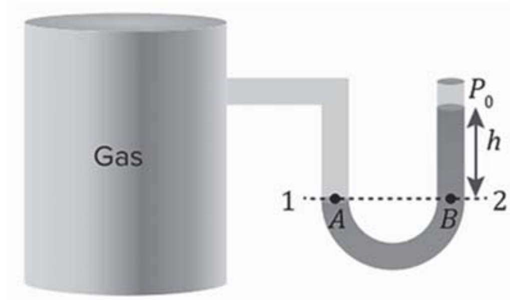
MANOMETER AND LAWS OF FLOTATION

Manometer:

A U-tube manometer consists of a tube bent into a "U" shape and filled with a liquid. Its purpose is to gauge pressure disparities between various fluids. Comparable to a barometer which assesses atmospheric pressure across various locations, the manometer's internal liquid is referred to as the manometric fluid.

Working of a Manometer:

In the case of a gas confined within a container, the pressure remains uniform across different points, unlike the pressure of liquid, which can vary within the same container. This characteristic makes a manometer ideal for assessing gas pressure within a container.



The level of the manometric fluid fluctuates according to the gas pressure, Pg, when the pressure valve is opened. Suppose gas molecules enter limb 1, displacing the liquid and causing a rise in the height of the fluid in limb 2. For a container-bound gas,

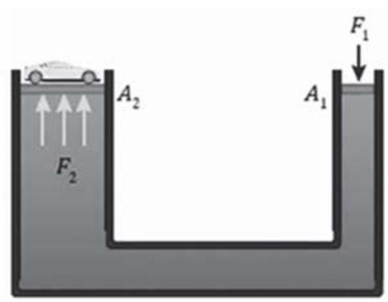
$$P_A = P_B$$
$$P_g = P_0 + \rho_m g h$$

Here, ρm represents the density of the manometric fluid. The level of the manometric fluid varies according to the pressure of the fluid to which it is linked.

Pascal's Law:

In a confined fluid, pressure applied from one end disperses equally in all directions throughout the fluid. This condition requires the fluid to be at rest and in equilibrium, with the effect of gravity disregarded.

As per Pascal's principle, $$P_1 = P_2$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

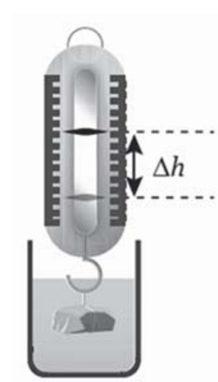


Utilizing this straightforward mechanism, also referred to as the piston and plunger system, one can elevate heavy objects positioned on a piston with a larger cross-sectional area by applying pressure to a smaller cross-sectional area. Let's take the example of lifting a car.

To raise a car weighing 1000 N with a force of 1 N, pressure is exerted on an area with a cross-section of 0.001 m^2 to lift the car placed on an area with a cross-section of 1 m^2. Hence, the pressure at both ends remains consistent.

Archimedes' Principle:

Archimedes' principle asserts that the total upward force exerted on an object submerged, either partially or entirely, in a fluid is equivalent to the weight of the fluid displaced by the object.



Imagine a stone initially suspended from a spring balance with a certain reading. When this stone is submerged in liquid, the reading on the spring balance decreases.

Therefore, when an object is submerged, partially or fully, in a fluid, the fluid applies an upward force on the object, commonly referred to as the buoyant force. This buoyant force diminishes the weight of the submerged stone. The resulting weight of the object is termed the apparent weight

When the buoyant force matches the body's weight, the body will float in the liquid.

To ensure the body remains stationary in the liquid, the buoyant force must balance the gravitational force.

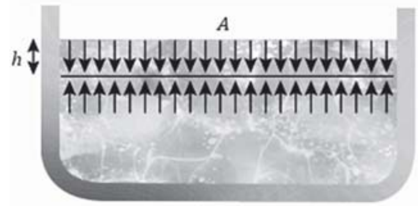
Increasing the volume of displaced fluid leads to a corresponding increase in the buoyant force, and vice versa.

In a container with a cross-sectional area A, at a depth of h, for a stationary layer, The total downward force exerted on the layer is expressed as follows:

$$F_{down} = F_g + F_{atm}$$

$\Rightarrow$ $$F_{down} = mg + P_{atm}A = \rho(Ah)g + P_{atm}A$$

Therefore, the pressure at any point within the same layer can be described as follows:



$$P = \frac{F_{down}}{A} = \frac{\rho(Ah)g + P_{atm}A}{A}$$

$\Rightarrow$ $$P = P_{atm} + \rho gh$$

Now, let's examine a scenario in which a solid is immersed in a liquid. The upward buoyant force is determined using the diagram.

$$F_{up} = P \times A = (P_{atm} + \rho gh)A$$

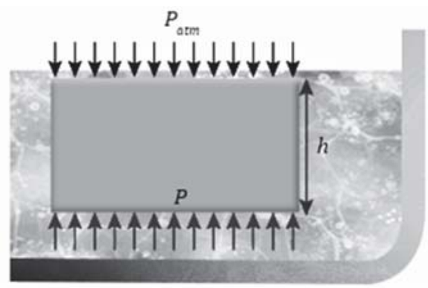
$$F_{down} = -P_{atm} \times A$$

$$F_{total} = F_{up} + F_{down}$$

$\Rightarrow$ $$F_{total} = A \times P_{atm} + \rho ghA - P_{atm} \times A$$

$\therefore$ $$F_{total} = \rho g(hA)$$

$\Rightarrow$ $$F_{total} = V\rho g(\because V = hA)$$



Here, V represents the volume of the solid.

The provided force, Ftotal, corresponds to the buoyant force exerted upwards on the body when it's immersed in the fluid. This buoyant force is determined by multiplying the volume of the submerged portion, the fluid's density, and the acceleration due to gravity. The apparent weight of the body (with density σ) when it's partially or fully submerged in the fluid is expressed as follows:

$$W_{app} = F_g - F_B = V\sigma g - V\rho g$$

$\Rightarrow$ $$W_{app} = Vg(\sigma - \rho)$$

Where, F_B = Buoyant force acting on the body

The following conclusions can be drawn from the provided equation:

a. If the density of the body exceeds that of the fluid, it sinks as the buoyant force cannot counteract its weight.

b. If the buoyant force surpasses the body's weight, it will float. This occurs when the body's density is lower than that of the fluid.

c. When the densities of both the body and the fluid are equal, the body remains suspended.