Chapter 16

Thermometry

TYPES OF EXPANSIONS

Temperature and its Measurement

The level of warmth or chilliness of an object is referred to as its body temperature.

High temperature

Low temperature

Heat naturally moves from hot things to cooler ones.
When two things are in thermal equilibrium, they don't exchange heat.
In this balanced state, their temperatures are the same.

Temperature Measurement

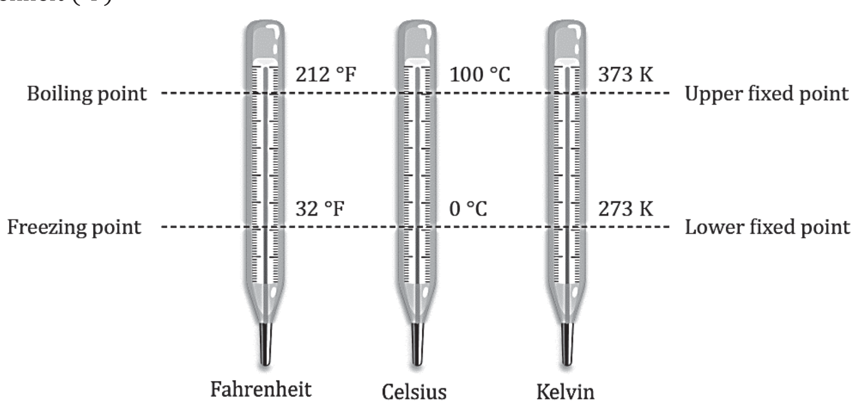
Temperature is determined with a tool known as a thermometer.

Digital thermometer

Mercury thermometer

Different scales of temperature measurement

Most used temperature scales are:
Centigrade (°C)
Kelvin (K)
Fahrenheit (°F)



Calibration for thermometers involves setting standard points. Typically, the boiling point of water marks the upper fixed point (UFP), while the freezing point of water marks the lower fixed point (LFP). The figures display the UFP and LFP on various scales.

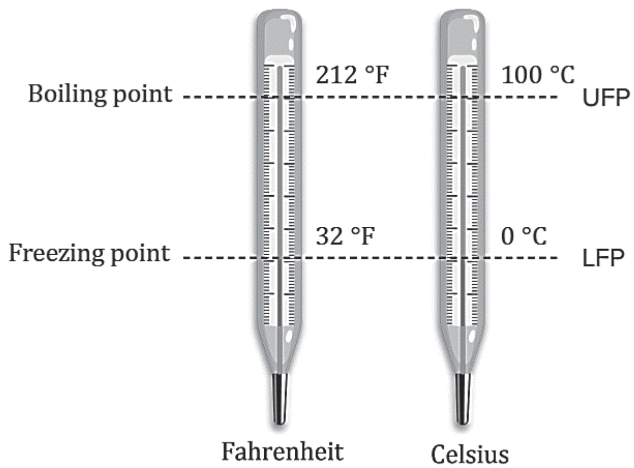
Conversion from one temperature scale to other

The formula used to convert readings from one scale to another is generally as follows:

$$\frac{\text{Reading of any scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales}$$

Example: To convert Fahrenheit (°F) to Celsius (°C), we use the following formula

$$\frac{°F - 32}{212 - 32} = \frac{°C - 0}{100 - 0}$$

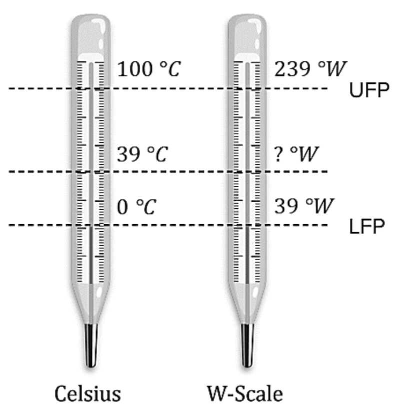


Formula for conversion between
Celsius - Fahrenheit - Kelvin

$$\frac{°C - 0}{100 - 0} = \frac{°F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

$$\Rightarrow \frac{°C}{5} = \frac{°F - 32}{9} = \frac{K - 273}{5}$$

Ex. On a new scale of temperature (which is linear) and called the W scale, the freezing and boiling points of water are 39 °W and 239 °W, respectively. What will be the temperature on the new scale, corresponding to a temperature of 39 °C on the Celsius scale?



Sol. Given,

Freezing point on scale = 39°W

Boiling point on scale = 239°W

Temperature on celsius scale = 39°C

$$\frac{\text{Reading of any scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales}$$

$$\frac{°C - 0}{100 - 0} = \frac{°W - 39}{239 - 39}$$

$$\frac{39}{100} = \frac{°W - 39}{200}$$

$$°W = 39 \times 2 + 39$$

$$°W = 117°W$$

Ex. A Centigrade and Fahrenheit thermometer are dipped in boiling water. The water temperature is lowered until the Fahrenheit thermometer registers 140 °F. What is the fall in temperature as registered by the Centigrade thermometer?

Sol. We have,

Reading in Fahrenheit scale = 140 °F

Let the reading on the centigrade scale corresponding to 140 °F be °C

$$\frac{\text{Reading of any scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales}$$

$$\frac{140 - 32}{212 - 32} = \frac{°C - 0}{100 - 0}$$

$$\frac{140 - 32}{180} = \frac{°C - 0}{100}$$

$$°C = 60°C$$

Fall on the Centigrade scale = $100^{o}C - 60^{o}C = 40^{o}C$

Thermal Expansion and its types

Thermal expansion is when stuff alters its size, area, bulk, and thickness because of temperature changes, usually without switching phases.

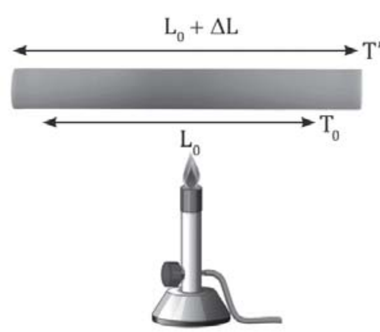
Linear Expansion

The alteration in a body's length because of temperature changes is called linear expansion. Suppose we have a rod with an initial length of L_0 at a temperature T_0. If we heat it to a new temperature T', causing its length to increase by ΔL, this demonstrates linear expansion.

The change in length is directly proportional to the original length and the change in temperature.

$$\Delta L \propto L_0$$

$$\Delta L \propto \Delta T$$



So,

$$\Delta L = L_0 \alpha \Delta T$$

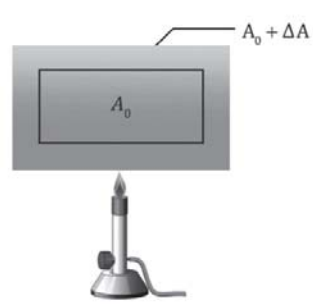
Here, α = Coefficient of linear expansion

$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$

Unit of α is K^{-1} or $°C^{-1}$.

∴ The final length of the rod is, $L = L_0(1 + \alpha\Delta T)$

Superficial or areal expansion



- The increase in surface area caused by temperature change is called superficial or areal expansion.
- Consider a flat surface with an initial area of A_0 at temperature T_0. When heated to a new temperature T', its area increases by ΔA.
- The change in area is directly related to both the initial area and the temperature change.

$$\Delta A \propto A_0$$

$$\Delta A \propto \Delta T$$

So,

$$\Delta A = A_0 \beta \Delta T$$

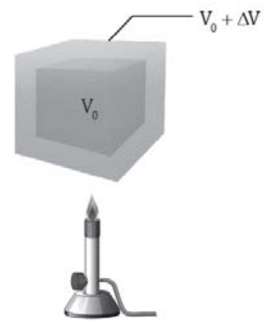
Here, β Coefficient of expansion

$$\beta = \frac{\Delta A}{A_n \Delta T}$$

Unit of β is K^{-1} or $^\circ C^{-1}$.

∴ The final area of the plate is, $A = A_0(1 + \beta\Delta T)$

Volume or Cubical Expansion



- The alteration in a body's volume because of temperature change is termed volumetric or cubical expansion.
- Imagine a rectangular box with an original volume of V_0 at a temperature of T_0. If heated to a new temperature, T', its volume expands by ΔV.
- The increase in volume is directly linked to both the initial volume and the temperature change.

$$\Delta V \propto V_0$$
$$\Delta V \propto \Delta T$$
$$\Delta V = V_0 \gamma \Delta T$$

Here, γ Coefficient of volumetric expansion

$$\gamma = \frac{\Delta V}{V_0 \Delta T}$$

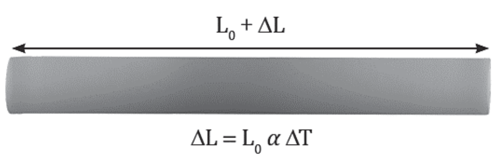
Unit of γ is K^{-1} or $^\circ C^{-1}$.

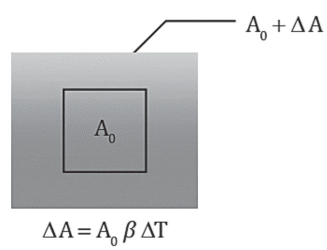
∴ The final volume of the cube is, $V = V_0(1 + \gamma\Delta T)$

Relation between Coefficient of Linear, Aerial and Cubical Expansion

Relation between α and β

Think of a rod with an original length of L_0, where the coefficient of linear expansion is α. When heated, its temperature rises by ΔT. Also, think of a square surface with each side measuring L_0, where the coefficient of areal expansion is β. When heated, its temperature increases by ΔT as well.





We have,

Area of the surface $(A_0) = L_0^2$

By differentiating with respect to length, we get the following

$$\frac{dA_0}{dL} = 2L_0$$
$$dA_0 = 2L_0 dL$$
$$\Delta A_0 = 2L_0 \Delta L$$
$$\Delta A_0 = 2L_0 \times L_0 \alpha \Delta T (\because \Delta L = L_0 \alpha \Delta T)$$
$$\Delta A_0 = 2L_0^2 \alpha \Delta T$$
$$\Delta A_0 = 2A_0 \alpha \Delta T \ldots\ldots (i)$$

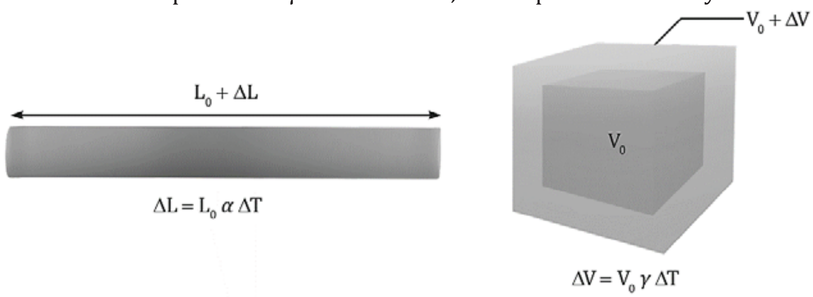
Also, $\Delta A_0 = A_0 \beta \Delta T \quad \ldots\ldots (ii)$

From equation (i) and (ii), we get the following:

$$A_0 \beta \Delta T = 2A_0 \alpha \Delta T$$
$$\beta = 2\alpha$$

Relation between α and γ

Imagine a rod with an initial length of L_0, where the coefficient of linear expansion is α. When it's heated, its temperature goes up by ΔT. Also, think of a cube with each side measuring L_0, where the coefficient of volumetric expansion is γ. When heated, its temperature rises by ΔT as well.



We have,

Volume of the cube $(V_0) = L_0^3$

$$\frac{dV_0}{dL} = 3L_0^2$$
$$\Delta V_0 = 3L_0^2 \Delta L$$
$$\Delta V_0 = 3L_0^2 L_0 \alpha \Delta T (\because \Delta L = L_0 \alpha \Delta T)$$
$$\Delta V_0 = 3L_0^3 \alpha \Delta T$$
$$\Delta V_0 = 3V_0 \alpha \Delta T \ldots\ldots (i)$$

Also, $$\Delta V_0 = V_0 \gamma \Delta T \ldots. (ii)$$

From equations (i) and (ii) we get the following:

$$V_0 \gamma \Delta T = 3V_0 \alpha \Delta T$$
$$\gamma = 3\alpha$$

Relation between α, β, and γ is given as follows:

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

For the same rise in the temperature of a body,

Percentage change in area = Percentage change in length × 2

Percentage change in volume = Percentage change in length × 3

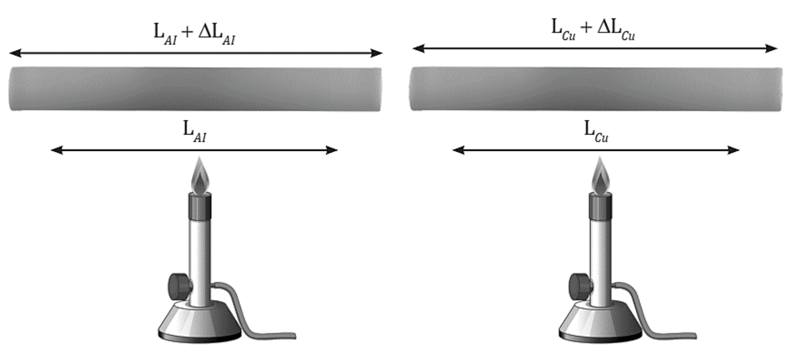
The values of α, β, and γ are independent of the units of length, area, and volume, respectively.

Ex. A copper rod of length 88 cm and an aluminum rod of unknown length have their increase in length independent of increase in temperature. The change in length of the copper rod is the same as that of the aluminum rod. Also, the temperature difference in both the rods is the same. The length of aluminum rod is:

$(\alpha_{Cu} = 1.7 \times 10^{-5} K^{-1}$ and $\alpha_{Al} = 2.2 \times 10^{-5}\ K^{-1})$

Sol. Given,

The change in length of the copper rod is the same as that of the aluminum rod. Also, the temperature difference in both the rods is the same.



For aluminum rod, change in length is as follows:

$$\Delta L_{Al} = L_{Al}\alpha_{Al}\Delta T$$

For copper rod, change in length is as follows:

$$\Delta L_{Cu} = L_{Cu}\alpha_{Cu}\Delta T$$

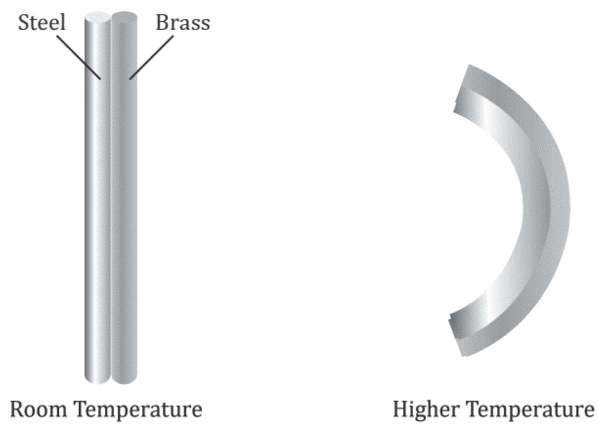
Now,

$$\Delta L_{Al} = \Delta L_{Cu}$$
$$L_{Al}\alpha_{Al}\Delta T = L_{Cu}\alpha_{Cu}\Delta T$$
$$L_{Al} \times 2.2 \times 10^{-5} = 88 \times 1.7 \times 10^{-5}$$
$$L_{Al} = 68 \text{ cm}$$

Thermal Expansion in Solid - Bi-Metallic Strip

A bimetallic strip joins two metal strips of the same length but different types of metal. They can't be separated, and each metal expands differently when heated.

Imagine a bimetallic strip with a thickness of d. It has strips of equal length and thickness made of steel and brass. Brass expands more than steel because its coefficient of expansion is higher. So, when heated, the brass expands more while the steel expands less. This unequal expansion causes the strip to bend, as depicted in the figure on the following page.

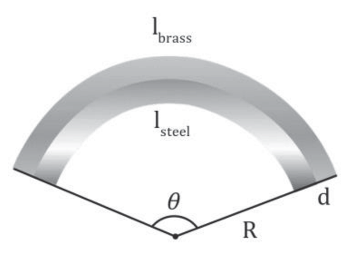


After expansion, let the radius of the curve formed by bending be R and sector angle be θ.

Length of the arc on the side of brass $(l_{brass}) = (R + d)\theta$

Length of the arc on the side of steel $I(l_{steel}) = (R)\theta$

As $\alpha_{brass} > \alpha_{steel}$, The increase in length of the brass side of the strip is more than that of the steel side strip.



The bimetalic strip will bend, with metal of greater coefficient of expansion (α) on the outer side, i.e., convex side