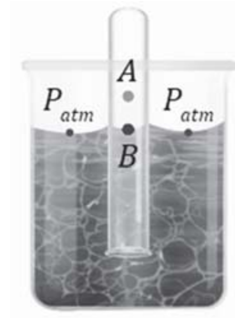
CAPILLARITY AND JURIN'S LAW

Capillarity:

Capillarity refers to the natural ascent or descent of a liquid within confined spaces like narrow tubes or porous materials. The direction of this movement hinges on the balance between the liquid's cohesion and its adhesion to the tube's walls.



Imagine a container filled with liquid, with a slender tube submerged in it. Let's focus on points A and B: A sits just above the liquid's surface, while B is just below it. The pressure at point A is.

$$P_A = P_{atm} \ \dots\dots\dots\dots (i)$$

Where *P*atm represents the atmospheric pressure, as depicted in the figure.

Referring to Figure 2, it's important to remember that the additional pressure existing within a liquid droplet can be expressed as follows:

Fig. 2

$$P_i - P_o = \frac{2T}{R}$$

$$P_{concave} - P_{convex} = \frac{2T}{R}$$

Hence, the pressure exerted on the concave side exceeds that on the convex side.

In Figure 3, it is evident that point PA lies on the concave side of the meniscus, while point PB is situated on the convex side.

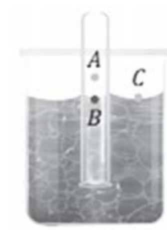


Fig. 3

Therefore,

$$P_A - P_B = \frac{2T}{R}$$

$\Rightarrow$ $$P_B = P_A - \frac{2T}{R}$$

So,

$$P_A > P_B$$

Now, let's focus our attention on point C, located on the unrestricted surface of the liquid. At this point, the pressure is equivalent to the atmospheric pressure because the liquid surface is open to the surrounding atmosphere. Therefore, the pressure at point C can be expressed as:

$$P_C = P_{atm} \ \dots\dots\dots\dots (iii)$$

Analyzing equations (i) and (iii), we observe that both points A and C correspond to atmospheric pressure (Patm). Furthermore, since PB is less than PA, it follows that PB is also less than Patm. Consequently, the pressure at point B is lower compared to the pressure at point C, denoted as $P_B < P_C$.

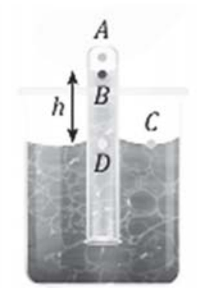


Fig. 4

This indicates that the pressure at the free surface of the liquid (point C) is higher than at point B. This pressure differential prompts the liquid surrounding the sides of the tube to exert a force, causing the liquid to rise within the tube in a capillary action.

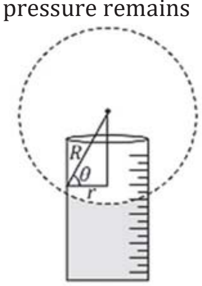
Following the ascent, let's designate point D at the same horizontal level as point C, as depicted in Figure 4.

The pressure at point D is determined by:

$$P_D = P_B + \rho g h \ \dots\dots\dots\dots (iv)$$

Similarly, the pressure at point D equals the pressure at point C, as they both lie on the same horizontal plane. In the context of a static liquid, the pressure remains uniform along the same horizontal level.

$\therefore$ $$P_D = P_C = P_{atm} \ \dots\dots\dots\dots (v)$$



From equations (iv) and (v), we get the following:

$\Rightarrow$ $$P_{atm} = P_B + \rho g h$$

$\Rightarrow$ $$P_{atm} = P_{atm} - \frac{2T}{R} + \rho g$$

(Using equations (i) and (ii), $$P_B = P_{atm} - \frac{2T}{R}$$

$\Rightarrow$ $$\frac{2T}{R} = \rho g h$$

The height by which the liquid will rise is as follows:

$\Rightarrow$ $$h = \frac{2T}{Dor}$$

Where, R is the radius of the meniscus.

Relation between the radius of the meniscus (R) and radius of the tube (r):

From the right-angled triangle as shown in the figure,

$$\cos\ \theta = \frac{r}{R}$$

$\Rightarrow$ $$R = \frac{r}{\cos\theta}$$

Substituting the value of in the equation of we get the following:

$\Rightarrow$ $$h = \frac{2T\cos\ \theta}{r\rho g}$$

Case 1: $\theta < 90°$

$$h = \frac{2T\cos\ \theta}{r\rho g}$$

Since $$\theta < 90°$$

$\Rightarrow$ $$\cos\ \theta = \text{ Positive}$$

The height of the liquid in the narrow tube will rise.

Case 2: $\theta = 90°$

$$h = \frac{2T\cos\ \theta}{r\rho g}$$

Since $$\theta = 90°,$$

$\Rightarrow$ $$\cos\ \theta = 0$$

There will be no change in the height of the liquid in the tube.

Case 3: $\theta > 90°$

$$h = \frac{2T\cos\ \theta}{r\rho g}$$

$\Rightarrow$ $$r = \frac{2T\cos\ \theta}{h\rho q}$$

Jurin's Law;

According to Jurin's law, the vertical height of a capillary column formed by a liquid at a specific temperature is inversely related to the radius of the tube.

Experiment:

Consider a scenario where a tube is equipped with a valve and connected to various capillary tubes of differing radii, as illustrated in the diagram. Initially, the valve is closed; upon opening it, the liquid within the tube disperses into all the connected tubes.

In this setup, the properties of the liquid, such as density (ρ), temperature (T), contact angle (θ), and gravitational acceleration (g), remain constant. Additionally, the capillary tubes are constructed from the same material.

Therefore

$$h = \frac{2T\cos\ \theta}{r\rho g}$$

$\Rightarrow$ $$hr = \frac{2T\cos\ \theta}{\rho q}$$

$\Rightarrow$ $$hr = \text{ Constant}$$

$\Rightarrow$ $$h \propto \frac{1}{r}$$

Therefore, the statement of Jurin's law is proved.

Note As the diameter of the tube decreases, the vertical ascent of the liquid within the tube increases proportionally.

Capillary Rise in a Tube of Insufficient Length:

Consider the following experimental setup: Begin by selecting a beaker containing a certain volume of liquid. Next, submerge a slender capillary tube vertically into the liquid. Let's assume that the liquid within the tube ascends to a height denoted by 'h'. Following this, introduce a second capillary tube into the beaker, ensuring its length ('L') is shorter than the height previously attained ('h'). Due to this discrepancy in length, according to the principle of energy conservation, the liquid will not exhibit a fountain-like expulsion. Instead, it will ascend within the second tube until reaching its uppermost point.

According to Jurin's law, when comparing the radii of the meniscus in both the long and short tubes, denoted as R and R' respectively, we can observe certain principles at play.

$$Hr = LR'$$

$$\Rightarrow \quad R' = \frac{h}{L}F$$

$$\Rightarrow \quad R' > R$$

In a tube of insufficient height, the water will rise to the top and the radius of the meniscus will increase

Capillary Rise in an Inclined Tube:

Consider the scenario where we have a capillary tube submerged vertically in a liquid, resulting in the liquid ascending to a height represented by 'h'. Now, if we tilt the tube at an angle 'α' away from the vertical position, an interesting phenomenon occurs: the height of the liquid within the capillary tube remains constant, neither rising nor falling. However, what does change is the length of the liquid column, denoted as 'l', which increases within the capillary tube.

$$h - l\cos\alpha$$

$$\Rightarrow \quad l = \frac{h}{\cos\alpha}$$

While the vertical height of the liquid column within the capillary tube remains constant, there is a notable change: the length of the liquid column within the tube extends or increases.

Ex. If water ascends a vertical capillary tube to a height of 10 cm, what length will it rise to when the tube is inclined at a 45° angle?

Sol. We have, The capillary rise in the tube, $h = 10$ cm

The angle of inclination of the capillary tube, $\alpha = 45°$.

The rise in the length of water in the capillary tube is as follows:

$$h = l\cos\alpha$$

$$\Rightarrow \quad l = \frac{h}{\cos\alpha}$$

$$\Rightarrow \quad l = \frac{10}{\cos 45°}$$

$$\Rightarrow \quad l = 10\sqrt{2}\text{ cm}$$