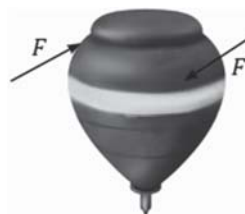


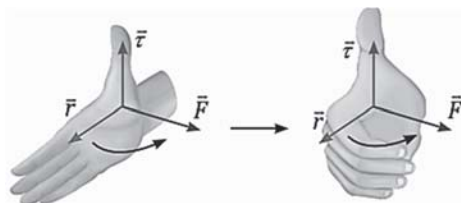
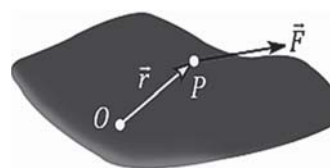
**ENTERING RIGID BODY DYNAMICS**

Imagine two equal and opposite forces, each with a magnitude of  $F$ , applied to opposite edges of a spinning top. Since the net force is zero, the center of mass of the top will experience no linear acceleration, meaning there's no translational motion induced by these forces. Nevertheless, despite the absence of linear motion, the top rotates due to the influence of these applied forces.

**Introducing Torque:**

Torque represents the rotational impact of a force.

- 1) It's denoted by the symbol  $\vec{\tau}$  and is also known as the moment of force.
- 2) Its SI unit is Newton meter (Nm).
- 3) The torque exerted by a force  $\vec{F}$  on a system about a point  $O$  is given by the equation:  $\vec{\tau} = \vec{r} \times \vec{F}$  where  $\vec{r}$  is the position vector of the point of application of the force.
- 4) Torque serves as the rotational equivalent of linear force.
- 5) It's an axial vector, and its direction is determined using the right-hand thumb rule: When the fingers, initially aligned along position vectors, are curled towards the force vector, the thumb indicates the direction of the torque vector.

**Magnitude of torque:**

Consider a force  $F$  applied to a body at point  $P$ , with a position vector of  $\vec{r}$ .

Let  $\theta$  represent the angle between the position and force vectors. In this scenario, the magnitude of torque can be expressed as follows:

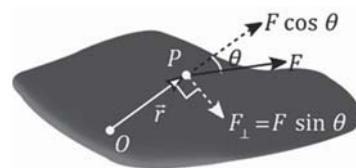
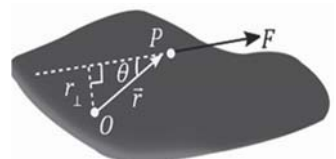
$$|\vec{\tau}| = r \times F \sin \theta = r_{\perp} F$$

Here,  $r_{\perp}$  (equal to  $r \sin \theta$ ) signifies the perpendicular distance from the line of action of the force to point  $O$ , also referred to as the force arm.

Alternatively, the magnitude of torque can be given by:

$$|\vec{\tau}| = r F \sin \theta = r F_{\perp}$$

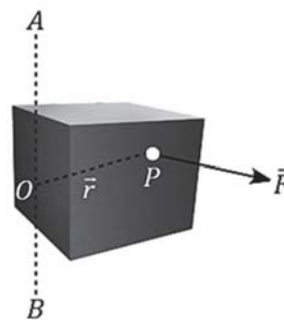
In this equation,  $F_{\perp}$  (equal to  $F \sin \theta$ ) denotes the component of force perpendicular to the position vector.

**Torque Around an Axis:**

We understand that the torque exerted by the force  $\vec{F}$  on the system illustrated in the figure about point  $O$  is determined by the equation:

$$\vec{\tau}_O = \vec{r} \times \vec{F}$$

In this context, the torque of force  $\vec{F}$  around line  $AB$  will be the component of  $\vec{\tau}_O$  along the same line.



**Force Couple:**

A force couple, consisting of two forces of equal magnitude but opposite directions, is a common concept in physics. We'll define the clockwise direction as positive orientation and the counterclockwise direction as negative orientation.

In this scenario:

The torque around point A is calculated as follows:

$$t_A = 0 + F(2d) \text{ (Clockwise)}$$

The torque around point B is calculated as follows:

$$t_B = F(2d) + 0 \text{ (Clockwise)}$$

The torque resulting from the force couple = the magnitude of one force  $\times$  by the distance between their lines of action ( $2d$ ).

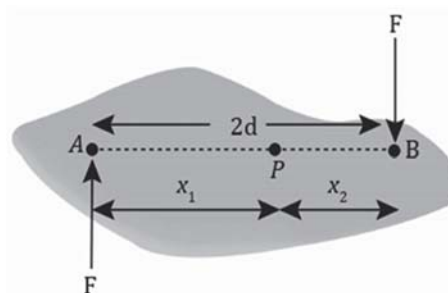
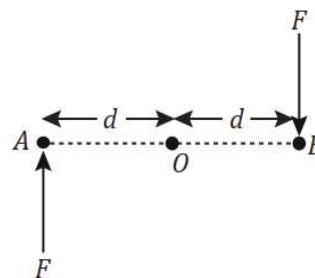
$$|\vec{\tau}| = F(2d)$$

In the provided diagram,

the torque around point P is determined as follows:

$$\begin{aligned} |\vec{\tau}| &= F(x_1) + F(x_2) \\ &= F(x_1 + x_2) \\ &= F(2d) \end{aligned}$$

Hence, the torque exerted by a force couple on a body remains constant regardless of the point of calculation. Put differently, when the net force acting on a system is zero, the torque remains consistent regardless of the reference point.

**Characteristics of a Force Couple:**

- 1) The resultant force applied to the body equals zero.
- 2) The body experiences a net torque.
- 3) The torque remains consistent regardless of the chosen point. When both the net force and net torque are zero, the torque about any point also equals zero.