Chapter 22

Sound

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Introduction to sound

A three-dimensional mechanical wave, known as a sound wave, involves the oscillation of particles in the medium along the direction of wave propagation, classifying it as a longitudinal wave. Its origin stems from vibrating sources such as guitar strings, human vocal cords, tuning fork prongs, or loudspeaker diaphragms.

The transmission of all mechanical waves relies on a medium possessing the qualities of inertia and elasticity. Since sound waves fall under the category of mechanical waves, they too require a medium, similar to other waves.

The progression of sound waves in any medium occurs through periodic compressions and rarefactions of pressure, generated by the vibrating source.

Condition for a longitudinal wave

The particles within a medium vibrate in alignment with the wave's propagation, resulting in parallel wave velocity and particle velocity. Due to their parallel nature, the cross product between these velocities is zero. In contrast, transverse waves exhibit a zero dot product between these two velocities.

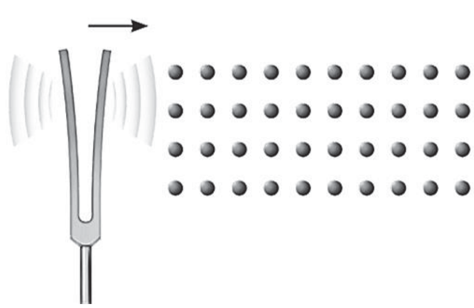
$$\vec{v}_w \parallel \vec{v}_p$$

$$\vec{v}_w \times \vec{v}_p = 0$$

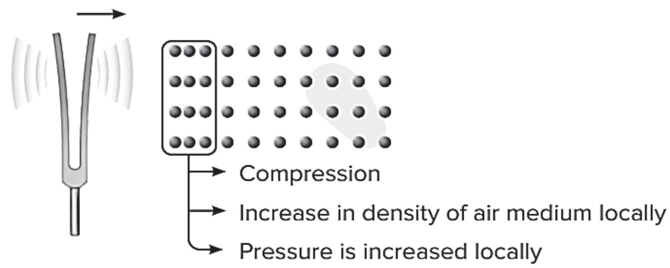
Propagation of Sound Wave

Imagine a tuning fork generating sound waves. Prior to the prongs of the tuning fork vibrating, the medium remains undisturbed, as depicted in the figure.

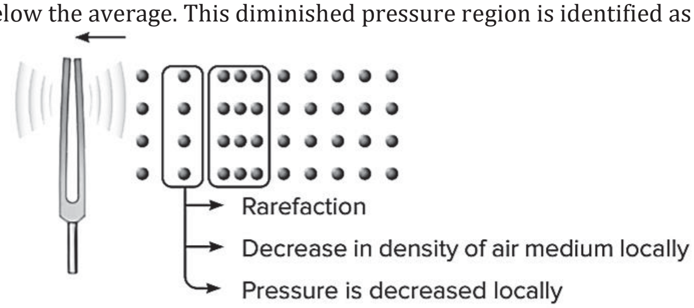
- As the prong moves outward to the right, it induces a disturbance in the medium, compressing the air ahead of it and leading to a slight increase in pressure.



- The area of heightened pressure is referred to as a compression pulse, and it propagates away from the prong at the speed of sound.



- ➢ Following the creation of the compression pulse, the prong then reverses its motion, moving inward. This action pulls away some air from the area in front of it, resulting in a slight decrease in pressure below the average. This diminished pressure region is identified as a rarefaction pulse.



Note:

- ➢ In the case of Simple Harmonic Motion (SHM) in the vibrating prongs, the particles in the nearby layer of the medium also undergo simple harmonic motion, specifically in the longitudinal direction.
- ➢ The progression of compression and rarefaction regions facilitates the transmission of sound waves throughout the medium.
- ➢ Maximum and minimum pressure variations occur in the compression and rarefaction regions, respectively.

Equation of displacement for the particle

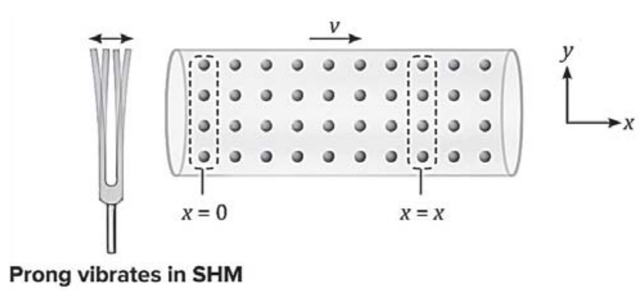
A sound wave essentially comprises variations in the existing parameters of the medium. These parameters include:

1. Position of the particle
2. Localized pressure

As sound travels through a medium, the particles of the medium undergo simple harmonic motion in the direction of wave propagation, leading to changes in their positions. This phenomenon can be expressed as follows.

$$S = S_0 \sin(kx - \omega t + \phi)$$

In this context, S denotes the immediate displacement of the x^{th} particle within the medium.



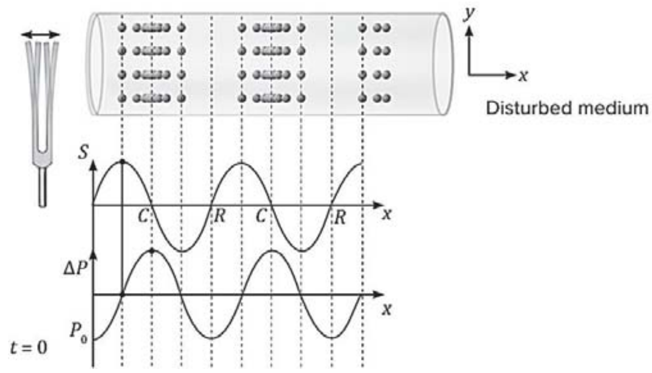
Variation of pressure during propagation of sound

Let's contemplate a particle positioned at x in a medium. When the source, represented by the prongs of the tuning fork, undergoes Simple Harmonic

Motion (SHM), it induces compressions and rarefactions within the medium. This indicates that the particles of the medium also undergo SHM in the direction of wave propagation. The displacement of the particle at position x, initially starting with a phase of zero, can be expressed as follows:

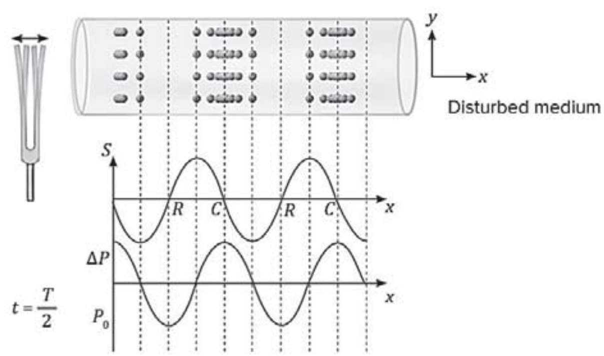
$$S = S_0 \sin(kx - \omega t) \quad \dots (1)$$

Substituting $t = 0$ into equation (1), we obtain $S = S_0 \sin kx$, which takes the form of a sinusoidal curve. Here, S represents the excess displacement at the xth position (Δx) of the medium particles. Analyzing the provided graph reveals that in regions where S is at its maximum, the corresponding excess pressure (ΔP) is at a minimum, indicating the presence of rarefactions. Conversely, in regions where S is at its minimum, the corresponding excess pressure (ΔP) reaches its maximum, signifying the occurrence of compressions.



The term "excess pressure" (ΔP) indicates that the pressure at a specific point exceeds the average pressure (P_0) of the medium.

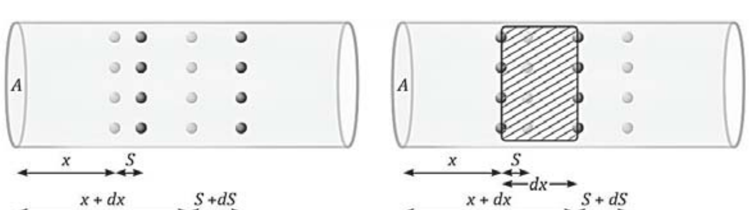
At $t = \frac{T}{2}$ During half of the time period, the sine curve experiences a phase shift of π, as illustrated in the figure. This illustrates that when S is at its maximum, the corresponding excess pressure (ΔP) is at a minimum, and conversely, when S is at its minimum, the corresponding excess pressure (ΔP) reaches its maximum.



Variation of Excess Pressure in the Gas Medium

As a sound wave travels through a medium, the immediate displacement of a particle at the x^{th} position is described by,

$$S = S_0 \sin(\omega t - kx) = S_0 \sin[\omega(t - \frac{x}{v})]$$



Where S represents the instantaneous displacement of the particle.

Let's examine particles located at positions x and x + dx within the medium. As the wave progresses, suppose the displacements of the particles at x and x + dx are S and S + dS, respectively. The volume between two particles separated by dx is denoted as V = Adx. The alteration in volume resulting from a minor displacement dS is given by,

$$dV = AdS$$
$$\frac{dV}{V} = \frac{A(dS)}{A(dx)}$$

From the definition of the bulk modulus,

$$B = \frac{-P}{\frac{dV}{V}}$$
$$P = -B(\frac{dV}{V})$$
$$P = -B(\frac{\partial S}{\partial x})$$
$$P = -BS_0\cos[\omega(t - \frac{x}{v})] \times (-\frac{\omega}{v})$$
$$P = \frac{BS_0\omega}{v}\cos[\omega(t - \frac{x}{v})]$$
$$P = BkS_0\cos[\omega(t - \frac{x}{v})]$$

On comparing this equation with $P = P_0\cos[\omega(t - \frac{x}{v})]$, we get,

$$P_0 = BkS_0$$

Where S_0 is excess displacement in the xth position (Δx) of the particles of the medium and P_0 is the excess pressure (ΔP).

Equation of sound propagation

The propagation equation for sound waves can be expressed in the following two forms:

$$S = S_0\sin(\omega t - kx)$$
$$P = P_0\cos(\omega t - kx) = P_0\sin(\omega t - kx + \frac{\pi}{2})$$

- $P_0 = BkS_0$ is the excess pressure amplitude.
- There is a phase difference of 90° between the displacement and pressure variation in the propagation of a sound wave.

Ex. The equation of a travelling sound wave along the x-axis is given as S = 6.0 sin (600t – 1.8 x), where S is measured in 10^{-5} m, t in seconds, and x in meters.

(a) Find the ratio of the displacement amplitude of the particles to the wavelength of the wave.

(b) Find the ratio of the velocity amplitude of the particles to the wave speed.

Sol. Given, $S_0 = 6 \times 10^{-5}m, \omega = 600s^{-1}, k = 1.8m^{-1}$(S is measured in$10^{-5}$)

$$\frac{S_0}{\lambda} = \frac{S_0}{(\frac{2\pi}{k})} = \frac{S_0k}{2\pi}$$
$$\frac{S_0}{\lambda} = \frac{(6\times10^{-5}\ m)(1.8m^{-1})}{2\pi}$$
$$\frac{S_0}{\lambda} = 1.7 \times 10^{-5}$$

(b)
$$\frac{(v_p)_{max}}{(v_p)_{wave}} = \frac{(S_0\omega)}{v}$$
$$\frac{(v_p)_{max}}{(v_p)_{wave}} = kS_0 = 1.8 \times 6 \times 10^{-5}$$
$$\frac{(v_p)_{max}}{(v_p)_{wave}} \approx 1.1 \times 10^{-4}$$

Ex. A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 1.0×10^{-3} Nm^{-2}, then find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4 × 105 Nm^{-2}.

Sol. Given $P_{max} - P_{min} = 1.0 \times 10^{-3}Nm^{-2}, B = 1.4 \times 10^5Nm^{-2}, \lambda = 40cm = 0.4m$

The maximum pressure and minimum pressure can be written as

$$P_{max} = P + P_0$$

$$P_{min} = P - P_0$$

Therefore, $$P_0 = \frac{P_{max} - P_{min}}{2}$$

$$P_0 = \frac{1.0 \times 10^{-3}}{2} = 5 \times 10^{-4} Nm^{-2}$$

$$P_0 = BkS_0$$

$$S_0 = \frac{P_0}{Bk}$$

$$S_0 = \frac{P_0}{B(\frac{2\pi}{\lambda})} = \frac{P_0 \lambda}{2\pi B}$$

$$S_0 = \frac{(5 \times 10^{-4} \times 0.4)}{(1.4 \times 10^5 \times 2\pi)}$$

$$S_0 = \frac{(2 \times 10^{-4})}{(1.4 \times 10^5 \times 2\pi)}$$

$$S_0 = \frac{10^{-9}}{(1.4 \times \pi)}$$

$$S_0 = 2.2 \times 10^{-10}\ m$$