BEATS

Resonance column method

The method of resonance column is an experimental approach used for determining the speed of sound in air. This technique employs a simple apparatus consisting of an elongated cylindrical glass tube, known as the resonance tube, and a container. Both the glass tube and the vessel are filled with water. Adjustments to the water level in the tube can be made with a cork attached to the connecting channel of the two containers. Furthermore, the water level can be measured using the calibration scale attached to the glass tube. The diagram provides a schematic depiction of the apparatus. The air column enclosed within the cylindrical glass tube is delimited by one open end and one closed end, determined by the water level. This configuration imparts characteristics akin to a closed organ pipe.

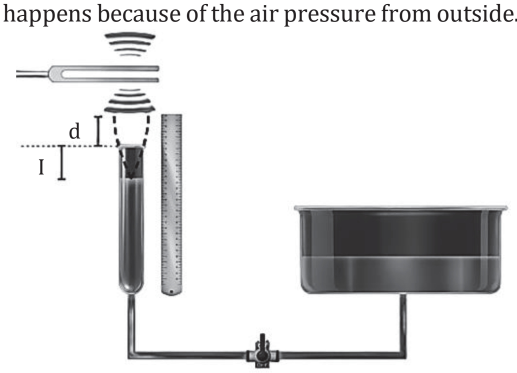
Hence, the length of the air column is, $L = (2n+1)(\frac{\lambda}{4})$

And the frequency is $f_n = (2n+1)(\frac{v}{4L}) = (2n+1)f_0$

Where f_0 is the frequency of the fundamental mode.

Imagine a situation in which a vibrating tuning fork is placed close to the open end and set into motion, generating longitudinal waves (sound waves) transmitted into the tube. This tuning fork acts as a source for the air column, resulting in the formation of a standing wave within the region of the air column in the tube. By carefully adjusting the water level in the resonance tube, let's presume that the air column vibrates in the first resonance mode, constituting its fundamental mode, as illustrated in the figure.

A displacement antinode (pressure node) should form at the open end of the tube, but by observing the figure, we can see that the antinode is created at a slightly higher position from the exact open end of the tube. This happens because of the air pressure from outside. It is known as end correction.



If the length of the air column for the first resonance is l_1 and the end correction is d, then the condition for the first resonance to occur is,

$$l_1 + d = \frac{\lambda}{4} \quad \dots (1)$$

Assume that the water level is further adjusted to perceive the second maximum, resulting in the length of the air column becoming l_2. This represents the second resonance in the air column or the first overtone. Therefore, the condition for the second resonance can be expressed as follows:

$$l_2 + d = \frac{3\lambda}{4} \quad \dots (2)$$

By subtracting equation (1) from equation (2), we get,

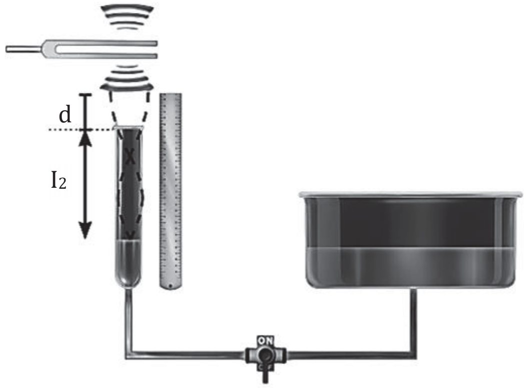
$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4}$$

$$l_2 - l_1 = \frac{2\lambda}{4} = \frac{\lambda}{2}$$

$$\lambda = 2(l_2 - l_1)$$

Since $\lambda = \frac{v}{f}$ and f are known, we get,

$$v = 2(l_2 - l_1)f$$



Hence, if the lengths of the air column for successive resonances and the frequency of the tuning fork are known, we can determine the speed of the sound wave in air.

Note Formula for end correction: The end correction is expressed as d = 0.6r or 0.3D, where r and D represent the radius and diameter of the tube, respectively.
If the lengths of the air column for successive resonances are denoted as l_1 and l_2, and the frequency of the tuning fork is f, then the velocity of the sound wave can be expressed as: $v = 2(l_2 - l_1)f$

Ex. A tuning fork, vibrating at a frequency of 800 Hz, induces resonance in a column tube. The upper end is open, while the lower end is closed by a water surface, the height of which can be adjusted. Successive resonances are detected at lengths of 9.75 cm, 31.25 cm, and 52.75 cm. Calculate the speed of sound in air based on the provided data.

Sol. Given,
The frequency of the tuning fork, f = 800 Hz
The lengths of the air column for successive resonances are as follows:

$$l_1 = 9.75\text{cm}$$
$$l_2 = 31.25\text{cm}$$
$$l_3 = 52.75\text{cm}$$

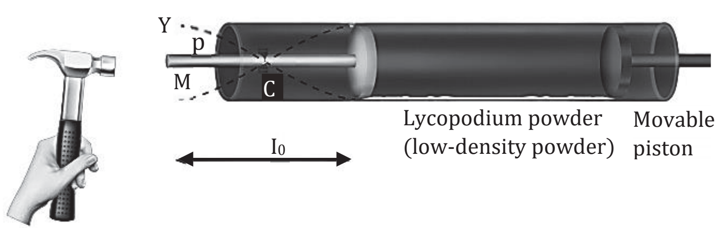
Therefore, $$l_2 - l_1 = 31.25 - 9.75 = 21.5 = 0.215\text{m}$$
$$l_3 - l_2 = 52.75 - 31.25 = 21.5\text{cm} = 0.215\text{m}$$

Hence, the speed of the sound in air is

$$v = 2(l_2 - l_1)f$$
$$v = 2(0.215)800$$
$$v = 344\text{ms}^{-1}$$

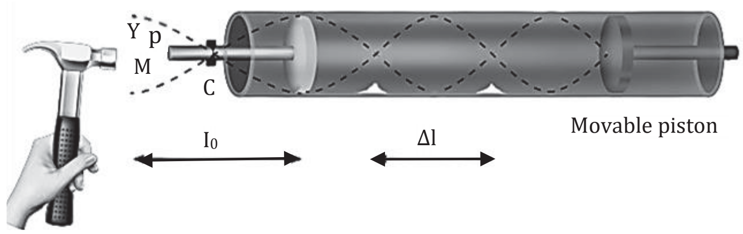
Kundt's tube method

Imagine a gas contained within an elongated tube sealed at both ends— one end featuring a movable piston, and the other capped by a plate containing a solid rod of length l0 welded to it. The rod is securely fastened precisely at its midpoint, and a powder of low density is dispersed within the chamber where the gas is contained, as illustrated in the figure.



Consider the distance between any two successive mounds as Δl, the velocity of the gas in the chamber as v_g, and the velocity of the rod as v_r. If ρ and Y represent the density and Young's modulus of the solid rod, then the velocity of the rod can be expressed as:

$$v_r = \sqrt{\frac{Y}{\rho}}$$



As the clamp is positioned at the midpoint of the rod, it functions as a point of displacement node, giving rise to a fundamental mode of longitudinal vibration in the rod.

Since the distance between an antinode and node is $\frac{\lambda}{4}$,

$$\frac{l_0}{2} = \frac{\lambda}{4}$$
$$l_0 = \frac{\lambda}{2}$$

If f is the frequency of vibration of the rod, then,

$$l_0 = \frac{v_r}{2f}$$
$$f = \frac{v_r}{2l_0} \quad \dots (1)$$

Now, the separation between two successive nodes within the air column, $\Delta l = \frac{\lambda'}{2}$ (Where λ ' is the wavelength of the standing wave in the tube)

Given that the frequency of the standing waves aligns with the frequency of the rod's vibration become.

$$\Delta l = \frac{v_g}{2f}$$
$$f = \frac{v_g}{2\Delta l}$$
$$f = \frac{v_g}{2\Delta l} \quad \dots (2)$$

Equating equation (1) with equation (2), we get,

$$\frac{v_r}{2l_0} = \frac{v_g}{2\Delta l}$$
$$v_g = \frac{\Delta l}{l_0} v_r \quad \dots (3)$$

Ex. A Kundt's tube apparatus consists of a copper rod, 1 meter in length, clamped at a distance of 25 cm from one of its ends. The tube is filled with air, and the speed of sound in air is known to be 340 m/s. The powder collected in heaps is spaced 5 cm apart. Determine the speed of sound waves in copper.

Sol. Given,

Velocity of air in the tube, v g = 340 ms^{-1}

Length of the copper rod, l = 1 m

Distance between the consecutive heaps (node), Δl = 5 cm = 0.05 m

The copper rod is clamped (displacement node) 25 cm away from the free end of the rod. Since we know that the distance between a consecutive node and an antinode is $\frac{\lambda}{4}$,

$$\frac{\lambda}{4} = 25$$
$$\lambda = 100\text{cm}$$

If the speed of the sound waves in the copper rod is v_r, then,

$$\frac{v_r}{f} = 100$$
$$f = \frac{v_r}{100} \quad \dots (1)$$

Now, since the distance between the consecutive heaps is Δl,

$\Delta l = \frac{\lambda'}{2}$ (λ s the wavelength of standing wave in the tube.

$$\lambda' = 2\Delta l$$
$$\frac{v_g}{f} = 2 \times 5$$
$$f = \frac{v_g}{10}$$

From equation (1), we get

$$f = \frac{v_r}{100}$$

Therefore, $\frac{v_r}{100} = \frac{v_g}{10}$

$$v_r = 10v_g$$
$$v_r = 10 \times 340$$
$$v_r = 3{,}400 ms^{-1}$$

Therefore, the speed of the sound waves in a copper rod is 3,400 ms^{-1}.

Beats

Suppose a sound wave with a frequency f of 264 Hz is considered, and the corresponding equation of the sound wave is $p_1 = p_0 \sin[\omega_1(t - \frac{x}{v})]$ Now, assume another sound wave with a frequency f is 270 Hz, and the corresponding equation of the sound wave is $p_2 = p_0 \sin[\omega_2(t - \frac{x}{v})]$ prove that $|\omega_1 - \omega_2| = \Delta\omega \ll \omega_1$ and ω_2

Hence, in the event of constructive interference between these two waves, we obtain:

$$p = p_1 + p_2$$
$$p = p_0[\sin \omega_1(t - \frac{x}{v}) + \sin \omega_2(t - \frac{x}{v})]$$
$$p = p_0[2\sin(\frac{\omega_1+\omega_2}{2})(t - \frac{x}{v})\cos(\frac{\omega_1-\omega_2}{2})(t - \frac{x}{v})]$$

By defining $\frac{\omega_1+\omega_2}{2} = \omega$ and $|\omega_1 - \omega_2| = \Delta\omega$, we get,

$$p = p_0[2\cos\frac{\Delta\omega}{2}(t - \frac{x}{v})\sin \omega(t - \frac{x}{v})]$$

Also, it can be written as follows:

$$p = A\sin \omega(t - \frac{x}{v})$$

Therefore, the amplitude of the resultant wave is given by,

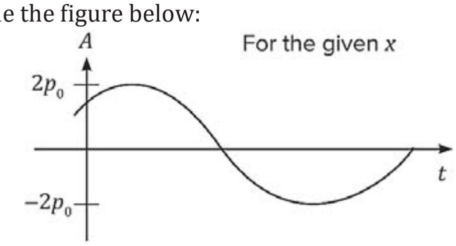
$$A = 2p_0\cos\frac{\Delta\omega}{2}(t - \frac{x}{v})$$

It is dependent on both x and t.

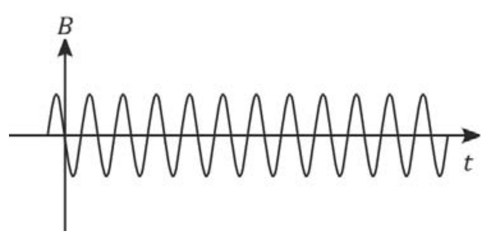
The angular frequency of the resultant wave is,

$$\omega = \frac{\omega_1+\omega_2}{2}$$

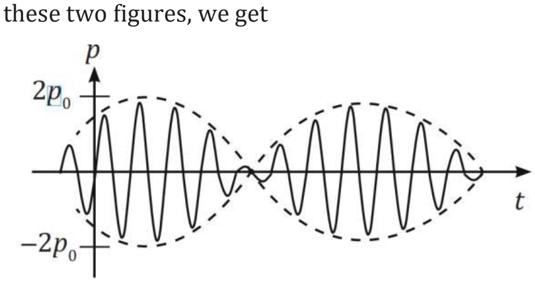
If we stabilize the position at x and solely take into account the variation of A, the resulting depiction will resemble the figure below:



Similarly, the Boolean expression describing the form of the sinusoidal component in the resulting wave would be:



If we superimpose these two figures, we get



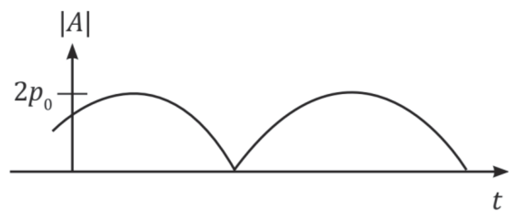
The cosine function in A makes the amplitude to vary between $+2p_0$ to $-2p_0$, and the variation of linear frequency of A is given by.

$$\frac{(\frac{\Delta\omega}{2})}{2\pi} = \frac{\Delta\omega}{4\pi}$$

Nevertheless, the amplitude is restricted from being negative. Consequently, the amplitude of the resulting wave is determined by the following:

$$|A| = 2p_0\cos[\frac{\Delta\omega}{2}(t - \frac{x}{v})]$$

The shape of |A| looks like the following:



Hence, the linear frequency of amplitude modulation is expressed as:

$$f_A = 2 \times \frac{\Delta\omega}{4\pi}$$
$$f_A = \frac{\Delta\omega}{2\pi}$$
$$f_A = \frac{|\omega_1 - \omega_2|}{2\pi}$$
$$f_A = \frac{|2\pi f_1 - 2\pi f_2|}{2\pi}$$
$$f_A = |f_1 - f_2|$$

As the intensity is directly proportional to the square of the amplitude, and the amplitude undergoes changes in accordance with the frequency, $|f_1 - f_2|$ the intensity of the resultant sound also becomes$|f_1 - f_2|$.

- The occurrence of periodic intensity variations in sound, resulting from the interference of two sound waves with slightly different frequencies, is termed beats.
- A single cycle encompassing maximum and minimum intensity is referred to as one beat. Therefore, the frequency of beats is determined by, $f_{beats} = |f_1 - f_2|$
- The inverse of the beat frequency is equivalent to the time interval between two successive maxima.
- To perceive the beat frequency, $|f_1 - f_2|$ should not be excessively large. In practice, for the beat frequency to be audible, , $|f_1 - f_2| < 16$ Hz

Ex. A tuning fork labeled A, with a frequency of 384 Hz, produces 6 beats in a 2 s interval when sounded simultaneously with another tuning fork, B. What might be the frequency of tuning fork B?

Sol. Given, Frequency of tuning fork A, $f_A = 384$ Hz

There are 6 beats produced in 2 s.

Therefore, the beat frequency is as follows:

$$f_{beats} == 3 \text{ Hz}$$

If f_B is the frequency of tuning fork B, then we get,

$$|f_A - f_B| = 3$$
$$f_A - f_B = \pm 3$$
$$f_B = f_A \pm 3$$
$$f_B = 384 \pm 3\text{Hz}$$

Therefore, the frequency of tuning fork B is either 381Hz or 387 Hz

Ex. A tuning fork generates a beat frequency of 4 beats per second when paired with another tuning fork with a frequency of 256 Hz. After adding a small amount of wax to the first tuning fork, the beat frequency increases to 6 beats per second. What was the initial frequency of the tuning fork?

Sol. Given,

Frequency of one tuning fork (B), $f_B = 256$ Hz

Frequency of beats, $f_{beats} = 4$ Hz

Therefore, if the frequency of the other tuning fork (A) is f_A, f_A becomes (256 ± 4). It means that f_A is either 252 Hz or 260 Hz.
It is said that tuning fork A is loaded with a little wax. As a result, its frequency decreases a little. Let the new frequency be $'f_A$ ($< f_A$) and the frequency of tuning fork B remains 256 Hz.
The increase in beat frequency for this case becomes, $f'_{beats} = 6$ Hz
Suppose the frequency of tuning fork A is 260 Hz. Now, if we decrease this frequency by 1 Hz or 2 Hz, the frequency will be closer to 256 Hz. Hence, the beat frequency for this case can never be 6 Hz.

Now, if the frequency of tuning fork A is 252 Hz and we decrease this frequency by 1 Hz or 2 Hz, then the frequency will go further away from 256 Hz (i.e., 251 Hz or 250 Hz). Hence, the beat frequency of 6 Hz must be achievable.
Therefore, the original frequency of the tuning fork was 252 Hz.

Ex. A tuning fork of unknown frequency makes 5 beats per second with another tuning fork that can cause a closed organ pipe of length 40 cm to vibrate in its fundamental mode. The beat frequency decreases when the first tuning fork is slightly loaded with wax. Find its original frequency. The speed of sound in air is 320 ms^{-1}.

Sol. It is given that the second tuning fork can cause a closed organ pipe of length 40 cm to vibrate in its fundamental mode. Therefore, if the frequency of the fundamental mode is f_0, then we get

$$f_0 = \frac{v}{4L}$$

Given that, $v = 320 ms^{-1}$ and $L = 40$ cm $= \frac{2}{5}$ m

Therefore, $$f_0 = \frac{320}{4 \times \frac{2}{5}}$$

$$f_0 = \frac{320 \times 5}{8}$$

$$f_0 = 200 \text{ Hz}$$

Therefore, the frequency of the second tuning fork is as follows: $f_2 = 200$ Hz
Now, if the frequency of the first tuning fork is f_1, then the beat frequency is 5 Hz.

Therefore, $$|f_1 - f_2| = 5$$

$$f_1 - f_2 = \pm 5$$

$$f_1 = f_2 \pm 5$$

Therefore, the frequency of the first tuning fork becomes either 205 Hz or 195 Hz.
Since the first tuning fork is slightly loaded with wax, its frequency decreases and as a result, the beat frequency also decreases.

- Suppose the frequency of the first tuning fork is 195 Hz. Now, if we decrease this frequency, the frequency will go further away from f_2 (= 200 Hz). Hence, the beat frequency for this case can never be less than 5 Hz.
- Now, if the frequency of the first tuning fork is 205 Hz and we decrease this frequency further, then the frequency will go towards f_2 (= 200 Hz). Hence, the beat frequency less than 5 Hz must be achievable.

Therefore, the original frequency of the tuning fork was 205 Hz.