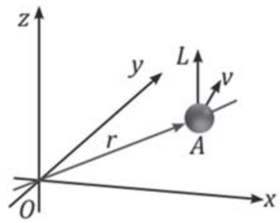
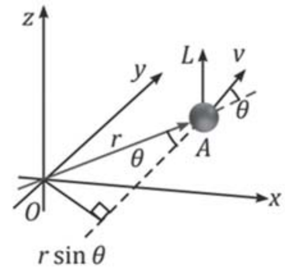
INTRODUCTION TO ANGULAR MOMENTUM

The torque or moment of force is defined as, $\vec{\tau} = \vec{r} \times \vec{F}$

The angular momentum about a specific point, or moment of linear momentum, is described as,

$$\vec{L} = \vec{r} \times \vec{p}$$





The angular momentum of a particle A, with velocity v and mass m, around the origin O, is given

by: $$\vec{L} = \vec{r} \times \vec{p}$$

or, $$\vec{L} = m(\vec{r} \times \vec{v})$$

The angular momentum's magnitude can alternatively be expressed as:

$$L = rp \sin\theta = (r \sin\theta)p = r(p \sin\theta)$$

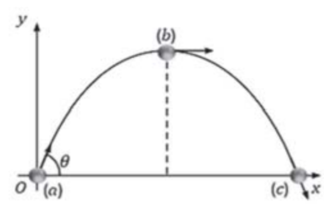
Angular momentum always lies perpendicular to the plane formed by $\vec{r}$ and $\vec{P}$. The SI unit for angular momentum is kg m^2 s^{-1}.

Ex. A particle with mass m is launched on a horizontal surface with an initial velocity of u, forming an angle θ with the horizontal. Determine the angular momentum of the particle relative to the point of projection in the given scenarios:

(a) At the onset of its motion

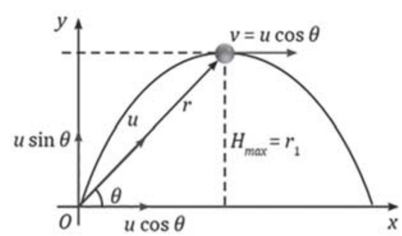
(b) At the peak of its trajectory

(c) Upon impact with the ground



Sol. (a) The angular momentum of the particle with respect to the point of projection at the beginning of its motion

Since the particle's initial velocity is zero, its angular momentum is likewise zero at its starting position.



(b) The angular momentum of the particle relative to the point of projection when it reaches the highest point of its trajectory at this juncture, H_{max} represents the perpendicular distance between the trajectory's path and the point of projection.

At this juncture, the velocity of the particle is given by $v = u \cos\theta$.

It is understood that, $$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

Therefore, the angular momentum can be expressed as:

$$L = r_\perp p$$

$$\Rightarrow \quad L = (H_{max})m(u\cos\theta)$$

$$\Rightarrow \quad L = \frac{u^2 \sin^2\theta}{2g} \times mu\cos\theta$$

$$\Rightarrow \quad L = \frac{mu^3 \sin^2\theta \cos\theta}{2g}$$

Since the direction of angular momentum is along the axis, it can be expressed as follows:

$$\vec{L} = \frac{mu^3 \sin^2\theta \cos\theta}{2g}(-\hat{k})$$

(c) The angular momentum of the particle relative to the point of projection at the moment it impacts the ground

At this juncture, the range (R) denotes the perpendicular distance between the trajectory's path and the point of projection.

The velocity of the particle in the y-direction at this point is represented by: $v = u \sin \theta$

It is understood that $R = \frac{u^2 \sin(2\theta)}{g}$

Therefore, the angular momentum can be expressed as:

$$L = r_\perp p = r p_\perp$$

$$\Rightarrow L = R(mu \sin \theta)$$

$$\Rightarrow L = \frac{u^2 \sin(2\theta)}{g}(mu \sin \theta)$$

$$\Rightarrow L = \frac{mu^3 \sin \theta \sin(2\theta)}{g}$$

Given that the direction of angular momentum aligns with the axis, we can represent it as follows:

$$\vec{L} = \frac{mu^3 \sin \theta \sin(2\theta)}{g}(-\hat{k})$$

Angular Momentum of System of Particles:

The collective angular momentum of a particle system adheres to the principle of superposition. Let's consider particles with linear momenta p_1, p_2,…,pn, each with respective position vectors r_1, r_2,…,rn.

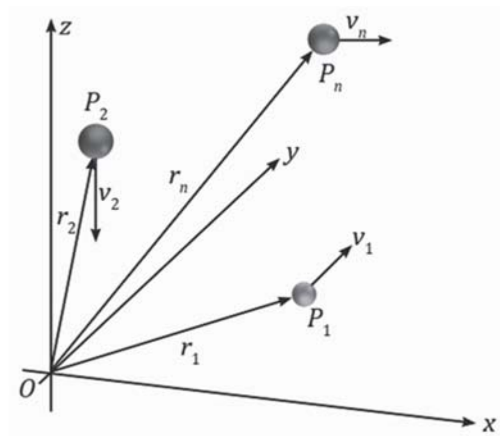
The angular momentum of this particle system around the origin O is described as follows:

$$\vec{L}_{system,0} = \vec{L}_{1,0} + \vec{L}_{2,0} + \vec{L}_{3,0} + \cdots \dots \vec{L}_{n,0}$$

$$\vec{L}_{system,0} = \sum_{i=1}^{n} \vec{L}_{i,0}$$

$$\vec{L}_{system,O} = \sum_{i=1}^{n} (\vec{r}_i \times \vec{p}_i)_0$$

$$\vec{L}_{system,0} = \sum_{i=1}^{n} m_i(\vec{r}_i \times \vec{v}_i)_0$$



A system of continuous particles refers to a rigid body. The angular momentum of a rigid body undergoing pure rotational motion, where the axis of rotation remains fixed, is determined in the following manner: Since each particle within the body is engaged in circular motion, the angular momentum of the system around an axis is the summation of the angular momenta of all individual particles around that same axis.

$$\vec{L}_{sys,0} = \sum_{i=1}^{n} m_i(\vec{r}_i \times \vec{v}_i)$$

or, $(L_{sys})_{Axis} = \sum(r_\perp p)_i$

$$= \sum[r_\perp m(\omega r_\perp)]_i$$

$$= \omega \sum(m r_\perp^2)_i$$

$\Rightarrow L_{sys} = I\omega[\because \sum(m r_\perp^2)_i = I]$

or, $(\vec{L}_{sys})_{Axis} = I_{Axis}\vec{\omega}$