Chapter 14

Fluid Mechanics

PRESSURE AND ITS MEASUREMENT

Density and Relative Density:

Density:

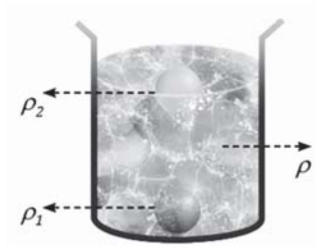
The density of any substance is defined as the mass per unit volume.

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$$

Imagine two spheres with densities ρ1 and ρ2,

In which, $\rho_2 < \rho < \rho_1$, are immersed in water of density ρ.

As illustrated in the diagram, it's evident that the sphere with a density lower than that of water floats, while the one with a greater density sink.



Relative density:

It represents the proportion of the substance's density to that of water at 4°C.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}}$$

Key points regarding relative density:

1) Relative density is dimensionless, indicating it has no units.
2) It is synonymous with specific gravity.
3) In the CGS system, the density of water at 4°C is 1 g cm^{-3}, serving as a reference point.

Density of mixture:

When two liquids with densities ρ_1 and ρ_2, masses m1 and m_2, and volumes V_1 and V_2 are combined, the resulting density of the mixture is determined as follows:

$$\rho_{\text{mix}} = \frac{m_1+m_2}{V_1+V_2}$$

Ex. When two liquids with densities ρ and 3ρ, volumes 3V and V, respectively, are blended together, determine the density of the resulting mixture.

Sol. Consider the following:

$$\rho_{\text{mix}} = \frac{m_1+m_2}{V_1+V_2}$$

In which, $m_1 = \rho(3\text{V})$ and $m_2 = 3\rho V$.

$$\rho_{\text{mix}} = \frac{\rho(3V)+3\rho V}{4V} = \frac{6\rho}{4}$$

$$\Rightarrow \quad \rho_{\text{mix}} = \frac{3}{2}\rho$$

Density of a Mixture of Two or More Liquids:

Scenario 1: Two liquids with densities ρ_1 and ρ_2, possessing identical masses, are combined.

$$\rho_{\text{mix}} = \frac{m_1+m_2}{V_1+V_2}$$

In which, $V_1 = \frac{m_1}{\rho_1}$ and $V_2 = \frac{m_2}{\rho_2}$

$$m_1 = m_2 = m$$

$$\rho_{\text{mix}} = \frac{2m}{\frac{m}{\rho_1}+\frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$$

Situation 2: Two liquids with densities ρ_1 and ρ_2, having identical volumes, are blended together.

$$\rho_{\text{mix}} = \frac{m_1+m_2}{V_1+V_2}$$

In which, $m_1 = \rho_1 V$ and $m_2 = \rho_2 V$

$$\rho_{\text{mix}} = \frac{\rho_1 V+\rho_2 V}{2V} = \frac{\rho_1+\rho_2}{2}$$

Put differently, if the volumes of both liquids are identical, the density of the mixture equals the arithmetic mean of the densities of the individual liquids. This principle extends to situations involving more than two liquids.

Pressure and its variation with depth:

Pressure refers to the force applied perpendicular to the surface of an object per unit area. It is a fundamental concept in physics and fluid mechanics, crucial in understanding various phenomena like buoyancy, fluid flow, and atmospheric conditions.

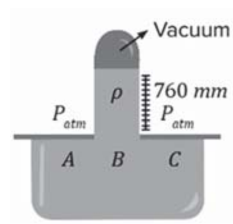
When considering pressure variation with depth, it refers to how the pressure exerted by a fluid change as one moves deeper into the fluid. This relationship is governed by Pascal's principle, which states that pressure increases with depth in a fluid due to the weight of the fluid above. This means that the deeper one goes into a fluid, the greater the pressure exerted on an object submerged within it. This principle is essential in various fields such as hydrodynamics, engineering, and geology.

Torricelli's Experiment:

In line with the notion of atmospheric pressure,

$$P_P = P_C = P_{atm}$$

Nonetheless, the pressure remains constant at all points along the same horizontal line.



$$P_A = P_B = P_C$$

Define P' as the pressure of the vacuum.

$$P_B = P' + Pgh$$

$\Rightarrow$ $P_B = 0 + \rho gh$

$$P_B = \rho_{Hg} g \times (760\text{mm})$$

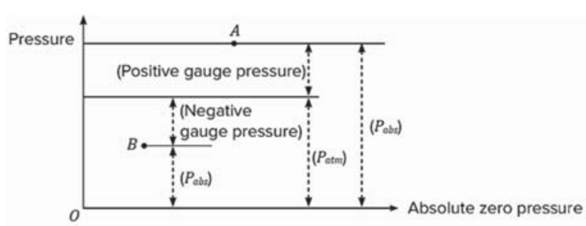
One atmosphere of pressure is also characterized as the pressure exerted by a column of mercury measuring 760 mm in height.

At sea level, $1 \text{ atm} = 0.76 \times 13.6 \times 10^3 \times 9.8$

$$= 1.013 \times 10^5 \text{ Nm}^{-2}$$

Relation between Atmospheric, Absolute and Gauge Pressure:

Atmospheric Pressure:



The pressure measured from a zero-reference point is termed absolute pressure.

From the graph,

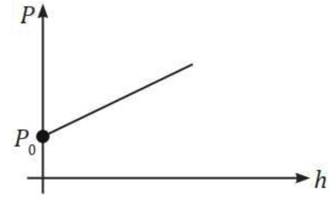
$$\text{At } A, P_{abs} = P_{atm} + P_{gauge}$$

$$\text{At } B, P_{abs} = P_{atm} - P_{gauge}$$

The force exerted by the air's weight over the Earth's surface is termed atmospheric pressure. Pressure measured in relation to atmospheric pressure serves as a reference point, known as gauge pressure.

Gauge pressure = Absolute pressure – Atmospheric pressure

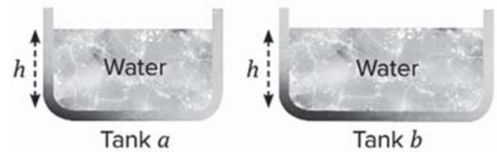
The pressure measured from absolute zero is referred to as absolute pressure. The graph illustrates pressure variation in direct correlation with altitude.



$$P = \rho g h + P_0$$

When comparing the provided equation to y = mx + c, it's evident that pressure P exhibits linear variation with height h. The primary purpose of gauge pressure lies in determining absolute pressure. In everyday scenarios, all pressure measuring instruments gauge pressure.

Ex. Based on the provided figure, what is the accurate observation?



Sol. The water column's height in both containers is identical.

According to the equation, $P = P_0 + \rho g h$

The pressure, P, is influenced by the height within a stationary fluid and the atmospheric pressure at the specified location.

Thus, the pressure at the bottom of tank a and tank b is the same.

Ex. What is the pressure experienced by a swimmer located 20 meters beneath the surface of water at sea level? (Assume $g = 10\ m/s^2$)

Sol. Given,

$$h = 20\ m, g = 10\ ms^{-2}, \rho = 1000\ kg\ m^{-3}$$

$$P = P_{atm} + \rho g h$$

$$P = (1 \times 10^5) + (1000 \times 10 \times 20)$$

$\Rightarrow$ $$P = (1 \times 10^5) + (2 \times 10^5)$$

$$P = 3\ atm$$