Chapter 13

Mechanical Properties of Solids

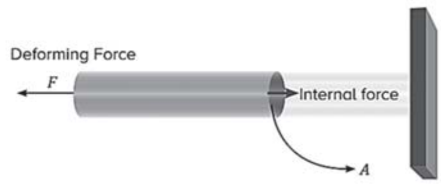
- Stress and Strain
  1. Definition and types of Stress
  2. Definition and types of Strain
  3. Hooke's Law
  4. Modulus of Elasticity
  5. Young's Modulus
- Hooke's Law and Modulus of Elasticity
  1. Modulus of Rigidity
  2. Bulk Modulus
  3. Stress and Strain Curve
  4. Ductile and Brittle Material
  5. Poisson's Ratio
  6. Work done in stretching a wire

STRESS AND STRAIN

Definition and types of Stress:

Definition of Stress:

Stress refers to the internal restoring force exerted per unit area of the cross-section.



Imagine a rod with a cross-sectional area A connected to a wall as depicted in the figure.
When a force F is applied to pull the free end of the rod, stress is induced within the rod.

In a state of equilibrium:

The influence of the external forces equals the impact of the internal restoring force. Consequently, the stress induced in the rod can be described as:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

SI unit: Nm^{-2}
CGS unit: dyne cm^{-2}

Ex. A 1 kN load is applied to a bar with a cross-sectional area of 0.8 cm^2 and a length of 10 cm. What is the stress experienced by the bar?

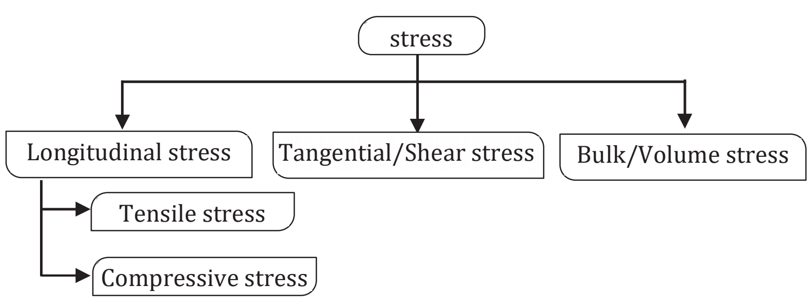
Sol. Provided:

The external force exerted on the bar,

$$F = 1 \text{ kN} = 1000 \text{ N Cross-sectional area of the bar,}$$
$$A = 0.8 \text{ cm}^2 = 80 \text{ mm}^2$$

Therefore,

$$\text{The stress induced} = \frac{\text{Force}}{\text{Area}}$$
$$\Rightarrow \quad \text{Stress} = \frac{1000 \text{ N}}{80 \text{ mm}^{-2}}$$
$$\Rightarrow \quad \text{Stress} = 12.5 \text{ N mm}^2$$



Types of Stress:

Normal stress arises from a normal force applied to a surface.

1. **Longitudinal stress:**

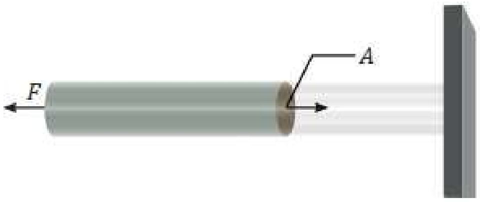
Stress generated by a normal force acting along the length of the object is referred to as longitudinal stress.

Longitudinal stress is of two types:

(a) **Tensile stress (σ_T):**

Tensile stress, induced by a tensile force, is the form of longitudinal stress. Suppose a tensile force F is applied to a rod with cross-sectional area A. The resulting tensile stress in the rod can be expressed as:

$$\sigma_T = \frac{F}{A}$$



(b) **Compressive stress (σ_C):**

Compressive stress, resulting from a compressive force, is a type of longitudinal stress. Suppose a compressive force F is applied to a rod with cross-sectional area A. The compressive stress induced in the rod can be described as:

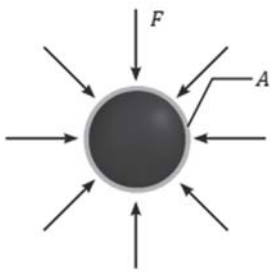
$$\sigma_c = \frac{F}{A}$$

2. **Bulk or Volume stress (P):**

The stress arising from a change in volume of a body due to the normal deforming force acting on each side of the body is referred to as bulk or volume stress. The stress developed can be expressed as:

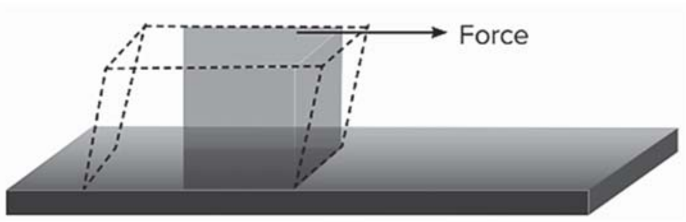
$$P = \frac{F}{A}$$

Wherein, P denotes the pressure.



Shear or Tangential Stress:

The stress induced by a deforming force applied tangentially to one of the faces of the body is referred to as shear or tangential stress. Let's consider a block affixed to a surface. When a force is applied tangentially to the upper surface of the block, the resulting deformation in the block is depicted in the figure.



Definition and types of Strain:

Strain, a dimensionless quantity, represents the ratio of the change in configuration to the original configuration.

Strain

(i) **Linear strain**

(ii) **Shearing strain**

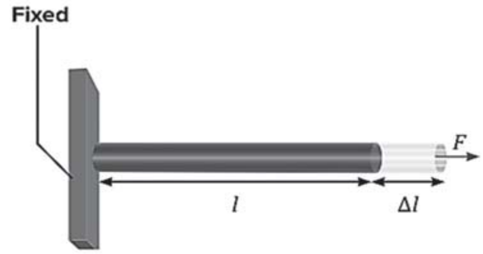
(iii) **Volumetric strain**

(1) **Longitudinal strain or Linear strain (ε):**

Linear strain refers to the ratio of the change in length to the original length of the object in the direction of the applied force.

Let's consider a rod of length l fixed against a vertical surface, as illustrated in the figure. When a force F is applied to the opposite section, its length increases by Δl.

The resulting strain due to the change in length can be expressed as:

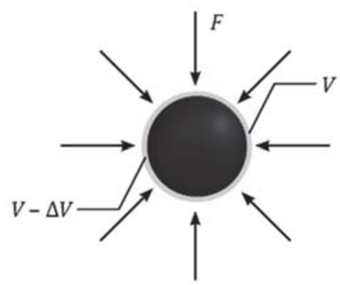


$$\varepsilon = \frac{\Delta l}{l}$$

(2) **Volumetric strain (θ):**

Volumetric strain is defined as the ratio of the change in volume of the body to its original volume. Imagine a sphere with volume V experiencing a force F acting uniformly across its surface. Suppose the volume of the sphere decreases by ΔV. The resulting strain due to the change in volume can be described as:

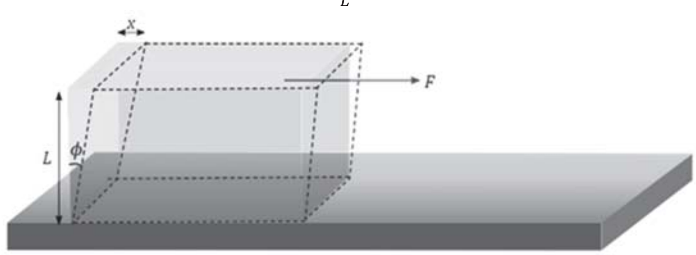
$$\theta = \frac{\Delta V}{V}$$



Shearing Strain (ϕ):

When a deforming force alters only the shape of a body without affecting its volume, the resulting strain is referred to as shearing strain. Let's consider a block secured on a surface. When a force is applied tangentially to the upper surface of the block, the deformation in the block is illustrated in the figure on the following page. Suppose the vertical face of the block is deformed by an angle, ϕ. The shear strain induced by the change in shape can be expressed as:

$$\phi = \frac{x}{L}$$

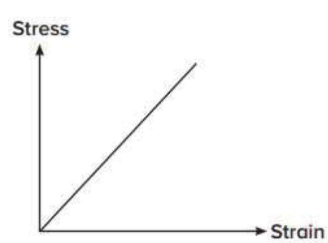


Elastic limit of a body:

The elastic limit refers to the maximum stress level at which a body can recover its original shape.

Hooke's Law:

Per Hooke’s law, the stress generated within a body under the elastic limit is directly proportional to the strain. The provided graph illustrates the correlation between stress and strain within this limit.



Modulus of Elasticity:

The modulus of elasticity is defined as the ratio of stress to strain.

Modulus of Elasticity

↓

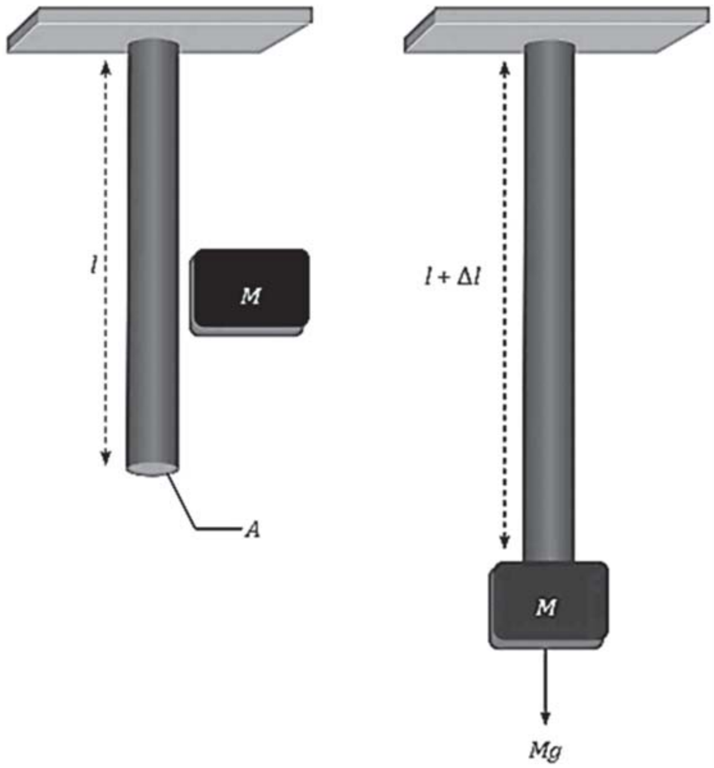
Young's modulus (Y) Modulus of rigidity (G) Bulk modulus (K)

Young's Modulus:

Young's modulus, within the elastic limit, represents the ratio of normal stress to longitudinal strain. Consider a rod with cross-sectional area A and length l suspended from a fixed support. A mass M is connected to the free end of the rod.

In this scenario, the deforming force is the weight, Mg, exerted downwards, resulting in tensile stress on the rod.

$$\sigma_T = \frac{Mg}{A}$$



The strain along the length caused by the force is given by $\varepsilon = \frac{\Delta l}{l}$

When the stress remains within the elastic limit, the Young's modulus of the rod is given by:

$$\text{Young's modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$\text{Young's modulus} = \frac{\sigma_\tau}{\varepsilon}$$

$$\Rightarrow \quad Y = \frac{\frac{Mg}{A}}{\frac{\Delta l}{l}}$$

$$\Rightarrow \quad Y = \frac{Mgl}{A\Delta l}$$

Young's Modulus (Y):

Young's modulus is the ratio of normal stress to longitudinal strain within the elastic limit. Let's envision a rod of length l and cross-sectional area A secured at one end, with a block of mass M affixed to the opposite end. As a result of this attachment, the rod undergoes elongation by Δl.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

Presently, the rod's new length is l + Δl.

The force exerted on the rod's cross-section is,

$$F_n = Mg$$

The alteration in the rod's length is, $\quad l + \Delta l - l = \Delta l$

The perpendicular stress exerted on the rod is, $\sigma_s = \frac{F_n}{A}$

$$\Rightarrow \quad \sigma_n = \frac{Mg}{A} \quad \ldots\ldots\ldots\ldots\text{(i)}$$

The strain along the length of the rod is, $\varepsilon = \frac{\Delta l}{l}$ ………….(ii)

$$\text{Young's modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

By combining equations (i) and (ii), we obtain,

$$Y = \frac{(\frac{Mg}{A})}{(\frac{\Delta l}{l})} = \frac{Mgl}{A\Delta l}$$