Chapter 12

Rigid Body

INTRODUCTION TO RIGID BODY

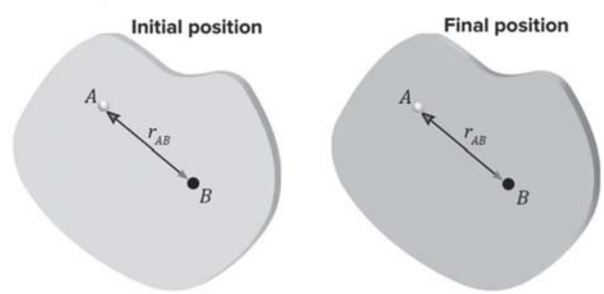
Introduction to rigid body:

The expression of a rigid body's characteristics encompasses several key aspects:

1) The rigid body maintains its original shape and dimensions throughout its motion.
2) The distances between all pairs of particles constituting the body remain constant, experiencing no alteration.
3) Within the rigid body, there is an absence of both separation velocity and approach velocity among its constituent particles.

To put it differently, in any form of motion exhibited by the body, there exists a complete absence of relative motion between any pair of particles forming the rigid structure.

Ex. Let's examine two specific points, labeled A and B, which belong to a rigid body and are initially situated at a distance denoted as r_{AB}. As the body undergoes a transition from its initial state to a final state, it is notable that the separation between these two points, A and B, persists unchanged, retaining its original value of r_{AB} throughout this motion.



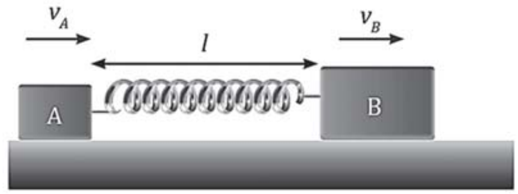
Special cases of rigid body:

Let's Explore Specific Scenarios Involving Rigid Bodies:

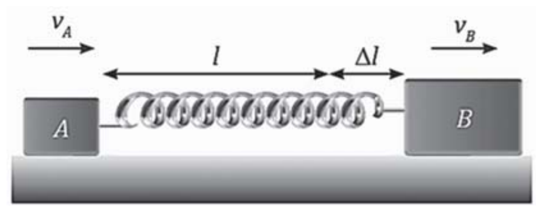
Special Case 1:

Imagine A Configuration Where a Spring, Possessing A Length Denoted As L, Is Affixed to Two Blocks Labeled a and B, As Depicted in The Accompanying Diagram. When Both Blocks, A And B, Exhibit Identical Velocities ($V_a = V_b$), The Collective System Comprising the Two Blocks and The Interconnected Spring Exhibits Characteristics Akin to A Rigid Body.

This Behavior Emerges Due to The Consistent Maintenance of Both Distance and Velocity Between Any Pair of Particles Within the System.

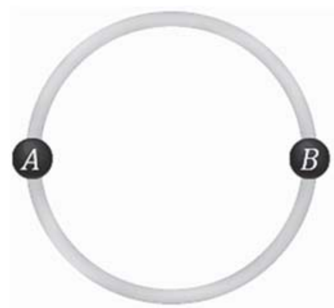


In Instances Where the Blocks Are in Motion with Varying Velocities, Albeit in The Same Direction, The Spring Undergoes Deformation, Either Elongating or Compressing. Specifically, When the Velocity of Block B Exceeds That of Block A (Vb > Va), The Spring Experiences an Increase in Length, Extending from Its Original Length Denoted as L To a New Length Represented as L + Δl. Consequently, Under These Conditions, The System Deviates from Rigid Body Behavior.



Case 2: Let's delve into another scenario involving two beads, labeled A and B, engaged in rotational motion around a circular ring, as illustrated in the provided diagram.

In the event where both beads possess identical angular velocities ($\omega_A = \omega_B$), the separation between them remains constant throughout their rotation. Under these circumstances, the system comprising the two beads exhibits characteristics akin to a rigid body.



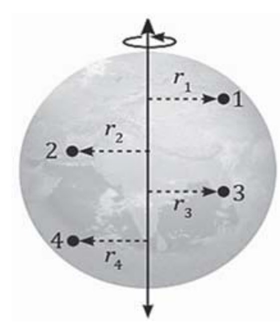
However, if we extend our analysis to include the entire system encompassing the two beads and the ring, the situation changes. Due to the altering relative positions of the beads concerning any given point on the ring as they rotate, the collective system of the beads and the ring fails to adhere to rigid body behavior.

Furthermore, when the beads rotate with differing angular velocities, their relative positions undergo variation. Consequently, in this scenario, the overall system fails to conform to the principles of a rigid body.

Difference between Circular and Rotational Motion:

Here's a detailed rephrasing:

Circular motion pertains specifically to an individual particle, while rotational motion pertains to a grouping of particles that collectively constitute a rigid body.



Ex. Consider the particles of Earth, labeled as 1, 2, 3, and 4, each engaging in individual circular motions, as depicted in the provided illustration. In this scenario, Earth itself undergoes rotational motion, while the individual particles execute circular motions.

Axis of Rotation:

The axis of rotation refers to the path traced by the centers of all particles within a rigid body engaged in circular motion.

It is an essential concept utilized to illustrate and define rotational motion accurately.

Characteristics of the Axis of Rotation:

1) The axis of rotation is not obligated to traverse through the body itself.
2) It is not necessarily stationary; it can vary in position.
3) The axis of rotation is not mandated to be perpendicular to the surface plane of the object.

Revision of Circular Kinematics:

The angular velocity of any point within a rigid body, concerning any other point within the same body, remains constant. This uniformity in angular velocity across all particles enables the definition of the overall angular velocity of the rigid body.

In both circular and rotational motion, certain parameters and their relationships play crucial roles.

These terms are fundamental to understanding both types of motion.

Angular Position		Rate of angular position change		Angular velocity		Rate of angular velocity change		Angular acceleration
θ	$\longrightarrow$	$\frac{d\theta}{dt}$	$\equiv$	ω	$\longrightarrow$	$\frac{d\omega}{dt}$	$\equiv$	α
	$l = r\theta$		$v = r\omega$		$a_t = r\alpha$		$\alpha_C = \frac{v^2}{r}$	

Equations of angular kinematics:

The equations of kinematics for constant tangential and angular acceleration magnitude are as follows:

$$\omega = \omega_0 + \alpha t$$
$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$