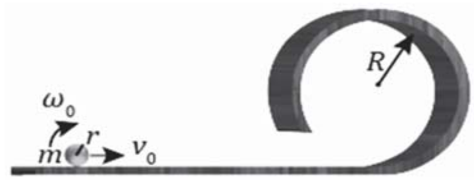
APPLICATION OF ROLLING

Ex. The provided illustration depicts a rough track, featuring a section shaped like a cylinder with a radius of R. What is the minimum linear speed required to initiate rolling motion for a sphere with a radius of r and a mass of m, ensuring it completes a full revolution around the cylindrical portion when initially set in motion on the horizontal segment?

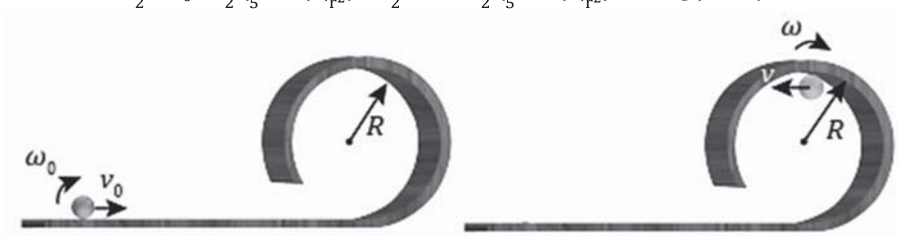


Sol. By employing the principle of energy conservation at both the starting and highest points, we derive the following equation:

The sum of translational kinetic energy, rotational kinetic energy, and potential energy at the starting point equals the sum of translational kinetic energy, rotational kinetic energy, and potential energy at the highest point.

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I_{com}\,\omega_0^2 + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I_{com}\,\omega^2 + mg[2(R-r)]$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_0^2}{r^2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v^2}{r^2}\right) + 2mg(R-r)$$



When at the highest point, equilibrium of forces in the radial direction occurs.

$$mg + 0 = \frac{mv^2}{R-r}$$

$$v^2 = g(R-r)$$

Further simplification results in the following:

$$\frac{7mv_0^2}{10} = \frac{7mv^2}{10} + 2mg(R-r)$$

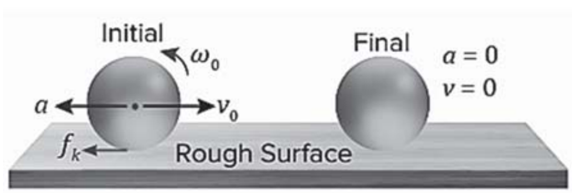
$$\Rightarrow \quad v_0^2 = g(R-r) + \frac{20}{7}g(R-r)$$

$$\Rightarrow \quad v_0^2 = \frac{27}{7}g(R-r)$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{20}{7}g(R-r)}$$

Ex. A hollow sphere with a radius of R is launched horizontally on a rough surface with an initial speed of v_0 and an initial angular velocity of ω_0. Determine their ratio to ensure the sphere comes to a halt after a certain period of time.

Sol. The diagram below illustrates the starting and ending conditions of the sphere. When projected along a rough surface, the sphere possesses linear momentum, which remains constant at all points on the surface.



Therefore, we can express this as:

$$\vec{L}_i = \vec{L}_f$$

$$I\omega_0 + mv_0R = 0 + 0$$

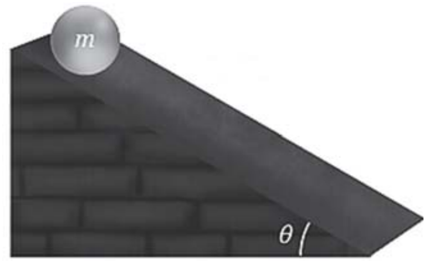
$$\frac{2}{3}mR^2\omega_0 + mv_0R = 0$$

Taking into account solely the magnitude,

$$\frac{2}{3}R\omega_0 = v_0$$

$$\Rightarrow \quad \frac{v_0}{\omega_0} = \frac{2R}{3}$$

Ex. On an inclined plane with an angle of inclination θ, a sphere of mass m rolls without slipping. Determine the linear acceleration of the sphere and the frictional force exerted on it. Additionally, ascertain the minimum static frictional force required to maintain pure rolling.



Sol. At the summit of the incline,

$$t = 0, u = 0, \omega_0 = 0$$

when time is t=0,
initial velocity is u=0,
and initial angular velocity is ω_0=0.
At the base of the incline, velocity is determined by

$$v = u + at,$$
$$v = at.$$

And $\omega = \omega_0 + \alpha t$

⇒ $\omega = \alpha t$

Since $v = \omega R$

⇒ $at = (\alpha t)R$

⇒ $a = \alpha R$

The total force exerted on the sphere in the direction of the incline is expressed as follows:

$$\sum \vec{F}_{ext} = m\vec{a}_{\text{cume}}$$

⇒ $\text{mgsin } \theta - \text{f} = \text{ma}$ ……. (i)

$$\sum \vec{\tau}_{\text{com}} = \text{I cos}$$

⇒ $\text{f} = \frac{2\text{mR}\alpha}{5}$

And, $a = \alpha R$

From equations (ii) and (iii),

$$\text{f} = \frac{2\text{ma}}{5}$$

⇒ $mg\sin\,\theta - f = ma$

⇒ $mg\sin\,\theta = \frac{7ma}{5}$

⇒ $a = \frac{5g\sin\,\theta}{7}$

and, $f = \frac{2mg\sin\,\theta}{7}$