt{F} = \frac{340}{4\times1}$$
$$\sqrt{F} = \frac{340}{25\times4\times1}$$
$$\sqrt{F} = \frac{340}{100}$$
$$\sqrt{F} = 3.4$$
$$F = (3.4)^2$$
$$F = 11.56\ \text{N}$$

Therefore, the tension in the wire is 11.56 N.

Ex. The vibration of air in an open organ pipe of length 60 cm is represented by the equation: $p = 4\sin(\frac{\pi x}{15})\cos(96\pi t)$ where x, p, and t are in cm, Nm^{-2}, and s, respectively.

(a) What is the excess pressure amplitude at x = 5 cm?

(b) Where are the pressure nodes located along the pipe?

Sol. By comparing the given equation, $p = 4\sin(\frac{\pi x}{15})\cos(96\pi t)$ with the general equation of standing waves, $p = 2p_0\sin(kx)\cos(\omega t)$, we get,

$k = \frac{\pi}{15}\text{cm}^{-1}$ (The unit is cm^{-1} because x is given in cm)
$$\omega = 96\pi s^{-1}$$
$$2p_0 = 4$$
$$p_0 = 2Nm^{-2}$$

(a) The excess pressure amplitude at any position x is, $p(x) = 4\sin(\frac{\pi x}{15})$

Therefore, at x = 5 cm, the excess pressure amplitude becomes,
$$p(x = 5) = 4\sin(\frac{\pi\times5}{15})$$
$$p(x = 5) = 4\sin(\frac{\pi}{3})$$
$$p(x = 5) = 4\times\frac{\sqrt{3}}{2}$$
$$p(x = 5) = 2\sqrt{3}Nm^{-2}$$

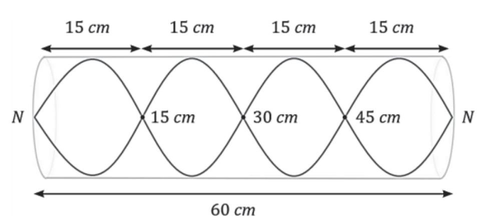
(b) Here, the concerned organ pipe is an open organ pipe. Therefore, at both the ends of the organ pipe, a displacement antinode or a pressure node is formed.

Given, The length of the open organ pipe is, L = 60 cm

Now,
$$k = \frac{\pi}{15}$$
$$\frac{2\pi}{\lambda} = \frac{\pi}{15}$$
$$\frac{\lambda}{2} = 15\ \text{cm}$$

We know that the distance between any two consecutive nodes is $\frac{\lambda}{2}$Since the length of the open organ pipe is, L = 60 cm and $\frac{\lambda}{2} = 15$ cm , there will be a node after every 15 cm.

Thus, there are three nodes or four loops in between the open ends of the organ pipe. Therefore, it is the fourth harmonic or third overtone.



Hence, the pressure nodes are located at 0 cm, 15 cm, 30 cm, 45 cm, and 60 cm.

Ex. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. The end corrections may be neglected. Let P0 denote the mean pressure at any point in the pipe and ΔP_0 be the maximum amplitude of pressure variation.

(a) Find the length (L) of the air column.

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressures at the open end of the pipe?

(d) What are the maximum and minimum pressures at the closed end of the pipe?

Sol. Since the pipe is closed at one end, it behaves as a closed organ pipe. Also, we know that for a closed organ pipe, $L = (2n - 1)\frac{\lambda}{4}$ and $f = (2n - 1)\frac{v}{4L}$ Hence, for the second overtone $(n = 3)$, the length of the air column is $L = \frac{5\lambda}{4}$ and the frequency is $f_2 = 5(\frac{v}{4L})$.

Given,

The frequency of the tuning fork that sets the air column to vibrate in its second overtone mode is, $f_2 = 440$ Hz

The speed of the sound in air is, $v = 330\ ms^{-1}$

Therefore, the wavelength is,

$$\lambda = \frac{v}{f}$$
$$\lambda = \frac{330}{440}m$$
$$\lambda = \frac{3}{4}m$$

(a) The length of the air column is,

$$L = 5(\frac{\lambda}{4})$$
$$L = \frac{5}{4} \times \frac{3}{4}$$
$$L = \frac{15}{16}m$$

(b) Since it is a closed organ pipe, there should be a pressure node at the open end. Suppose that the closed organ pipe is placed in such a way that the open end is at $x = 0$. The equation of pressure should be,

$p = \Delta p_0 \sin(kx)\cos(\omega t)$, where $\Delta p_0 \sin(kx)$ is the amplitude of pressure.

Now, the angular wave number is given by

$$k = \frac{2\pi}{\lambda}$$
$$k = \frac{2\pi}{(\frac{3}{4})} \text{ (Since } \lambda = \frac{3}{4}m)$$
$$k = \frac{8\pi}{3}m^{-1}$$

We also know that the length of the air column is, $L = \frac{15}{16}m$

Therefore, at the middle of the air column, i.e., at $x = \frac{L}{2} = \frac{15}{32}m$ the amplitude of the pressure becomes,

$$p = \Delta p_0 \sin(kx)$$
$$p = \Delta p_0 \sin(\frac{8\pi}{3} \times \frac{15}{32})$$
$$p = \Delta p_0 \sin(\frac{5\pi}{4})$$
$$p = \Delta p_0 \sin(\pi + \frac{\pi}{4})$$
$$p = -\Delta p_0 \sin(\frac{\pi}{4})$$
$$p = -\frac{\Delta p_0}{\sqrt{2}}$$

Since amplitude cannot be negative, the amplitude of the pressure will is, $p = \frac{\Delta p_0}{\sqrt{2}}$

(c) At the open end (pressure node), there will not be any fluctuation in the pressure. So, the amplitude of the pressure is constant at the open end.

(d) At the closed end, the amplitude of the pressure is,

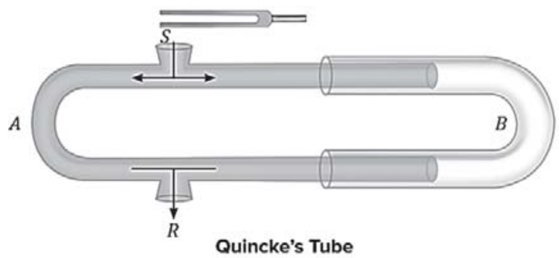
$$p = \Delta p_0 \sin(kL)$$
$$p = \Delta p_0 \sin(\frac{8\pi}{3} \times \frac{15}{16})$$

$$p = \Delta p_0 \sin(\tfrac{5\pi}{2})$$
$$p = \Delta p_0 \sin(2\pi + \tfrac{\pi}{2})$$
$$p = \Delta p_0 \sin(\tfrac{\pi}{2})$$
$$p = \Delta p_0$$

Therefore, the maximum amplitude is $p_{max} = (p_0 + \Delta p_0)$ and the minimum amplitude is $p_{min} = (p_0 - \Delta p_0)$, where p_0 is the average or mean value of the pressure.

Quincke's Experiment

The Quincke's tube comprises two U-tubes, labeled A and B. Tube B is movable, allowing it to slide along tube A, as depicted in the figure.



Quincke's Tube

Tube A is equipped with two openings: (i) S, serving as the source generating sound waves, and (ii) R, designated as the receiver or detector. The sound waves generated by the source split within tube A, and the resulting two beams of sound waves combine at the receiver. Due to their common origin, these two beams are coherent. Consequently, interference occurs at the receiver or detector. The overall phase difference between these two sound wave beams is then determined by.

$$\phi = \phi_x + \phi_0 + \phi_r$$
$$\phi = \phi_x \quad \text{... (1)}$$

[Since there is no reflection of the beams of sound waves, they just travel in two different directions inside the tubes. Also, they are coherent, and ϕ_0 and ϕ_r are zero.]

Therefore, the total phase difference is only due to the path difference between the beams. We know that for interference phenomenon,

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$
$$I_r = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

The respective path differences for the maxima and the minima are,

$$\Delta x = n\lambda$$
$$\Delta x = (2n \pm 1)\frac{\lambda}{2}$$

If tube B is displaced by a distance x from its initial position, the additional distance covered by the sound wave beam following that route is 2x. Consequently, the supplementary path difference between the two beams becomes $\Delta x_{extra} = 2x$ Adjusting the position of tube B on either side induces a path difference resulting in a phase difference. This phase difference, in turn, can lead to varying intensities, manifesting as either maximum or minimum intensity. In essence, it has the capability to generate constructive or destructive interference based on the specific phase difference.

Ex. In a Quincke's experiment, the sound intensity has a minimum value I at a particular position. As the sliding tube is pulled out by a distance of 16.5 mm, the intensity increases to a maximum of 9*I*. Take the speed of sound in air to be 330 ms^{-1}.

(a) Find the frequency of the sound source.

(b) Find the ratio of the amplitudes of the two waves arriving at the detector, assuming that it does not change much between the positions of minimum intensity and maximum intensity.

Sol. Given,

$$I_{min} = I$$
$$I_{max} = 9I$$

The speed of sound in air is, v = 330 ms^{-1}

The sliding tube is pulled out by a distance x = 16.5 mm

(a) The extra path difference suffered by one of the two interfering beams is,

$$\Delta x_{extra} = 2x = 33mm = 33 \times 10^{-3}m$$

Because of this extra path difference, the minima gets converted into the maxima and we know that the distance between the consecutive maxima and minima is $\frac{\lambda}{2}$ Therefore,

$$\Delta x_{extra} = \frac{\lambda}{2}$$
$$\Delta x_{extra} = \frac{v}{2f}$$
$$f = \frac{v}{2\Delta x_{extra}}$$
$$f = \frac{330}{2\times 33\times 10^{-3}}$$
$$f = \frac{10\times 10^{3}}{2}$$
$$f = 5\times 10^{3}\,Hz$$
$$f = 5KHz$$

Therefore, the frequency of the source is 5KHz

(b) We know that the resultant intensity of two interfering sound waves is,

$$I_r = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

Now, for maximum intensity, $\cos\phi = +1$ and for minimum intensity, $\cos\phi = -1$. Therefore,

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$$
$$\frac{9I}{I} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$$
$$\frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = (\frac{3}{1})^2$$
$$\frac{(\sqrt{I_1}+\sqrt{I_2})}{(\sqrt{I_1}-\sqrt{I_2})} = (\frac{3}{1})$$

Now by applying componendo-dividendo rule, we get,

$$\frac{2\sqrt{I_1}}{2\sqrt{I_2}} = \frac{4}{2}$$
$$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{2}{1}$$
$$\frac{A_1}{A_2} = \frac{2}{1}$$

Therefore, the ratio of the amplitudes of the two waves arriving at the detector is 2:1.