HOOKE'S LAW AND MODULUS OF ELASTICITY

Modulus of Rigidity:

Within the elastic limit, the relationship between tangential stress and shearing strain is defined by the modulus of rigidity (G).

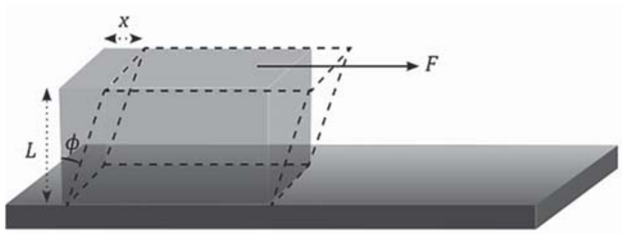
$$G = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

Let's examine a cube with its lower face fixed, while a tangential force F is applied to the upper face, which has an area A, as depicted in the figure. The displacement of the upper face is denoted by x.

Tangential stress $= \frac{F}{A}$

Shearing strain $= \frac{x}{L} = \tan\phi \approx \phi$

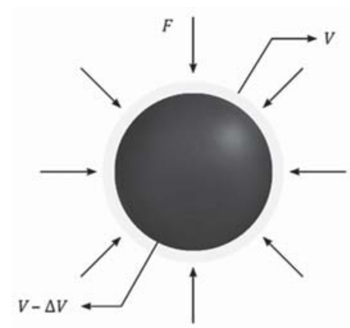
$$G = \frac{\frac{F}{A}}{\phi} = \frac{F}{A\phi}$$



Bulk Modulus:

Within the elastic limit, the relationship between normal stress and volumetric strain is described by the bulk modulus (K).

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$



Let's consider a ball with a volume V subjected to a force F applied uniformly from all directions perpendicular to the surface of the ball.

$$K = \frac{\frac{F}{A}}{-\frac{\Delta V}{V}} = -\frac{\Delta P \times V}{\Delta V}$$

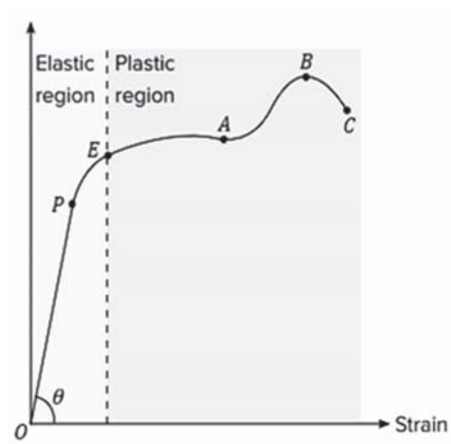
Where ΔP represents the rise in pressure, with the '-' sign indicating the ball's compression.

Stress and Strain Curve:

The data depicted in the graph (refer to the subsequent page) corresponds to specific points denoted by the following labels:

P: Proportional limit

C: Fracture point



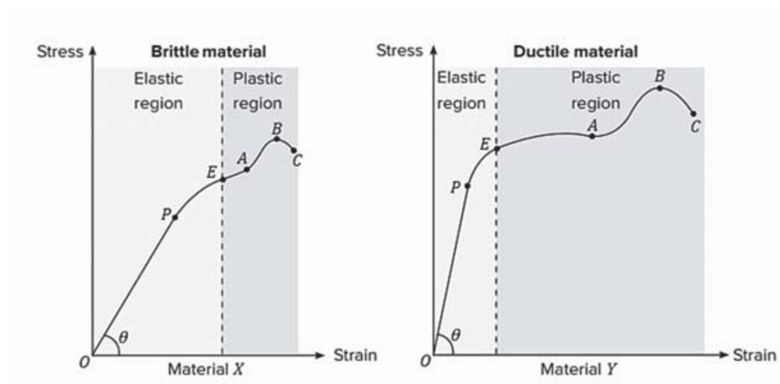
Within the OP segment, stress exhibits a proportional relationship with strain. Hooke’s law, which asserts that stress (σ) within the elastic limit is directly proportional to the corresponding strain (ε), is applicable: Stress ∝ Strain. Hence, it can be deduced that Hooke’s law holds true within this range.

After surpassing point E, the body undergoes irreversible deformation. The stress associated with point E is identified as the material's yield strength. Point B denotes the stress known as the ultimate tensile st rength of the material. For brittle materials, the plastic region between points E and C is minimal. Conversely, for ductile materials, this plastic region extends significantly between points E and C.

Ductile and Brittle Material:

Ex. Stress-strain curves have been plotted for two distinct materials, denoted as X and Y. Upon examination, it becomes evident that for material X, the ultimate strength point and the fracture point are in close proximity, whereas for material Y, these points are significantly distant from each other. From this observation, it can be inferred that material X is more likely to be (respectively),

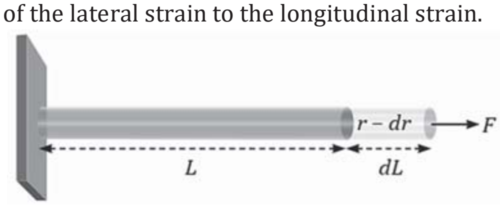
Sol. In ductile materials, the fracture point is situated below the ultimate tensile strength, indicating a notable separation between these two points. Conversely, in brittle materials, the fracture point closely aligns with the ultimate tensile strength, resulting in minimal distance between them. Based on this characteristic, it can be concluded that material X exhibits brittleness, while material Y demonstrates ductility.



Poisson's Ratio:

Poisson's ratio (v) is defined as the negative ratio of the lateral strain to the longitudinal strain.

$$v = \frac{-\frac{dr}{r}}{\frac{dL}{L}}$$

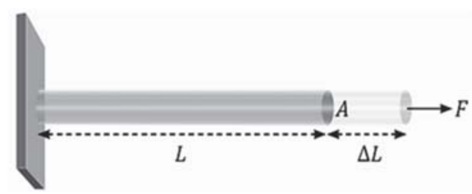


Work Done in Stretching a Wire:

Let's consider a scenario where a force denoted as F is applied along the length of a rod with a length of L and a cross-sectional area of A.

Stress, represented as "σ", is calculated as the ratio of the applied force F to the cross-sectional area A:

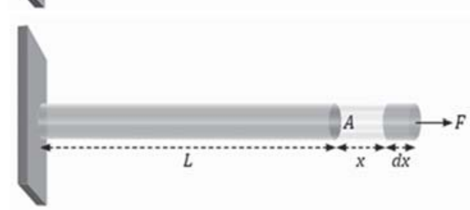
$$\text{Stress} = \frac{F}{A}$$



Similarly, strain, denoted as "ε", is determined by the change in length ΔL relative to the original length L:

$$\text{Strain} = \frac{\Delta L}{L}$$

These expressions allow us to quantify the stress and strain experienced by the rod under the applied force.



$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

⇒ $F = \frac{YA\Delta L}{J}$ ……………(i)

Using equation (i), we derive the following:

⇒ $F = \frac{YAx}{L}$ ………………..(ii)

The energy expended for a further increase (dx) in length is,

$$dW = F\,dx$$

The cumulative work performed in extending the length by ΔL is,

$$W = \int_0^{\Delta L} dW = \int_0^{\Delta L} F dx$$

⇒ $$W = \int_0^{\Delta L} \frac{YAx}{L} dx$$

⇒ $$W = [\frac{YA\ ^2}{2L}]_0^{\Delta L}$$

⇒ $$W = \frac{YA(\Delta L)^2}{2L}$$