LAWS OF RADIATION

Stefan's law

The radiant energy emitted per unit area per unit time by a perfectly black body is directly proportional to the fourth power of its absolute temperature.

The formula for the energy emitted by a black body per unit area per unit time is expressed as:

$$E \propto T^4$$

$$E = \sigma T^4$$

Where, $\sigma =$ Stefan's constant $= 5.67 \times 10^{-8} Wm^{-2}K^{-4}$

For an ordinary body, $E = \varepsilon\sigma T^4$

Where, ε is the emissivity of the material.

Radiant energy (Q)

If Q is the total radiant energy radiated by the body, then we get,

$$E = \frac{Q}{A \times t} = \varepsilon\sigma T^4$$

$$Q = A\varepsilon\sigma T^4 t$$

Radiant power (P)

The radiant power is defined as the radiant energy per unit time.

$$\frac{Q}{t} = A\varepsilon\sigma T^4$$

$$P = \frac{Q}{t} = A\varepsilon\sigma T^4$$

When a body is at temperature T and its surroundings are at temperature T_0, the body undergoes absorption and emission of energy concurrently.

The energy emitted by the body is given as follows:

$$E_{out} = \varepsilon\sigma T^4$$

The energy absorbed by the body is given as follows:

$$E_{in} = \varepsilon\sigma T_0^4$$

The net energy emitted is given as follows:

$$E_{net} = \varepsilon\sigma(T^4 - T_0^4)$$

Ex. A body of emissivity 0.75, surface area 300 cm2, and temperature 227 °C is kept in a room at temperature 27 °C. Calculate the initial value of the net power emitted by the body.

(a) 50 W (b) 59.4 W (c) 69.4 W (d) 79.4 W

Sol. Given, Emissivity of the body, $\varepsilon = 0.75$

Surface area of the body, $A = 300cm^2 = 0.03m^2$

Temperature of the body, $T = 227°C = 227 + 273 = 500K$

Temperature of the surrounding, $T_0 = 27°C = 27 + 273 = 300K$

The net radiant power of the body is given as follows:

$$P_{net} = \varepsilon\sigma A(T^4 - T_0^4)$$

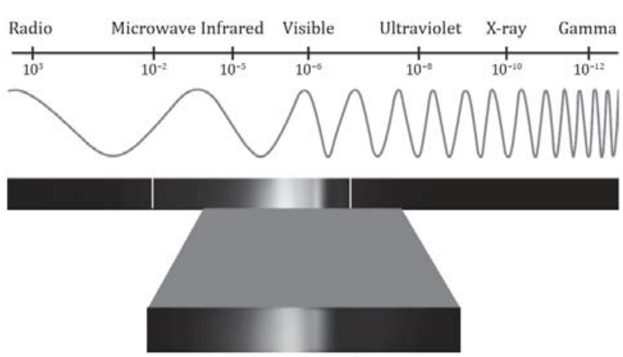
$$P_{net} = 0.75 \times 5.67 \times 10^{-8} \times 0.03 \times (500^4 - 300^4)$$

$$P_{net} = 69.4W$$

Thus, option (C) is the correct answer.

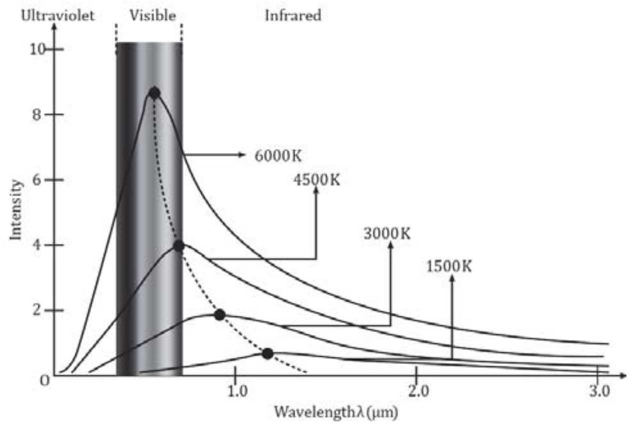
Electromagnetic Spectrum

We receive infrared waves, visible light, and ultraviolet rays from the entire electromagnetic spectrum, specifically from wavelengths within the range of 7.8×10^{-7}m to 4×10^{-4}m on Earth. Together, they are referred to as thermal radiation.



Thermal radiation

The graph illustrates the wavelengths (λ) of radiations emitted from a body at a specific temperature plotted against their intensity (I). From this graph, we can infer how the distribution of emitted energy from a body varies with temperature, as explained below:



1. The overall shape of the curve remains consistent across all temperatures.
2. With an increase in the body's temperature, the most probable wavelength (λ_{max}) in the spectrum shifts towards shorter wavelengths, resulting in a leftward shift of the plot.
3. Higher temperature bodies emit radiation with shorter wavelengths (λ_{max}), while lower temperature bodies emit radiation with longer wavelengths (λ_{max}).

Sun as a black body

The Sun's temperature is extremely high, leading it to emit a wide range of radiations, making it an exemplary instance of a black body.

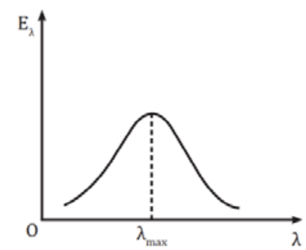
Distribution of energy in the spectrum of a black body

A perfect black body emits radiation of all the possible wavelengths.

- At a set temperature, energy is unevenly spread across different wavelengths.
- At this temperature, heat radiation grows in intensity as wavelength extends. There's a peak intensity at a specific wavelength, diminishing as wavelength continues to increase.

The total intensity of radiation at a specific temperature is represented by the area under the curve.

The area under the curve is given as follows:



$$A = E = \int E_\lambda d\lambda$$

Wien's Displacement Law

This law asserts that the multiplication of the wavelength corresponding to the peak intensity of radiation and the absolute temperature of the body (measured in Kelvin) remains constant.

$$(\lambda_{max}) \propto \frac{1}{T}$$

$$\lambda_{max} T = b = \text{Constant}$$

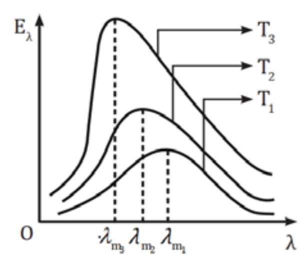
Where, b is known as Wien's constant and has a value of 2.89×10^{-3} mK.

Effect of temperature on a black body radiation curve

As temperature rises, the wavelength where the spectral intensity (E_λ) peaks shifts towards shorter values (towards the left).

In the graph, the temperatures follow the sequence: $T_3 > T_2 > T_1$.

It's noticeable that as temperature increases, the wavelength corresponding to maximum intensity decreases.



$$\lambda_{m_1} > \lambda_{m_2} > \lambda_{m_3}$$

Ex. The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0. If the temperature of the black body is now changed so that it radiates the maximum energy at wavelength $\frac{3}{4}\lambda_0$ the power radiated by it becomes nP. Find the value of n.

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{256}{81}$ (d) $\frac{81}{256}$

Sol. According to Wien's displacement law,

$$\lambda_{max}T = b = \text{Constant}$$

$$\frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}}$$

$$\frac{T_1}{T_2} = \frac{\frac{3}{4}\lambda_o}{\lambda_o} = \frac{3}{4}$$

Power radiated by a blackbody, $P = \sigma A T^4$

$$\frac{P_1}{P_2} = \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4$$

$$\frac{P}{nP} = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

$$n = \frac{256}{81}$$

Thus, option (C) is the correct answer.

Temperature of the Sun and Solar Constant

Consider R as the radius of the Sun and T as its temperature.

Area of the Sun. $A = 4\pi R^2$

The energy emitted per second through radiation is expressed as:

$$P = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

Upon reaching the Earth, this energy (P) will disperse across a sphere with a radius denoted as r, representing the average distance between the Earth and the Sun.

Solar constant (S)

The solar constant refers to the intensity of solar radiation at the Earth's surface.

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{\frac{1}{4}}$$

Here, $r = 1.5 \times 10^8$ km

$$R = 7 \times 10^5 \text{ km}$$

$$S = 2 \text{calcm}^{-2}\text{min}^{-1} = 1.4 \times 10^3 \text{Wm}^{-2}$$

$$\sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{k}^{-4}$$

Substituting the values, $T = [(\frac{1.5\times10^8}{7\times10^5})^2(\frac{1.4\times10^3}{5.67\times10^{-8}})]^{\frac{1}{4}}$

$$T \approx 5800 \text{ K}$$

Rate of loss of heat (R_H) and rate of cooling (R_C)

When an ordinary body at temperature T is situated in an environment of temperature T_0, the heat loss due to radiation is expressed as follows:

$$\Delta Q = Q_{emmision} - Q_{absorption}$$

$$\Delta Q = A\varepsilon\sigma(T^4 - T_0^4)$$

The rate of heat loss is given as follows:

$$R_H = \frac{dQ}{dt} = A\varepsilon\sigma(T^4 - T_0^4)$$

The rate of cooling is given as follows:

$$R_C = \frac{A\varepsilon\sigma}{V\rho c}(T^4 - T_0^4)$$

V = Volume of the body ρ = Density of the body c = specific heat of the body

Factors affecting the rate of cooling, R_c

1. **Nature of radiating surface (ε)**
   The higher the emissivity, the quicker the cooling process.
2. **Area of radiating surface (A)**
   The larger the surface area for radiation, the faster the cooling process.
3. **Mass of radiating surface (m = ρV)**
   The more substantial the mass of the radiating body, the slower the cooling rate.
4. **Specific heat of radiating body (c)**
   The higher the specific heat of the radiating body, the slower the cooling process.
5. **Temperature of radiating body (T)**
   The higher the temperature of the radiating body, the quicker the cooling process.

6. Temperature of surrounding (T_0)

The higher the temperature of the surroundings, the slower the cooling process.

Newton's Law of Cooling

Newton's law of cooling asserts that when the temperature of a body is relatively close to that of its surroundings, the rate of cooling is directly proportional to the temperature difference between the body and its environment.

Consider T as the temperature of the body and T0 as the temperature of the surroundings. When the body is cooling, indicating heat flow from the body, it means the body's temperature is higher than that of its surroundings.

Suppose the temperature gap between the body and its surroundings is minimal.

$$T - T_0 = \Delta T$$

$$T = T_0 + \Delta T \quad \ldots (1)$$

From Stefan's law,

$$\frac{dQ}{dt} = A\varepsilon\sigma(T^4 - T_0^4)$$

Also, $dQ = mcdT$

$$mc\frac{dT}{dt} = A\varepsilon\sigma(T^4 - T_0^4)$$

Substituting $T = T_0 + \Delta T$, we get the following:

$$mc\frac{dT}{dt} = A\varepsilon\sigma((T_0 + \Delta T)^4 - T_0^4)$$

$$\frac{dT}{dt} = KT_0^4((1 + \frac{\Delta T}{T_0})^4 - 1)$$

$$\frac{dT}{dt} = KT_0^4(1 + 4\frac{\Delta T}{T_0} - 1)(\because \frac{\Delta T}{T_0} << 1)$$

$$\frac{dT}{dt} = 4KT_0^3\Delta T$$

$$\frac{dT}{dt} \propto -\Delta T \text{ or, } \frac{d\theta}{dt} \propto -(\theta - \theta_0)$$

The negative sign indicates that the temperature of the body decreases over time. The larger the temperature disparity between the body and its surroundings, the faster the rate of cooling.

If $\theta = \theta_0$,

$$\frac{d\theta}{dt} = 0$$

Therefore, a body can never be cooled to a temperature lesser than its surrounding by radiation.

From Newton's law of cooling,

$$\frac{d\theta}{dt} \propto -(\theta - \theta_0)$$

If a body cools down by radiation from θ_1 °C to θ_2 °C,

$$-\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$$

$$\theta = \theta_{avg} = \frac{(\theta_1 + \theta_2)}{2}$$

$$\frac{d\theta}{dt} = -K(\frac{\theta_1 + \theta_2}{2} - \theta_0)$$

θ_1 Is the initial temperature of the body. θ_2 Is the final temperature of the body.

θ_0 Is the temperature of the surrounding.

Ex. An object is kept in a large room which has a temperature of 25 °C. It takes 12 minutes to cool from 80 °C to 70 °C. How much time will be taken by the same object to cool from 70 °C to 60 °C?

(a) 15 minutes (b) 10 minutes (c) 12 minutes (d) 20 minutes

Sol. Given,

Temperature of the surrounding, $\theta_0 = 25$ °C

Temperature of the body, $\theta_1 = 80$°C

Time taken to cool from 80°C to 70 °C = t = 12 minutes

Final temperature after 12 minutes $= \theta_2 = 70$ °C

Let the time taken to cool from 70 °C to 60°C be t′.

By Newton's law of cooling (from 80°C to 70°C), we get the following:

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

$$\frac{(80 - 70)}{12} = K(75 - 25)$$

$$10 = K \times 600$$

$$K = \frac{1}{60} \quad \ldots (1)$$

By Newton's law of cooling (from 70°C to 60°C), we get the following:

$$\frac{(70-60)}{t'} = K(65 - 25)$$

$$K = \frac{1}{t'(4)}$$

$$K = \frac{1}{t'(4)} \quad \ldots (2)$$

From equation (1) and equation (2), we get the following:

$$\frac{1}{4t'} = \frac{1}{60}$$

$$t' = \frac{60}{4}$$

$t' = 15$ minutes

Thus, option (A) is the correct answer.