DOPPLER EFFECT

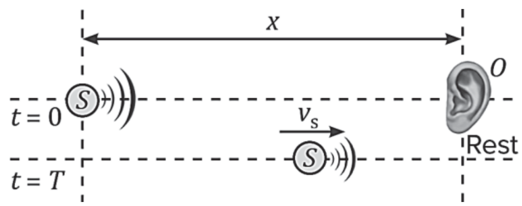
Introduction to Doppler Effect

A person standing on a railway platform experiences a louder sound from a train horn as it approaches and a fading sound as the train departs. This scenario exemplifies the Doppler Effect, which is an observer-dependent phenomenon. The Doppler Effect becomes noticeable only when there is a changing distance between the observer and the sound source over time. Consequently, an observer positioned at the center of a circular railway track, where the distance to a moving train remains constant, does not perceive any variations in the horn's loudness. The Doppler Effect refers to the apparent change in the frequency of a sound wave due to the motion of the source, the observer, or both.

Observer is Stationary and Source is Moving

Case I: Source is moving towards the observer

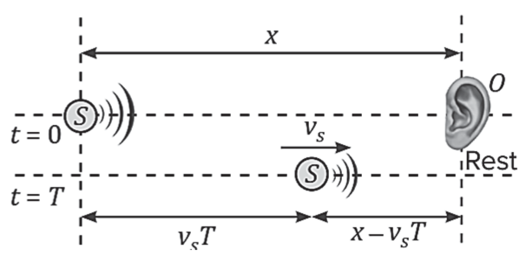
Consider an observer at rest while the source of sound waves moves toward the observer with a velocity v. At time t = 0, position the source at some initial location with a separation x between the source and the observer. By time t = T, the source has approached the observer, as illustrated in the figure.



Let T be the time interval between two consecutive pulses emitted by the source. Therefore, the frequency of the source is given by $f_0 = \frac{1}{T}$

To determine the frequency received by the observer, one can measure the time interval between consecutive pulses received by the observer. The inverse of this time interval yields the apparent frequency received by the observer. The calculation can be performed in the following manner:

Suppose the separation between the source and the observer is x at time t = 0. Consider the first pulse emitted by the source occurring at t = 0. Given that the time interval between consecutive pulses emitted by the source is T and the source's velocity is v_s, by the time the second pulse is emitted, the source has moved to a distance v_sT, as depicted in the figure.



Assuming the velocity of the sound wave in the medium is v, the observer receives the first pulse after $T_1 = \frac{x}{v}$ time. As the second pulse is emitted from the source after a time interval T from the first one, and at that moment, the source is at a distance of $(x - v_sT)$ from the observer, the time for the second pulse to reach the observer can be expressed as:

$$T_2 = T + \frac{x - v_sT}{v}$$

Hence, the time gap between the two successive pulses received by the observer can be expressed as:

$$T' = T_2 - T_1$$

$$T' = \left(T + \frac{x - v_sT}{v}\right) - \frac{x}{v}$$

$$T' = T + \frac{x}{v} - \frac{v_sT}{v} - \frac{x}{v}$$

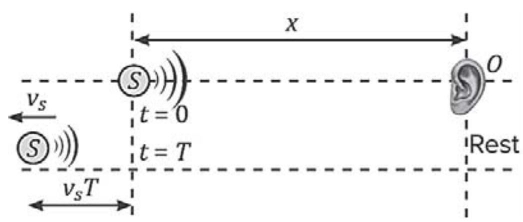
$$T' = (1 - \frac{v_S}{v})T$$
$$T' = (\frac{v - v_S}{v})T$$
$$\frac{1}{T'} = (\frac{v}{v - v_S})\frac{1}{T}$$
$$f' = (\frac{v}{v - v_S})f_0 \quad \text{... (1)}$$

Thus, the observer's perceived frequency of the sound is determined by equation (i). As observed in this expression, the denominator is smaller than the numerator, leading to $f' > f_0$. Consequently, the apparent frequency of the sound heard by the observer is higher than the original frequency. This aligns with our practical experience, such as when a train approaching a stationary passenger on a platform blows its horn, the pitch of the horn appears to increase.

Note A detailed examination of equation (i) reveals its independence from x. This condition holds true only when the observer and the source align in the same straight line. Additionally, as all velocities are assumed to be constant, the apparent frequency remains constant as well.

Case II: Source is moving away from the observer

Imagine the sound wave source is moving in the opposite direction from the observer with a velocity v_s. at t = 0, position the source at an initial location with a separation x between the source and the observer. By t = T, the source has moved away from the observer by a distance v_sT, as illustrated in the figure.



In this scenario, the initial pulse reaches the observer after a certain period $T_1 = \frac{x}{v}$ However, as the second pulse is emitted from the source T time after the first one, and the source is now at a distance (x + v_sT) from the observer, the time taken for the second pulse to reach the observer is expressed as:

$$T_2 = T + \frac{x + v_S T}{v}$$

Hence, the time gap between the two successive pulses received by the observer can be expressed as:

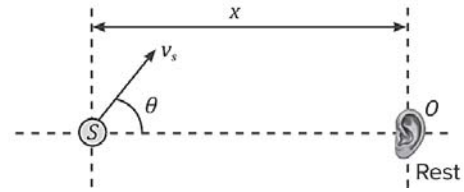
$$T' = T_2 - T_1$$
$$T' = (T + \frac{x + v_S T}{v}) - \frac{x}{v}$$
$$T' = T + \frac{x}{v} + \frac{v_S T}{v} - \frac{x}{v}$$
$$T' = (1 + \frac{v_S}{v})T$$
$$T' = (\frac{v + v_S}{v})T$$
$$\frac{1}{T'} = (\frac{v}{v + v_S})\frac{1}{T}$$
$$f' = (\frac{v}{v + v_S})f_0 \quad \text{... (2)}$$

Thus, in this instance, the observer's perceived frequency of the sound is determined by equation (2). As evident from this expression, the denominator is greater than the numerator, resulting in $f' < f0$, as anticipated.

Case III: Source is moving in an arbitrary direction

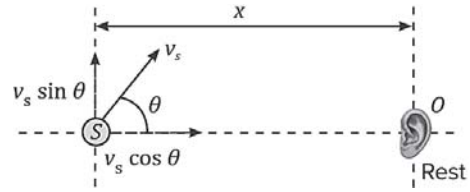
Consider the source is moving at a velocity v_s at an angle θ relative to the line connecting the source and the observer, as depicted in the figure.

From the preceding discussion, it is evident that only the velocity component along the line connecting the source and the observer contributes to the alteration in the separation distance over time. This alteration results in the apparent change in the source's frequency. Let's decompose the velocity vector into its components, as illustrated in the figure.

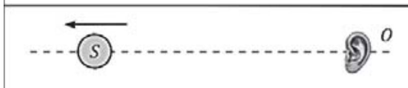
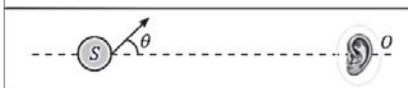


It is observed that $v_s \cos\theta$ lies along the line connecting the source and the observer. Notably, $v_s \cos\theta$ points towards the observer. Consequently, equation (1) can be applied, with v_s replaced by $v_s \cos\theta$. Thus, the apparent frequency perceived by the observer is expressed as:

$$f' = \frac{v}{v - v_s \cos\theta} f_0$$



Summary

	$f' = \frac{v}{v - v_s} f_0$
	$f' = \frac{v}{v + v_s} f_0$
	$f' = \frac{v}{v - v_s \cos\theta} f_0$

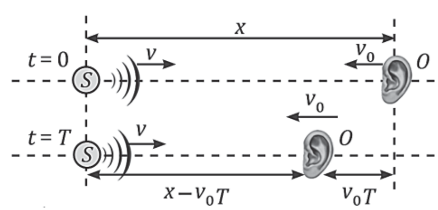
Source is Stationary and Observer is Moving

Case I: Observer is moving towards the source

Assume the source is stationary, and the observer is approaching the source of sound waves with a velocity v_0. At $t = 0$, position the observer at some initial location. By $t = T$, the observer has moved closer to the source, as illustrated in the figure.

In this context, T represents the time gap between two successive pulses emitted by the source. Hence, the frequency of the source can be expressed as:

$$f_0 = \frac{1}{T}$$



If the speed of the sound wave in the medium is v, the separation between two successive pulses is v_T. Now, considering that the sound wave (rather than the source) is traveling through the medium at a velocity v and the observer is also moving toward the source with a velocity v_0, the moment when the first pulse becomes audible to the observer can be determined by.

$$T_1 = \frac{x}{v + v_0}$$

Suppose the separation between the source and the observer is x at $t = 0$. Now, considering that the second pulse is emitted T time after the first one, and the observer is located at a distance $(x - v_0T)$ from the second pulse at that moment, the time when the observer can hear the second pulse is expressed as:

$$T_2 = T + \frac{x - v_0T}{v + v_0}$$

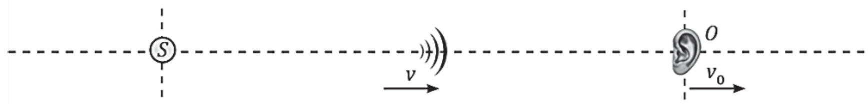
Hence, the time gap between the two successive pulses received by the observer can be expressed as:

$$T' = T_2 - T_1$$
$$T' = (T + \frac{x - v_0 T}{v + v_0}) - \frac{x}{v + v_0}$$
$$T' = T + \frac{x}{v + v_0} - \frac{v_0 T}{v + v_0} - \frac{x}{v + v_0}$$
$$T' = (1 - \frac{v_0}{v + v_0})T$$
$$T' = (\frac{v}{v + v_0})T$$
$$\frac{1}{T'} = (\frac{v + v_0}{v})\frac{1}{T}$$
$$f' = (\frac{v + v_0}{v})f_0$$
$$f' = (\frac{v + v_0}{v})f_0 \quad \text{... (1)}$$

Thus, in this scenario, the observer perceives the apparent frequency of the sound according to equation (1). Given that the numerator exceeds the denominator, $f' > f_0$.

Case II: Observer is moving away from the source

In this situation, the observer is moving away from the source at a velocity v0. Consequently, the direction of sound wave propagation aligns with the direction of the observer's motion.



Hence, v must be greater than v0; otherwise, the pulse will not be audible to the observer. Other than this condition, everything else remains consistent with case I. Consequently, the moments when the observer can hear the first and second pulses are:

$T_1 = \frac{x}{v - v_0}$ and, $T_2 = T + \frac{x + v_0 T}{v - v_0}$, respectively.

Hence, the time gap between the two successive pulses received by the observer can be expressed as:

$$T' = T_2 - T_1$$
$$T' = (T + \frac{x + v_0 T}{v - v_0}) - \frac{x}{v - v_0}$$
$$T' = T + \frac{x}{v - v_0} + \frac{v_0 T}{v - v_0} - \frac{x}{v - v_0}$$
$$T' = (1 + \frac{v_0}{v - v_0})T$$
$$T' = (\frac{v}{v - v_0})T$$
$$\frac{1}{T'} = (\frac{v - v_0}{v})\frac{1}{T}$$
$$f' = (\frac{v - v_0}{v})f_0 \quad \text{... (2)}$$

Thus, in this scenario, given that the numerator is smaller than the denominator, $f' < f_0$.

Case III: Observer is moving in an arbitrary direction

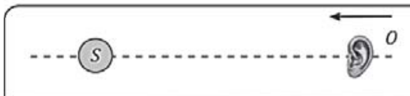
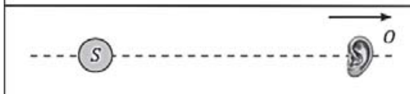
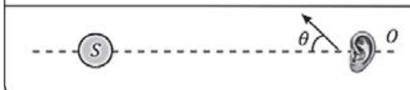
Imagine that the observer is moving with a velocity v_0 at an angle θ relative to the line connecting the source and the observer, as illustrated in the figure.

Given that the alteration in frequency is solely influenced by the component of velocity along the line connecting the source and the observer, let's break down the components of the velocity vector, as depicted in the figure.

As $v_0 \cos\theta$ aligns with the line connecting the source and the observer and points toward the source, equation (1) can be employed to determine the apparent frequency by substituting v_0 with $v_0 \cos\theta$. Consequently, the apparent frequency perceived by the observer is expressed as:

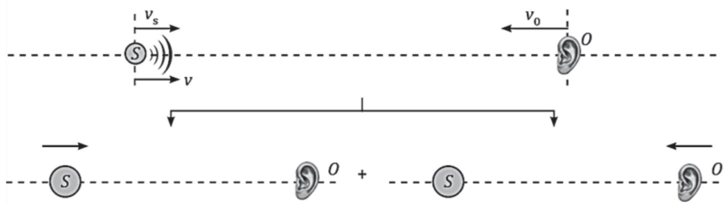
$$f' = \frac{v+v_0\cos\theta}{v} f_0$$

Summary

	$f' = \frac{v+v_0}{v} f_0$
	$f' = \frac{v-v_0}{v} f_0$
	$f' = \frac{v+v_0\cos\theta}{v} f_0$

Source and Observer Moving Towards Each Other

This scenario can be divided into two parts: one where the source is moving towards the observer with velocity v_s while the observer remains stationary, and the other where the observer is moving towards the source with velocity v_0 while the source is stationary.



In this scenario, where the source is approaching the stationary observer with a velocity v_s, the frequency expression is provided by:

$$f' = \frac{v}{v-v_s} f_0$$

In this instance, where the observer is advancing towards the stationary source with a velocity v_0, the frequency expression is provided by:

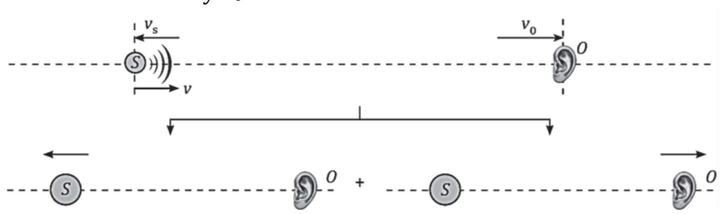
$$f' = \frac{v+v_0}{v} f_0$$

When these two situations combine, the frequency expression becomes:

$$f' = \frac{v+v_0}{v-v_s} f_0$$

Source and Observer Receding from Each Other

This scenario can be divided into two parts: one where the source is moving away from the stationary observer with velocity v_s, and the other where the observer is moving away from the stationary source with velocity v_0.



In this situation, where the source is moving away from the stationary observer with velocity v_s, the frequency expression is given by:

$$f' = \frac{v}{v+v_s} f_0$$

In this instance, where the observer is moving away from the stationary source with a velocity v_0, the frequency expression is given by:

$$f' = \frac{v-v_0}{v} f_0$$

When these two scenarios superpose, the expression of frequency becomes.

$$f' = \frac{v-v_0}{v+v_s} f_0$$

Note In all these cases, in the apparent frequency (f’) relation, the effect of observer velocity to sound velocity should be in the numerator and the effect of source velocity to sound velocity should be in the denominator.

Mnemonics: Remember the word ‘onsd’, which reflects the fact that in the apparent frequency formula, if the observer is moving, correction should be in the numerator in apparent frequency formula and if source is moving, correction should be in denominator.

Generalized formula

f_0 = Actual frequency

f' = Apparent frequency

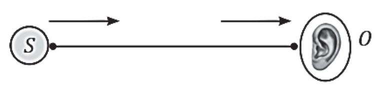
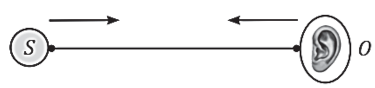
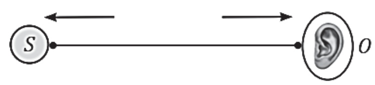
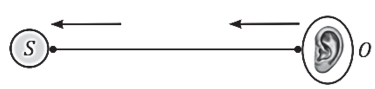
v = Speed of sound in the medium

v_0 = Speed of observer along the line joining the source and the observer

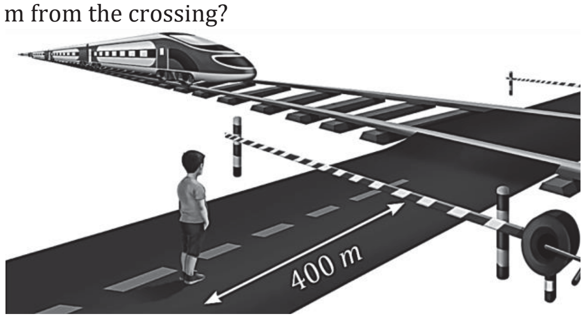
v_S = Speed of source along the line joining the source and the observer

The generalized formula of apparent frequency is given by,

$$f' = \left(\frac{v \pm v_0}{v \mp v_s}\right) f_0$$

	$f' = \frac{v - v_0}{v - v_s} f_0$
	$f' = \frac{v + v_0}{v - v_s} f_0$
	$f' = \frac{v - v_0}{v + v_s} f_0$
	$f' = \frac{v + v_0}{v + v_s} f_0$

Ex. A train approaching a railway crossing at a speed of 120 km h^{-1} blows a short whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 ms^{-1}. What is the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing?



Sol. Given

Frequency of the whistle of the train, $f_0 = 640$ Hz

Speed of the train, $v_s = 120\ \text{Kmh}^{-1} = (120 \times \frac{5}{18}) = \frac{100}{3}\ \text{ms}^{-1}$

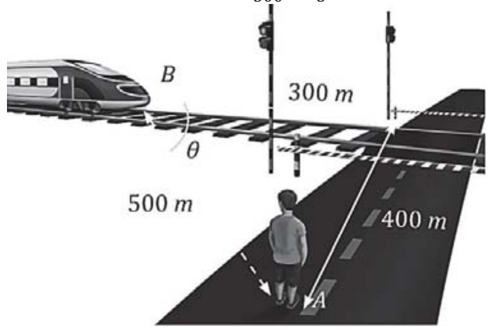
Speed of sound in air, $v = 340\text{ms}^{-1}$

Firstly, our task is to determine the velocity component aligned with the line connecting the source and the observer and ascertain the angle at which the train (source) is advancing towards the observer. Given that the road is perpendicular to the railway track, so by applying Pythagoras theorem, we get,

AB = 500 m Hence,

If θ is the angle of approach of the source to the observer, then,

$$\cos\theta = \frac{300}{500} = \frac{3}{5}$$



Now, since the observer is stationary and the source is moving at an angle θ, the formula for apparent frequency for this case is given by

$$f' = \frac{V}{V - v_s\cos\theta} f_0$$

$$f' = \frac{340}{340 - (\frac{100}{3}) \times (\frac{3}{5})} \times 640$$

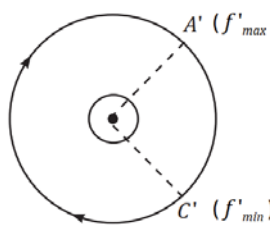
$$f' = \frac{340}{340 - 20} \times 640$$

$$f' = \frac{340}{320} \times 640$$

$$f' = 340 \times 2$$

$$f' = 680 \text{ Hz}$$

Note If we analyze the source's movement in a clockwise direction, points A' and C' witness an exchange in the intensity of sound waves—specifically, at A', the intensity reaches its maximum, while at C', the intensity diminishes to its minimum. Even in this scenario, moving from f'_{min} position (i.e., C′) to f'_{max} position (i.e., A'), the path taken by the source is along the major arc and tmin is 1.329 s.



Ex. A source of sound moving along the x-axis at a speed of 22 ms^{-1} is continuously emitting a sound of frequency 2.0 kHz travelling in air at a speed of 330 ms^{-1}. A listener, Q, stands on the y-axis at a distance of 330 m from the origin. At t = 0, the source crosses the origin P.

(a) When does the sound emitted from the source at P reach listener Q?

(b) For the sound emitted at P, what is the frequency heard by the listener?

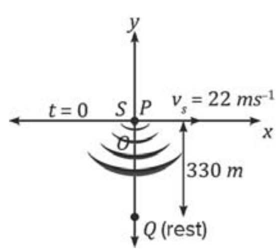
(c) Where is the source at the moment when the sound emitted at P is received by listener Q?

Sol. **(a)** Once the sound is emitted by the source at t = 0, it becomes independent of the source and the source keeps on moving.

The observer (Q) is at rest at a distance of 330 m below the point where the sound wave is emitted. Velocity of sound, v = 330 ms^{-1}.

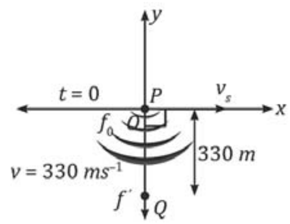
Thus, the time taken by sound to reach the listener at Q,

$$t = \frac{s}{v} = \frac{330\text{m}}{330\text{ms}^{-1}} = 1\text{s}$$



(b) The moment sound is emitted, the component of velocity of source along the line joining the source and the observer (i.e., along the y-direction) is zero, and the observer is at rest. Hence, the apparent frequency observed by the observer is the same as the original frequency of the source.

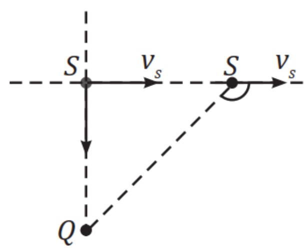
$$f' = f_0 = 2\text{kHz}$$



(c) Since the time taken by the sound to reach the listener at Q is t = 1 s, and the velocity of the source is, $v_s = 22\ ms^{-1}$,

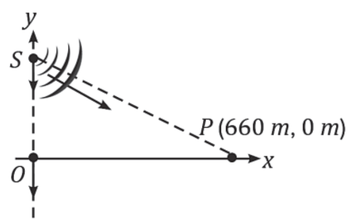
The horizontal displacement of the source during the interval of 1 s is,

$$S = v_s t = 22\ m$$

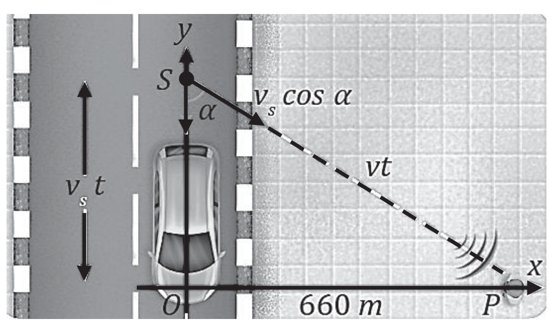


Ex. A source emitting sound at a frequency of 4,000 Hz is moving along the y-axis with a speed of 22 ms^{-1}. A listener is standing on the ground at position (660 m, 0 m). Find the frequency of the sound received by the listener at the instant the source crosses the origin. The speed of sound in air is 330 ms^{-1}.

Sol. At the moment when the source reaches the origin, the sound heard by the listener at that instant is the sound pulse that had been emitted earlier by the source. Thus, while the source was at some distance from the origin, the sound pulse that was emitted travelled to the listener as shown in the figure.



We know that the component of velocity of the source along the line joining the listener at the time when the first pulse is emitted is responsible for the apparent frequency heard.



Let t be the time taken by the source to reach the origin and the sound to reach the observer.

Thus, The distance covered by the sound pulse in time t, SP = vt

The distance covered by the source in time t, $SO = v_s t$

Given, $f_0 = 4{,}000$ Hz, $v_c = 22ms^{-1}$, $v = 330ms^{-1}$

Here, the observer is at rest and the source is approaching the observer. Thus, the apparent frequency is given by the following:

$$f' = \left(\frac{v}{v - v_s \cos\alpha}\right) f_0$$

$$f' = \left(\frac{v}{v - v_s\left(\frac{v_s}{v}\right)}\right) f_0 \left(\because \cos\alpha = \frac{v_s t}{vt} = \frac{v_s}{v}\right)$$

$$f' = \frac{v^2}{v^2 - v_s^2} f_0$$

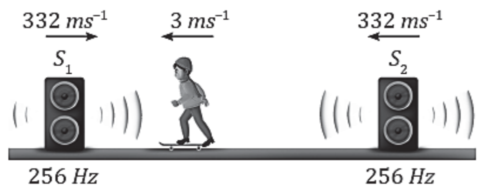
$$f' = \frac{(330)^2}{(330)^2 - (22)^2}(4000) = 4018 \text{ Hz}$$

Ex. Two identical sound sources vibrating at the same frequency of 256 Hz are kept fixed at some distance apart. A listener runs between the sources at a speed of 3.0 ms–1 so that he approaches one source and moves away from the other. Find the beat frequency observed by the listener. The speed of sound in air is 332 ms^{-1}.

Sol. The beat frequency (f_b) heard by the observer is the difference of the apparent frequency heard corresponding to the sound pulse emitted by both the sources.

$$f_b = |f_1' - f_2'|$$

Where f'_1 and f'_2 are the apparent frequencies for the observer corresponding to sources S_1 and S_2. The original frequencies of both the sources are the same, i.e., $f_0 = 256$ Hz



For source S_1, the observer is approaching it with a velocity of 3 ms^{-1}, and for source S_2, the observer is receding away from it with a velocity of 3 ms^{-1} (the source is at rest).

$$\left(\because f' = \left(\frac{v \pm v_0}{v \mp v_s}\right) f_0\right)$$

$$f_1' = \left(\frac{v + v_0}{v}\right) f_0$$

$$f_2' = \left(\frac{v - v_0}{v}\right) f_0$$

Note: If the observer is moving towards the source, then we take a positive sign for v_0, and if the observer is moving away from the source, then we take a negative sign for v_s.

If the observer is moving towards the source, then we take a positive sign for v_0, and if the observer is moving away from the source, then we take a negative sign for v_s.

$$f_b = |f_1' - f_2'|$$

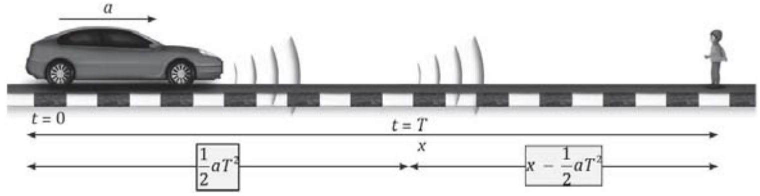
$$f_b = \left(\frac{v + v_0}{v}\right) f_0 - \left(\frac{v - v_0}{v}\right) f_0$$

$$f_b = \left(\frac{v + v_0 - v + v_0}{v}\right) f_0 = \frac{2 v_0 f_0}{v}$$

$$f_b = \frac{2 \times 3 \times 256}{330} = 4.6 \text{Hz}$$

Ex. A source emitting a sound of frequency f is placed at a large distance from an observer. The source starts moving towards the observer with a uniform acceleration of a. Find the frequency heard by the observer corresponding to the wave emitted just after the source starts. The speed of sound in the medium is v.

Sol. Here, we cannot use the formula of the Doppler effect because in this case, the source is accelerating, which means that its velocity is changing with respect to time. This implies that the apparent frequency also changes with respect to time. Let the large distance between the source and the observer be x. The observer and the source were at rest at t = 0.



The time taken by the first pulse to reach the observer, $T_1 = \frac{x}{v}$

The time taken by the second pulse to reach the observer $T_2 = T + \left(\frac{x-\frac{1}{2}aT^2}{v}\right)$

Here $\frac{1}{2}aT^2$ is the distance travelled by the source during the interval of t = 0 to t = T when the second pulse was emitted at t = T.

The time interval between the receptions of two successive sound pulses is,

$$T' = T_2 - T_1$$

$$\text{Or, } T' = T - \frac{\left(\frac{1}{2}aT^2\right)}{v}$$

$$\frac{1}{f'} = \frac{1}{f} - \frac{a}{2vf^2}$$

$$\frac{1}{f'} = \frac{2vf - a}{2vf^2}$$

$$f' = \frac{2vf^2}{2vf - a}$$