TORRICELLI'S THEOREM AND TERMINAL VELOCITY

Torricelli's Theorem:

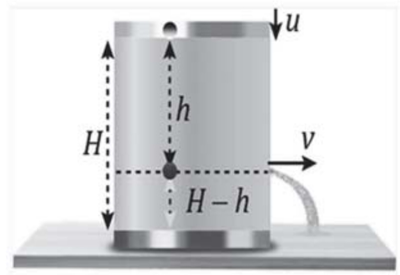
This principle asserts that the velocity of liquid efflux from an orifice equals the velocity attained by an object falling freely through a distance equivalent to the depth of the orifice beneath the liquid's free surface.

Using the equation of motion, this velocity can be determined as follows:

$$v^2 = u^2 + 2gh$$

$$\Rightarrow \quad v^2 = 0 + 2gh$$

$$\Rightarrow \quad v = \sqrt{2gh}$$



This relationship holds true only when the area of the orifice is significantly smaller than the area of the container.

In this scenario, 'h' (the height of the water level above the orifice) varies with time. Consequently, the velocity is also subject to change over time.

Range of Efflux (R):

The range of efflux refers to the horizontal distance traveled by a liquid stream discharged through the orifice.

It can be calculated by multiplying the speed of efflux by the time of fall (or flight).

$$R = v \times t$$

In this context, $v = \sqrt{2gh}$

Using the second equation of motion vertically, one can calculate the time of fall as follows:

$$H - h = u_y t + \frac{1}{2}gt^2$$

$$H - h = 0 + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

When substituting these values into the equation for range, $R = v \times t$, we obtain the following:

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$R = \sqrt{4h(H-h)}$$

Maximum range:

The range of efflux for an orifice located at a depth h beneath the free surface of water is determined by:

$$R = \sqrt{4h(H-h)}$$

$$\Rightarrow \quad R^2 = 4h(H-h)$$

For maximum R,

$$\frac{dR^2}{dh} = 0$$

$$\Rightarrow \quad \frac{d}{dh}(4Hh - 4h^2) = 0$$

$$\Rightarrow \quad 4H - 8h = 0$$

$$\Rightarrow \quad h = \frac{H}{2}$$

At $h = \frac{H}{2}$,

The maximum range of efflux occurs.

The maximum range of the liquid discharged from the orifice is as follows:

$$R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = 2 \times \frac{H}{2} = H$$

Motion of Solid Sphere Through Fluid

Stoke's Law:

As a solid moves through a fluid, it encounters a viscous force, also known as drag force, which opposes its velocity. This drag force (F_D) is influenced by the following parameters:

$$F_D \propto \text{Viscosity of the fluid } (\mu)$$
$$F_D \propto \text{Shape and size of the solid body } (r)$$
$$F_D \propto \text{Velocity of the solid body } (v)$$

In the case of a solid sphere with a radius r, the drag force can be expressed as:

$$F_D = 6\pi\mu r v$$

This principle is recognized as Stokes' law.

Note: The coefficient of viscosity, often represented by the symbol eta (η), can also be denoted by mu (μ).

Terminal Velocity:

A smooth, solid, and spherical object with a density of ρ_b and radius r descend vertically through a fluid with a density of ρ_l under the influence of gravity, encountering the following forces:

(a) Force due to gravity/weight (F_g)

(b) Buoyant force (F_B)

(c) Drag force (F_D)

The directions of these forces are depicted in the figure. Initially, the spherical object accelerates through the fluid. During this phase, the weight and buoyant forces remain constant while the drag force, $F_D = 6\pi\mu r v$, increases with the body's velocity. Thus, when these forces balance each other, the object reaches a state of equilibrium. Consequently, it achieves a constant maximum velocity known as the terminal velocity.

This velocity can be determined by equating all the forces acting on the object in the vertical direction.

$$m\frac{dv}{dt} = F_B + F_D - F_g$$
$$0 = V\rho_l g + F_D - mg$$

The drag force acting on a solid sphere with a radius r, $F_D = 6\pi\mu r v$

$$V\rho_l g + 6\pi\mu r v_T = V\rho_b g$$
$$v_T = \frac{V(\rho_b - \rho_l)g}{6\pi\mu r}$$
$$v_T = \frac{\frac{4}{3}\pi r^3(\rho_b - \rho_l)g}{6\pi}$$
$$v_T = \frac{2}{9}\frac{(\rho_b - \rho_l)r^2 g}{\mu}$$

Poiseuille’s Law:

Poiseuille's Law elucidates the correlation between fluid flow within a cylindrical conduit and the pressure disparity propelling the motion, in conjunction with the fluid's physical attributes and the pipe's geometry. This law originated from experimental verification and derivation by Sir Gotthilf Heinrich Ludwig Hagen in 1839 and Sir Jean Léonard Marie Poiseuille in 1838, eventually published in 1840.

In scenarios of laminar or smooth flow, the volumetric flow rate of a substance is typically represented by the pressure gradient divided by the resistance posed by viscosity. This resistance is contingent upon viscosity and length, with the fourth power being determined by the radius, exhibiting slight variation. Poiseuille's law is notably consistent with experiments involving uniform liquids like Newtonian fluids, which adhere to Newton's laws in conditions of minimal or zero turbulence.

According to Poiseuille's Law, the velocity of steady fluid flow through a narrow blood vessel or tube is directly proportional to the pressure and the fourth power of the tube's radius, and inversely proportional to the length of the tube and the coefficient of viscosity.

In fluid dynamics, the Hagen-Poiseuille equation, alternatively known as the Hagen-Poiseuille law, serves as another reference to Poiseuille's law. This principle finds application in predicting pressure drops in fluids coursing through lengthy cylindrical conduits.
Poiseuille's law finds practical application in various scenarios, such as analyzing airflow within lung alveoli or predicting fluid flow through materials like plastic, paper, or straws.

$$\Delta \mathrm{p} = \frac{8\mu L Q}{\pi R^4}$$

The alteration in diameter has the most profound impact on resistance. The vessel's radius correlates directly with the volume flow rate. Even slight alterations in radius can yield significant variations in volume flow. Poiseuille’s law is employed to determine the volumetric flow rate.