APPLICATIONS OF SURFACE TENSION

Work done in blowing a liquid drop and soap bubble:

Work Done in Blowing a Liquid Drop:

Let's take a liquid droplet with an initial radius of r_i. Subsequently, we attempt to enlarge the surface area, causing it to expand until the final radius reaches r_f.

The initial surface area of the liquid droplet is given by: $A_i = 4\pi r_i^2$

The eventual surface area of the liquid droplet, $A_f = 4\pi r_f^2$

Given that there's only one interface on the liquid droplet (the liquid-air interface),

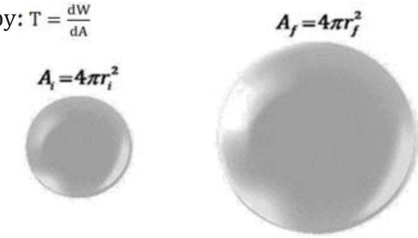
The surface tension of the liquid droplet is represented by: $T = \frac{dW}{dA}$

$\Rightarrow \quad dW = T \times dA$

$\Rightarrow \quad dW = T(A_f - A_i)$

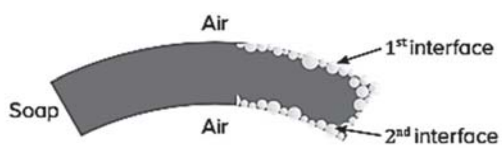
$\Rightarrow \quad dW = T \times 4\pi(r_f^2 - r_i^2)$

$\Rightarrow \quad dW = 4\pi T(r_f^2 - r_i^2)$



Work Done in Blowing a Soap Bubble:

The primary distinction between soap bubbles and liquid droplets is that soap bubbles feature two interfaces. This dual interface results in soap bubbles possessing double the surface tension compared to liquid droplets.



The soap bubble starts with an initial radius denoted as r_i and eventually reaches a final radius r_f.

The starting surface area of the soap bubble is calculated as follows: $A_i = 4\pi r_i^2$

Upon reaching its final state, the soap bubble's surface area is denoted as A_f. r_f

Since a soap bubble consists of two interfaces, its surface tension effectively doubles.

Variation in surface area, $dA = A_f - A_i = 4\pi(r_f^2 - r_i^2)$

Surface tension of the soap bubble, $T' = 2T$

$\Rightarrow \quad dW = T^1 \times dA$

$\Rightarrow \quad dW = 2T(A_f - A_i)$

$\Rightarrow \quad dW = 2T \times 4\pi(r_f^2 - r_i^2)$

$\Rightarrow \quad dW = 8\pi T(r_f^2 - r_i^2)$

Work done in splitting of bigger drop:

Let's examine a larger liquid droplet with a radius R, which splits into n smaller droplets, each with a radius r.

The surface area of the larger droplet is denoted as A_i=RAi=R,

while the combined surface area of all the identical smaller droplets is A_f=n×4πr^2.

It's understood that the volume of the larger droplet equals the total volume of all nn identical smaller droplets.

Therefore, $V_i = V_f$

$\Rightarrow \quad \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad r = \frac{R}{n^{\frac{1}{3}}} \ldots\ldots (i)$

Next, the accomplished work,

$dW = T \times dA$

$\Rightarrow \quad dW = T(A_f - A_i)$

$\Rightarrow \quad dW = T \times 4\pi(nr^2 - R^2) \ldots\ldots (ii)$

Inserting the value of ( ),

$\Rightarrow \quad dW = T \times 4\pi(n(\frac{R}{n^{\frac{1}{3}}})^2 - R^2)$

$\Rightarrow \quad dW = 4\pi R^2 T(n^{\frac{1}{3}} - 1)$

Work done in coalesce of smaller drops:

Ex. When two small drops of mercury, each with a radius R, merge to create a single large drop, what is the ratio of the total surface energies before and after the transformation?

Sol. Consider the following:
The radius of each mercury drop before merging is R.
The radius of the mercury drop after merging is r.
The combined volume of the two mercury drops equals the volume after merging.
The initial surface energy is represented by.

$$U_i = dW_i = TA_i = T \times 2 \times 4\pi R^2$$

Also,

$$V_i = V_c$$

$\Rightarrow$ $$2 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3$$

$\Rightarrow$ $$r = 2^{\frac{1}{3}}R \ldots\ldots (i)$$

The resulting surface energy

$$U_f = dW_f = TA_f$$

$\Rightarrow$ $$dW_f = T \times 4\pi \times r^2$$

$\Rightarrow$ $$U_f = dW_f = T \times 4\pi \times 2^{\frac{2}{3}} \times R^2$$

The proportion of initial to final surface energy is described as follows:

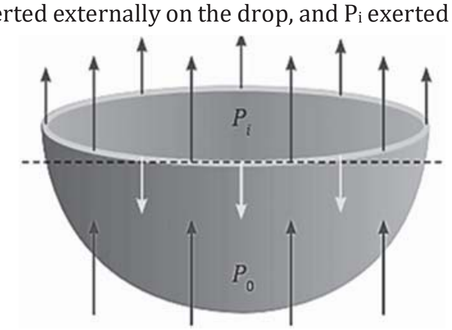
$$\frac{U_i}{U_f} = \frac{T\times2\times4\pi\ \ ^2}{T\times4\pi R^2\times2^{\frac{2}{3}}} = \frac{2}{2^{\frac{2}{3}}}$$

$$\frac{U_i}{U_f} = 2^{\frac{1}{3}} : 1$$

Excess Pressure inside a liquid drop and soap bubble:

Excess Pressure Inside a Liquid Drop:

Within a liquid droplet, two pressures are in play: P_0 exerted externally on the drop, and P_i exerted internally.



The pressure inside the drop must also accommodate surface tension. Given that P_i surpasses P_0, the surplus pressure ($P_i - P_0$) is termed as excess pressure.

Now, if we bisect the drop, we observe external pressure acting uniformly across the surface from all directions. The horizontal components of this external pressure nullify each other, leaving only the vertical components.

The pressure inside the surface counterbalances all external pressure.

For equilibrium, $\sum \vec{F}_{net} = \vec{0}$

$\Rightarrow$ $$P_0\pi R^2 + T(2\pi R) = P_i(\pi R^2)$$

$\Rightarrow$ $$P_i - P_0 = \frac{T(2\pi R)}{\pi R^2}$$

$\Rightarrow$ $$P_i - P_0 = \frac{2T}{R}$$

$\Rightarrow$ $$\Delta P = P_i - P_0 = \frac{2T}{R}$$

$$\frac{2T}{R} = \text{The surplus pressure within a liquid droplet}$$

Ex. For a spherical water droplet with a radius of 1 mm and a surface tension of 70×10^{-3} Nm^{-1}, what is the pressure difference between the interior and exterior of the droplet?

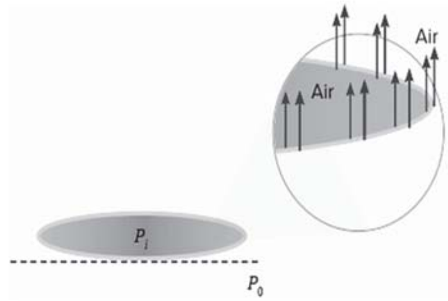
Sol. Consider the following:
The radius of the water droplet, R=1 mm = 1×10^{-3} m.
Surface tension of the water droplet, T=70×10^{-3} Nm^{-1}.
Now, let's determine the excess pressure.

$$\Delta P = \frac{2T}{R} = \frac{2\times70\times10^{-3}}{1\times10^{-3}}$$

$\Rightarrow$ $$\Delta P = 140 Nm^{-2}$$

Excess Pressure Inside a Soap Bubble:

If we cut the soap bubble in half, we observe external pressure (P_0) exerted uniformly across the surface from all directions. However, due to the soap bubble's dual interfaces, the surface tension doubles. Internally, pressure (P_i) acts upon the inner surface, effectively balancing the external pressure across the surface.



The surface tension exhibited by a soap bubble,

$$T' = 2T$$

The surface area exposed to pressure, $A = \pi R^2$

To maintain balance, $\sum \vec{F}_{net} = \vec{0}$

$\Rightarrow$ $$P_0\pi R^2 + 2T(2\pi R) = P_i(\pi R^2)$$

$\Rightarrow$ $$P_i - P_0 = \frac{2T(2\pi\ \ )}{\pi R^2}$$

$\Rightarrow$ $$P_i - P_0 = \frac{4T}{R}$$

$\Rightarrow$ $$\Delta P = P_i - P_0 = \frac{4T}{R}$$

$\frac{4T}{R}$ = The surplus pressure within a soap droplet

Ex. If the air pressure inside a soap bubble with a diameter of 0.7 cm exceeds the external pressure by 8 mm of water, what is the surface tension of the soap solution?

Sol. Consider the following measurements:

The soap bubble's diameter, d, is 0.7 cm.

Its radius, r, equals 0.35 cm.

The bubble's height, h, is 8 mm, equivalent to 0.8 cm.

The soap bubble's surface tension,

$$\Delta P = \frac{4T}{R}$$

$\Rightarrow$ $$\rho_w gh = \frac{4T}{R}$$

$\Rightarrow$ $$T = \frac{R\rho_w gh}{4}$$

$\Rightarrow$ $$T = \frac{0.35\times1\times981\times0.8}{4} = 68.67 \text{ dyne cm}^{-1}$$