Chapter 19

Heat Transfer

CONDUCTION AND FOURIER’S LAW

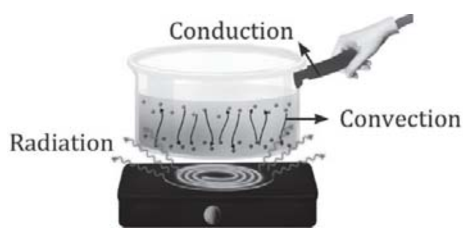
Mode Of Heat Transfer

Heat is a type of energy that moves between two bodies when they have different temperatures. This energy always moves from the body with a higher temperature to the one with a lower temperature.

Mode Of heat transfer
- → Conduction
- → Radiation
- → Convection

Conduction

This process involves the transmission of heat from one point to another within a substance without the particles themselves moving. In solids, heat transfer primarily occurs through conduction.



Convection

This process involves the transfer of heat from one point to another through the actual movement of heated particles within the medium. This method of heat transfer is more prevalent in fluids such as liquids and gases.

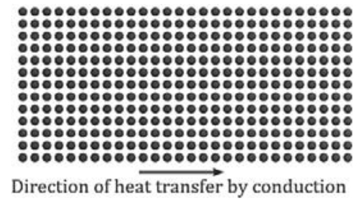
Examples of convection heat transfer include sea and land breezes. During the day, the land heats up faster than the sea, causing the air above the land to become less dense and rise. This creates low-pressure areas above the land, and cold air from the sea moves towards the land, resulting in a sea breeze. Conversely, at night, the land cools more rapidly than the sea, creating low pressure above the sea's surface. As a result, cold air from the land blows towards the sea, leading to a land breeze.

Radiation

This process involves the direct transmission of heat from one point to another without any intermediary within the medium. In this scenario, thermal energy is carried by electromagnetic waves, making it the fastest method of heat transfer.

Conduction and its Characteristics

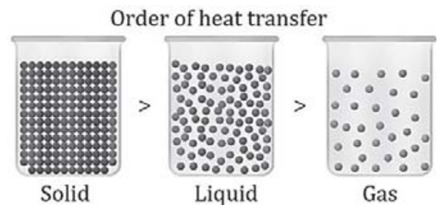
Conduction



This is a process where heat energy is transferred through collisions among neighboring atoms or molecules. In solids, atoms oscillate around their average positions in three-dimensional space, the extent of which depends on the temperature. As the temperature increases, so does the amplitude of oscillation.

In solids, a continuous array of atoms spans the entire material, as depicted in the illustration above. The left end of the material receives constant heat from an external source. As a result of this thermal energy input, the atoms on the left begin oscillating around their average positions and collide with adjacent colder atoms. Consequently, thermal energy (heat) is transferred in this process. The atoms that absorb thermal energy subsequently affect their neighboring colder atoms in a similar manner, perpetuating this cycle until a steady state is reached.

Characteristics of Heat Transfer by Conduction



Conduction requires a medium. The particles involved merely vibrate within their positions without displacing from them.

Conduction is most pronounced in solids because of their tightly packed structures. It occurs to a lesser extent in liquids and gases compared to solids.

Metallic solids excel in conducting heat because their free electrons transport the heat energy, whereas in non-metallic solids and fluids, conduction occurs solely through the vibrations of molecules.

Conduction in a Metallic Rod

Let's envision a setup featuring a metallic rod horizontally secured on a stand, with four thermometers arranged along its length, as depicted in the diagram. The left end of the rod is consistently supplied with heat from an external source. As the temperature rises, the metal atoms on the left begin to vibrate more vigorously. Consequently, thermal energy begins to transfer towards the right, which is the cooler region of the medium, due to collisions with neighboring atoms.

This creates a temperature gradient along the length of the rod. Moving from left to right, the temperature of the rod decreases, as indicated by the thermometers. The temperature gradient continues to fluctuate until it stabilizes at a steady state. At this point, the amount of heat crossing any cross-section of the slab per unit time becomes equal. Consequently, the rate of heat flow remains constant over time.

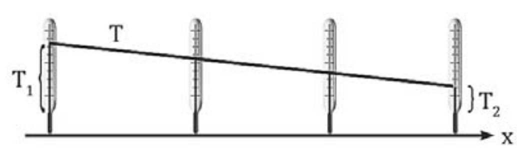
Transient heat transfer

In transient heat transfer, the temperature at any given point within the medium evolves over time.

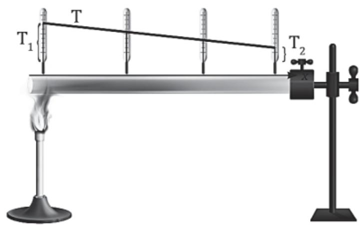
Steady heat transfer

In steady heat transfer, the temperature at any given point within the medium remains constant over time.

Temperature Gradient

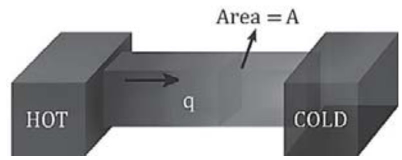


The temperature gradient represents the variation in temperature with respect to position.



Temperature gradient $= \frac{dT}{dx}$

Fourier's Law of Heat Conduction



The rate of heat transfer through a medium is directly proportional to the cross-sectional area perpendicular to the direction of heat flow and the temperature gradient across the medium.

Let the rate of heat transfer be q.

According to the definition, $q \propto A$

$$q \propto -\frac{dT}{dx}$$

$$q = -kA\frac{dT}{dx}$$

The negative sign in the temperature gradient expression arises because the temperature decreases in the direction of heat transfer. Here, 'k' represents the thermal conductivity of the material.

Fourier's law of heat conduction applies under these conditions:

- The medium conducting heat must be in a steady state.
- The thermal conductivity (k) of the medium remains constant.
- It is applicable solely to one-dimensional (1D) heat transfer.

About the thermal conductivity

In general, the sequence of thermal conductivity among various substances is as follows:

$$k_{\text{gases}} < k_{\text{liquids}} < k_{\text{solids}}$$

In the realm of solids, metals typically exhibit higher thermal conductivity compared to non-metals.

$$k_{\text{non-metals}} < k_{\text{metals}}$$

Units and dimensions of thermal conductivity

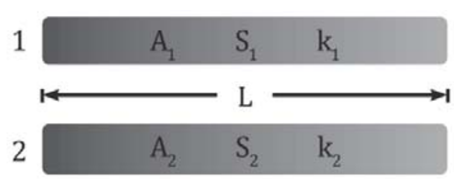
From $q = -kA\frac{dT}{dx}$

Watt (W) $= k(\text{ unit }) \times m^2 \times \frac{K}{m}$

Unit of k $= \frac{W}{m \cdot K} = \frac{J}{s \cdot m \cdot K}(\text{MKS}) = \frac{cal}{cm \cdot s \cdot {}^\circ C}(\text{CGS})$

Dimension of k $= \frac{J}{s \cdot m \cdot K} = \frac{ML^2T^{-2}}{TL\theta} = MLT^{-3}\theta^{-1}$

Ex. Consider two rods of same length but different specific heats (s_1, s_2), thermal conductivities (k_1, k_2), areas of cross section (A_1, A_2), and both having temperatures T_1 and T_2 at their ends. If the rate of loss of heat due to conduction is equal, then which of the following expressions is correct?



(a) $k_1A_1 = k_2A_2$ (b) $\frac{k_1A_1}{s_1} = \frac{k_2A_2}{s_2}$ (c) $k_2A_1 = k_1A_2$ (d) $\frac{k_2A_1}{s_2} = \frac{k_1A_2}{s_1}$

Sol. The rates of heat transfer (heat loss) are equal. Therefore,

$$q_1 = q_2$$

$$-\frac{k_1A_1(T_1 - T_2)}{L} = -\frac{k_2A_2(T_1 - T_2)}{L}$$

$$k_1A_1 = k_2A_2$$

Thus, option (A) is the correct answer.

Ex. Heat is flowing through two cylindrical rods of the same material. The diameters of the rods are in the ratio 1:2 and their lengths are in the ratio 2:1. If the temperature difference between their ends is the same, then what will be the ratio of the rate of flow of heat through them?

(a) 1:1 (b) 2:1 (c) 1:4 (d) 1:8

Sol. Since both the rods are made of the same material, their thermal conductivities are equal.

Therefore, $k_1 = k_2 = k$ (Say)

Moreover, $\frac{d_1}{d_2} = \frac{1}{2}$ and $\frac{l_1}{l_2} = 2$

Here, d and l represent the respective diameter and length of the rods. The temperature difference (ΔT) between their ends remains constant.

Rate of heat flow through the first rod, $q_1 = \frac{kA_1\Delta T}{l_1}$

Rate of heat flow through the second rod, $q_2 = \frac{kA_2\Delta T}{l_2}$

Ration, $\frac{q_1}{q_2} = \frac{\frac{A_1}{l_1}}{\frac{A_2}{l_2}}$

$$= \frac{A_1 l_2}{A_2 l_1}$$

$$= \frac{\pi(\frac{d_1}{2})^2 l_2}{\pi(\frac{d_2}{2})^2 l_1}$$

$$= (\frac{d_1}{d_2})^2 (\frac{l_2}{l_1})$$

$$= \left(\frac{1}{2}\right)^2 (\frac{1}{2}) = \frac{1}{8}$$