EQUATIONS OF SHM

Relationship between Velocity (v) and Displacement (x)

The particle's displacement, represented as $x = A\sin\omega t$, allows us to define its velocity.

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

On squaring both the sides, we get,

$$v^2 = (A\omega)^2\cos^2(\omega t)$$
$$v^2 = \omega^2[A^2 - A^2\sin^2(\omega t)]$$
$$v^2 = \omega^2(A^2 - x^2)[\text{ Since } x = A\sin(\omega t)]$$
$$v = \omega\sqrt{A^2 - x^2}$$

Thus, for $x = 0$, the velocity of the particle is $v = A\omega$ and for $x = A$, the velocity of the particle is, $v = 0$.

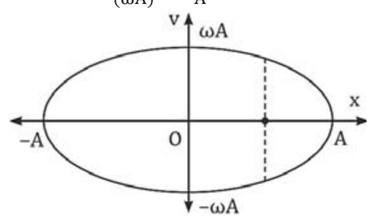
Velocity vs Displacement graph

The connection between velocity and displacement is established by the following relationship:

$$v = \omega\sqrt{A^2 - x^2}$$

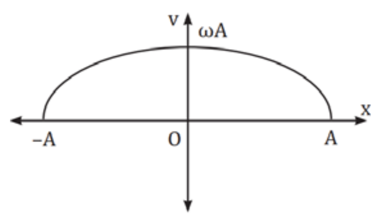
On squaring both sides of the equation, we get,

$$v^2 = \omega^2A^2 - \omega^2x^2$$
$$v^2 + \omega^2x^2 = \omega^2A^2$$
$$\frac{v^2}{(\omega A)^2} + \frac{x^2}{A^2} = 1$$



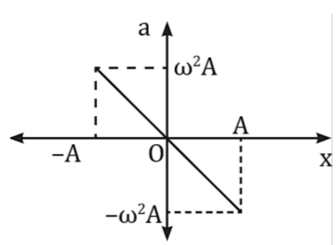
This represents the typical equation of an ellipse, and the curve closely resembles the one depicted in the illustration.

Examining the curve reveals that for a particular position of the particle, there exist two potential velocities—one for the particle moving to the right and another for the particle moving to the left. When considering the curve in relation to speed and displacement, it corresponds to the upper half section of the velocity versus displacement graph.



Acceleration vs Displacement graph

The connection between acceleration and displacement is established by the following relationship $a = -\omega^2 x$. Therefore, the graphical representation of this correlation will be a straight line that passes through the origin (O) and exhibits a negative slope, as depicted in the figure.

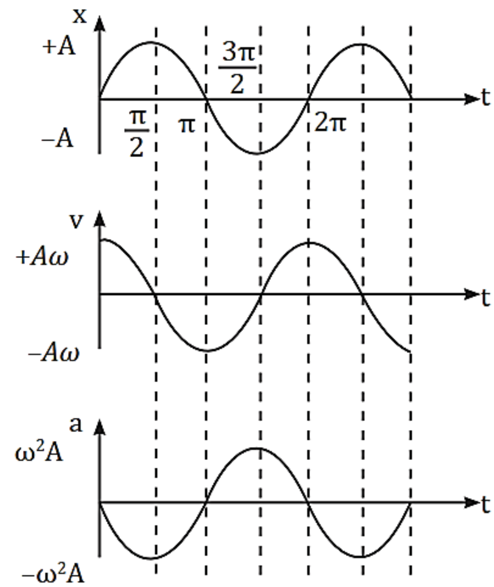


Graphical representation

When the displacement is specified as $x = A\sin\omega t = A\sin(\omega t + 0)$ in such a case, the velocity and acceleration of the particle are described as.

$$v = \frac{dx}{dt} = A\omega\cos\omega t = A\sin\left(\omega t + \frac{\pi}{2}\right) \text{ and}$$

$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t = A\omega^2 \sin(\omega t + \pi)$ Respectively. The graphs of these quantities are as shown:



Comparing the phase angle of displacement, velocity and acceleration, it is seen that the velocity leads the displacement by $\frac{\pi}{2}$ the acceleration precedes the velocity by $\frac{\pi}{2}$ it can also be stated that the velocity trails the acceleration by $\frac{\pi}{2}$ Furthermore, the acceleration precedes the displacement by π.

Physically, 'the displacement lags behind the velocity by $\frac{\pi}{2}$ this implies that when the displacement is zero (indicating the particle is at the mean position), the particle's velocity is at its maximum. Furthermore, 'the acceleration leads the displacement by π' signifies that both displacement and acceleration are zero at the mean position but exhibit opposite signs at the extreme ends.

Graphically, the advantage held by the velocity over the displacement in terms of $\frac{\pi}{2}$ is represented by shifting the origin of the displacement from $t = 0$ to $t = \frac{T}{4}$ to make the In the velocity graph, the precedence of velocity over displacement, and the lead taken by the acceleration over displacement by π, is depicted by adjusting the origin of displacement from. $t = 0$ to $t = \frac{1}{2}$ to make the acceleration graph.