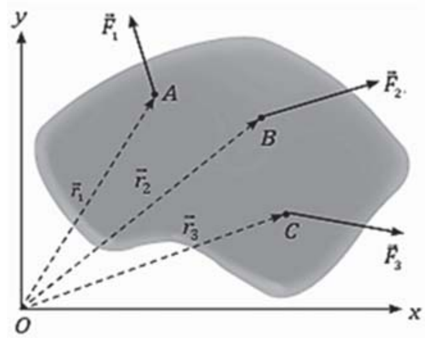
INTRODUCTION TO MOMENT OF INERTIA

The moment of inertia, commonly represented as I, stands as a cornerstone concept in both physics and engineering, especially within the realm of rotational dynamics. It serves as a metric for gauging an object's reluctance to alterations in its rotational state, much akin to how mass reflects an object's resistance to changes in linear motion.

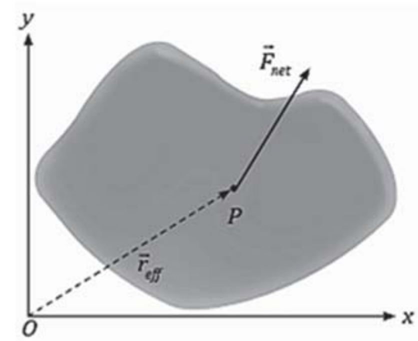
Point of Application of Force:

The point of force application refers to the precise location on an object where the net force must be exerted to achieve the same effects as the original force orientation.



Let's consider a scenario with three forces, $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ acting on a body at points A, B, and C, respectively, in the directions indicated in the diagram. Let $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ denote the position vectors of these three points.

In the case of a point mass, the application of multiple forces results in translational motion. However, in the case of a rigid body, since the points of force application may vary, the combined effect of these forces can induce both translational and rotational motions.



The collective impact of multiple forces can be consolidated into a single net force, net $\vec{F}_{net}$, which contributes to both translational and rotational effects on the body. Let P represent the point where this net force acts on the rigid body. The torque exerted by the applied force about this point is zero, as it corresponds to the point of application of the net force, where the force arm becomes zero. We can express the rotational effect of these forces as follows:

$$[\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3] = \vec{r}_{\text{eff}} \times \vec{F}_{\text{net}}$$

Toppling:

Consider a cuboid subjected to the application of force F, depicted in Figure A. In this scenario, four distinct forces act upon the cuboid:

1) The applied force.
2) The weight of the cuboid.
3) The normal reaction exerted by the ground.
4) The force arising from static friction.

It's noteworthy that both force F and the weight pass through point O. Consequently, their torque about point O amounts to zero. Upon increasing the magnitude of force F, the distribution of pressure undergoes uniform changes, causing the normal reaction to shift towards the right, as illustrated in Figure B. In this configuration, the torques generated by the normal reaction and friction counterbalance each other, ensuring the stability of the cuboid. However, the edge of the cuboid represents the maximum limit for the displacement of the normal reaction. When a force is applied beyond this limit, the body initiates the toppling motion.

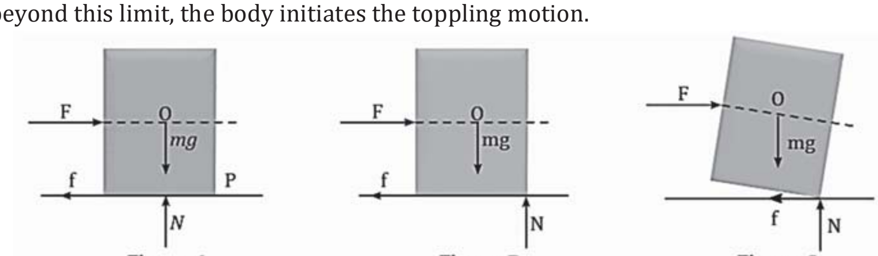
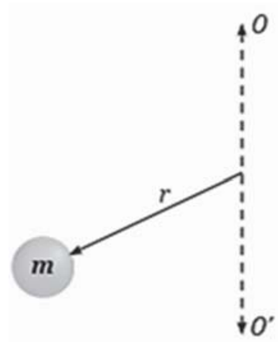


Figure A Figure B Figure C

Moment of Inertia of a Point Mass:

In rotational motion, the equivalent counterpart of mass is referred to as the moment of inertia. It serves as a measure of a body's resistance to changes in its state of rest or uniform rotation. The moment of inertia (I) for a particle with mass m situated at a distance r from an axis is defined by the following equation:

$$I = mr^2$$



The moment of inertia shares similarities with mass in that it is always positive. It is a scalar quantity, meaning it only has magnitude, and this magnitude is directly related to the square of the distance between the mass and the axis of rotation. Without an axis of rotation, the concept of moment of inertia lacks definition. Also referred to as the second moment of mass, the moment of inertia describes how mass is distributed around an axis.

Moment of inertia of multiple masses:

If the moments of inertia of multiple masses are defined with respect to the same axis of rotation, they can be summed together. The moment of inertia of n particles with masses m_1, m_2,..., mn, located at distances r_1, r_2,..., rn, respectively, from an axis OO', is expressed as follows:

$$I = \sum_{i=1}^{n} (m_i r_i^2)$$