VISCOSITY AND REYNOLD'S NUMBER

Relative Density:

The relative density of an object is the quotient of its density divided by the density of water at 4°C.

$$\text{R.D.} = \frac{\rho}{\rho_w}$$

When a body is entirely immersed and its weight decreases to W_a in a fluid with density ρ_a, and to W in water, the relative density (R.D.) is determined by the equation:

$$\frac{W_a}{W} = \frac{V\rho_a g}{V\rho_w g} = \frac{\rho_a}{\rho_w} = \text{R.D.}$$

Law of Floatation:

i. When a solid's density is lower than that of a liquid, it will float with a portion submerged in the liquid.
ii. When a solid's density matches that of a liquid, it will remain suspended within the liquid.
iii. If a solid's density exceeds that of a liquid, it will sink.

Ex. A wooden log with a mass of 120 kg floats in water (with a density of 1000 kg/m^3). What mass should be added to the log to precisely submerge it? (The density of wood is $\rho_{\text{wood}} = 600$ kg/m^3.)

Sol. When a mass m is placed on the wooden log, the combined weight equals the sum of the mass of the wooden log and mass m, multiplied by g. This force acts downward. Simultaneously, as the wooden log barely submerges, the buoyant force acts upward, counterbalancing the weight of the log and the additional mass.

$$V_{\log}\rho g = (m_{\log} + m)g$$

$$V_{\log} = \frac{m_{\log}+m}{\rho}$$

$$\frac{m_{\log}}{\rho_{\text{wood}}} = \frac{120+m}{1000}$$

$$\frac{120}{600} = \frac{120}{1000}$$

$$0.2 \times 1000 = 120 + m$$

$$\Rightarrow \quad m = 200 - 120 = 80$$

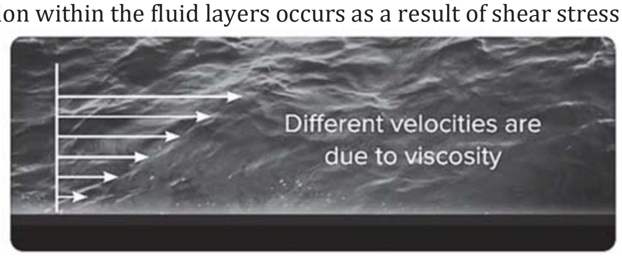
$$= 80\text{Kg}$$

Viscosity:

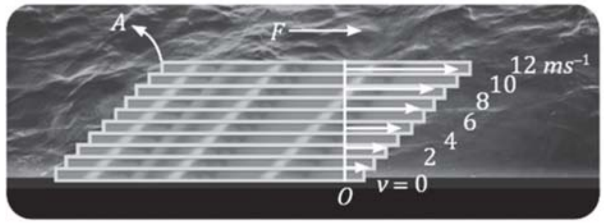
The characteristic that defines a fluid's resistance to flow is referred to as viscosity. For instance, in a comparison between honey and milk, honey exhibits higher viscosity, indicating greater resistance to flow compared to milk.

Newton's Law of Viscosity:

At the interface, there exists no relative movement between the solid particles and the fluid particles. Deformation within the fluid layers occurs as a result of shear stress and strain.



Applying a shear force to the upper layer of the fluid induces shear strain, which manifests as follows: Distinct layers begin moving at varying velocities. The ongoing deformation caused by viscous force is recognized as the rate of deformation or shear strain rate. Shear stress is directly proportional to the shear strain rate. Essentially, the continuous deformation of the fluid, even without an applied force, is termed flow.

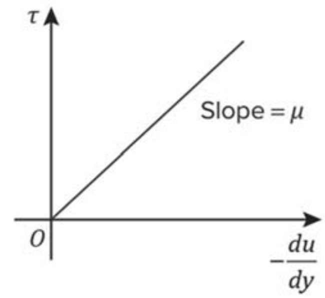


A velocity gradient will exist vertically, where the velocity magnitude differs across various layers and is depicted using velocity vectors. The shear stress is directly linked to the rate of velocity change along the vertical direction.

$$\tau \propto \frac{du}{dy}$$

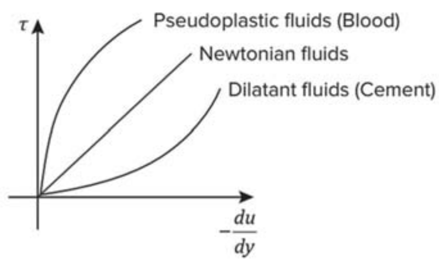
$\Rightarrow$ $$\tau = -\mu \frac{du}{dy}$$

$\Rightarrow$ $$\frac{F}{A} = -\mu \frac{du}{dy}$$



This is referred to as Newton's law of viscosity, where μ represents the viscosity coefficient. The negative sign indicates resistance to flow. The rate of shear strain equals the velocity gradient. The viscosity unit in the CGS system is poise (P) while in the SI system, it's expressed as pascal-second (Pa s) or newton per square meter-second (Nm^{-2} s).1 poise is equivalent to 0.1 pascal-second.

Note: The symbol eta (η) is alternatively utilized for the coefficient of viscosity in addition to mu (μ).
The graphs for fluids adhering to Newton's law of viscosity exhibit the following characteristics:
These fluids are termed as Newtonian fluids. Examples include water, starch solution, and others.
Fluids that deviate from Newton's law of viscosity are referred to as non-Newtonian fluids. Examples of non-Newtonian fluids include blood, cement, and others.



Ex. Water moves across an inclined surface, with the layer's thickness measuring 0.1 mm. The velocity of the uppermost surface registers at 0.1 ms^{-1}. Determine the force applied to the upper surface per unit area. (μ_{water} = 0.1 cP (centipoise)).

Sol. Provided that,

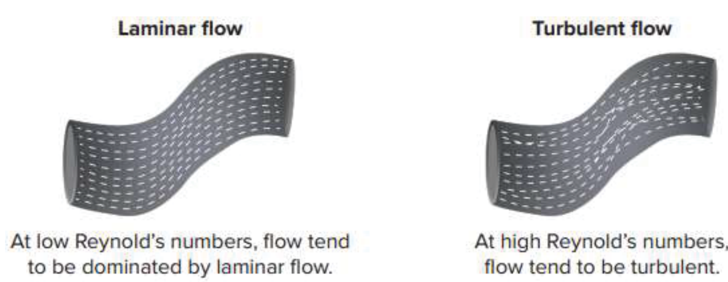
d = 0.1 mm,
v = 0.1 ms-1,
μwater = 0.1 cP = 0.0001 Pa s,
dy = 0.1 mm = 0.0001 m,
du = 0.1 ms-1

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \frac{F}{A} = 0.0001 \times \frac{0.1}{0.0001} = 0.1\text{Nm}^{-2}$$

Reynold's Number:

The term used to describe the smooth movement of a fluid is laminar flow. In this type of flow, each particle adheres to the trajectory of the preceding particle.



When the Reynolds number is low, the flow is classified as laminar, whereas higher values indicate turbulent flow. The Reynolds number primarily relies on the balance between viscous and inertial forces, making it a dimensionless quantity.

$$\text{Reynold's number} = \frac{\text{Inertial force}}{\text{Viscous force}}$$

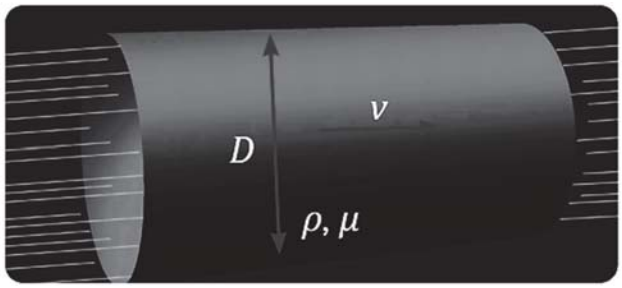
$$\text{Inertial force} = \frac{dp}{dt} = v\frac{dm}{dt}$$

Also, $dm = V \times \rho$

$\Rightarrow$ $\text{Inertial force} = v \times \frac{d}{dt}(V \times \rho) = v \times A \times \rho \times \frac{dx}{dt}$

$\Rightarrow$ $= v^2 A\rho$

The expression for viscous force is as follows:



$$\tau = \frac{F}{A} = \mu\frac{V}{D} \text{ (for pipe)}$$

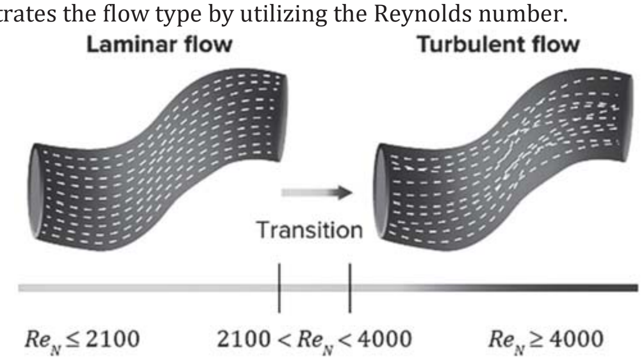
$\Rightarrow$ $F = \frac{\mu A v}{D}$

Formulating the Reynolds number involves dividing these two forces, resulting in the following formula:

$$Re_N = \frac{\rho A v^2}{(\frac{\mu A v}{D})} = \frac{D v \rho}{\mu}$$

Critical Reynolds Number:

The diagram illustrates the flow type by utilizing the Reynolds number.



If the Reynolds number is less than or equal to 2100, then the flow comes under the laminar flow. If it is between 2100 and 4000, then the flow comes under the transition flow. For turbulent flow, the Reynolds number is greater than or equal to 4000.

Types of Fluids:

Compressible fluids:

Fluids exhibiting fluctuating densities are referred to as compressible fluids, with gases being prime examples of highly compressible fluids.

Incompressible fluids:

Fluids characterized by consistent densities are termed incompressible fluids. Water serves as an example of an incompressible fluid, maintaining a constant density even under compression.

Viscous fluids:

Viscous fluids are those containing internal friction between their layers.

Non-viscous fluids:

Non-viscous fluids are characterized by the absence of internal friction between their layers.

Ideal fluids:

Ideal fluids are characterized by being both incompressible and non-viscous.

Real fluids:

Real fluids encompass compressible and viscous fluids.