ANGULAR IMPULSE AND COMBINED MOTION

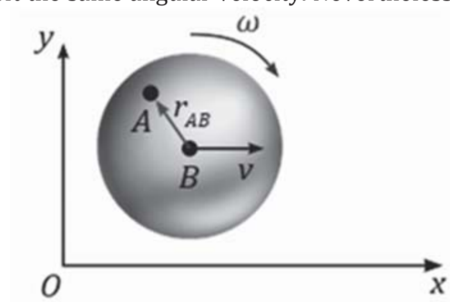
Angular Impulse:

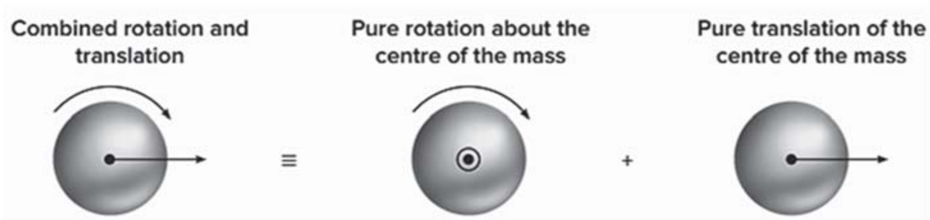
The angular impulse of a force signifies a alteration in angular momentum. The angular impulse of a torque, or the rotational force around a point or axis, is expressed as follows:

$$(\vec{J}_F)_A = \int_{t_1}^{t_2} \vec{\tau}\, dt$$
$$= \int_{L_1}^{L_2} d\vec{L} \quad [\because \vec{\tau} = \frac{d\vec{L}}{dt}]$$
$$= (\vec{L}_2 - \vec{L}_1)_A$$

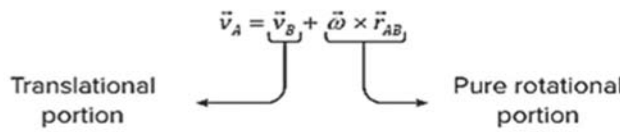
Combined Translation and Rotational Motion:

In pure translational motion, every particle in the system shares identical linear velocities, while pure rotational motion ensues when all particles exhibit the same angular velocity. Nevertheless, bodies can sometimes engage in both rotational and translational motions simultaneously. For instance, the wheels of a vehicle rotate around their axes while simultaneously moving linearly along the road. Analyzing combined motion involves breaking it down into its constituent parts of pure rotation and pure translation. Take, for example, a sphere rolling along the x-axis. This motion can be dissected as follows:





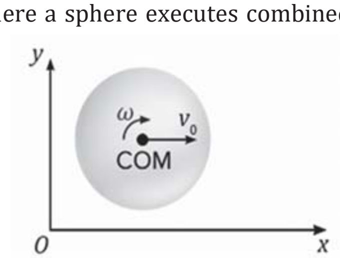
In mathematical terms, the velocity of any point, denoted as A, on this sphere, is expressed as follows:



Total kinetic energy in a combined motion:

The total kinetic energy of a body engaged in combined rotational and translational motion is the sum of these two components. Let's contemplate a scenario where a sphere executes combined rotational and translational motion.

$$(KE)_{\text{total}} = (KE)_{\text{rotational}} + (KE)_{\text{translational}}$$
$$= \frac{1}{2} I_{\text{com}}\, \omega^2 + \frac{1}{2} m v_{\text{com}}^2$$



Ex. For a sphere with mass and radius, the moment of inertia about an axis passing through the center is given by $I_{\text{com}} = \frac{2}{5} mR^2$

Therefore, the total kinetic energy can be expressed as:

$$(KE)_{\text{total}} = \frac{1}{2}\left(\frac{2mR^2}{5}\right)\omega^2 + \frac{1}{2} m v_0^2$$

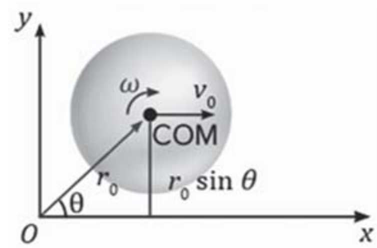
Where ω is the angular velocity of the sphere and $v0$ represents the linear velocity of its center of mass.

Total angular momentum in a combined motion:

The total angular momentum of a body around an axis is the combination of its angular momentum about its center of mass and the angular momentum of the center of mass around the selected axis.

$$\vec{L}_A = \vec{L}_{\text{gs, com}} + \vec{L}_{com,A}$$

$$\vec{L}_A = I_{com}\vec{\omega} + (\vec{r}_{com} \times m\vec{v}_{com})_A$$



Ex. For a sphere with mass and radius, the moment of inertia about an axis passing through the center is given by

$$\mathrm{I}_{\text{com}} = \frac{2}{5}\mathrm{mR}^2$$

Therefore, the overall angular momentum will be as follows:

$$(\vec{L}_{\text{sphere}})_0 = (\frac{2}{5}mR^2)\omega \otimes + mv_0(r_0 \sin\ \theta) \otimes$$

Since the angular momentum's direction is clockwise and perpendicular to the screen, the ⊗ symbol is employed.