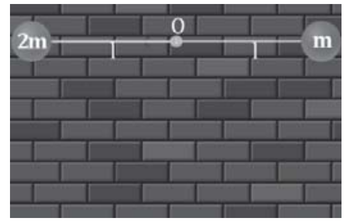
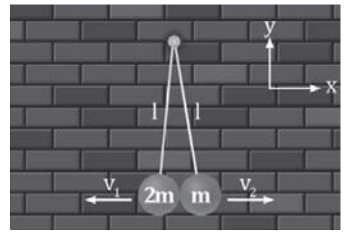
OBLIQUE COLLISION



Ex. Two balls having masses m and 2m are fastened to two light strings of the same length l. The other ends of the strings are fixed at point O. The strings are kept in the same horizontal line and the system is released from rest. The collision between the balls is elastic.

(a) Find the velocities of the balls just after their collision.

(b) How high will the balls rise after the collision?



Sol. **(a)** **Velocities of the balls just after their collision**

We know that the velocity of the bob of the pendulum of length l at lowest position is, $v = \sqrt{2gl}$ here, the two balls will strike each other with the same velocity. Let the velocities of the two balls having the masses 2m and m just after the collision be v_1 and v_2, respectively, in the directions as shown in the figure.

By using the principle of conservation of momentum, we get the following:

$$2mv - mv = -2mv_1 + mv_2$$

$$2v - v = -2v_1 + v_2$$

$$v_2 - 2v_1 = v \quad \text{... (1)}$$

Coefficient of restitution,

$$e = \frac{v_{sep}}{v_{app}} = 1$$

$$\frac{v_1 + v_2}{2v} = 1$$

$$v_1 + v_2 = 2v \quad \text{... (2)}$$

By subtracting equation (1) from equation (2), we get the following:

$$v_1 = \frac{v}{3}$$

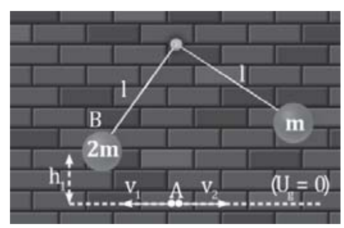
By putting it in equation (1), we get the following:

$$v_2 = \frac{5v}{3}$$

$$v = \sqrt{2gl},$$

$$v_1 = \frac{\sqrt{2gl}}{3} \text{ and } v_2 = \frac{5\sqrt{2gl}}{3}$$

(b) **Rise of balls after the collision**



Let the height through which the balls of masses 2m and m rise after collision be h_1 and h_2, respectively.

By using the principle of conservation of mechanical energy for the ball of mass, we get the following:

$$K_A + U_A = K_B + U_B$$

$$\frac{1}{2}(2m)v_1^2 + 0 = 0 + (2m)gh_1$$

$$v_1^2 = 2gh_1 = \frac{2gl}{9}\left[\because v_1 = \frac{\sqrt{2gl}}{3}\right]$$

$$h_1 = \frac{l}{9}$$

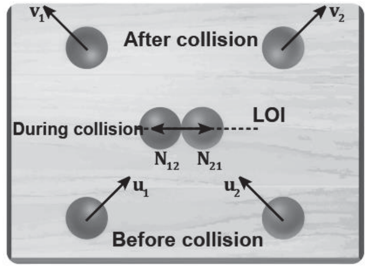
As v_2 is greater than the required velocity for the critical vertical circular motion, the ball of mass m will rise to a maximum height in VCM i.e., twice the radius of circle.

$$v_2 = \sqrt{\frac{50gl}{9}} > \sqrt{5gl}, h_2 = 2l$$

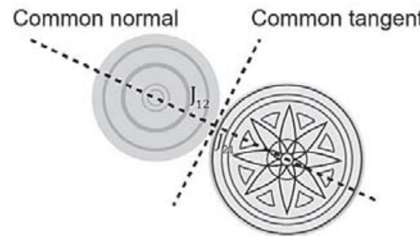
Line of Impact

- When two bodies collide, the line perpendicular to their surfaces at the point of contact is referred to as the line of impact.
- For a circular or spherical body, the line of impact always intersects its center point.
- The maximum impact force during the collision applies along this line to both bodies.
- Regardless of the orientation of the colliding objects, the line of impact (LOI) always aligns with the common normal.
- The maximum momentum is transferred along the line of impact (LOI) during a collision.

Ex. Consider two balls colliding with velocities u_1 and u_2 as depicted in the diagram. Despite their initial and final velocities being in different directions, the line of impact for this collision is horizontal.

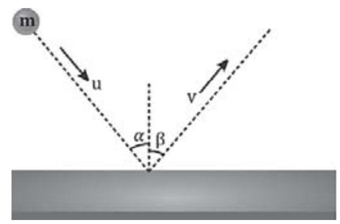


Oblique Collision



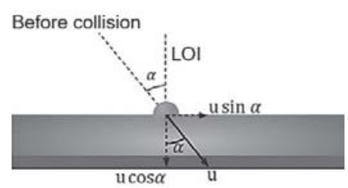
- Two impulses, equal in magnitude and opposite in direction, are applied along the common normal direction.
- The momentum of each particle undergoes a change in the direction of the common normal.
- There is no impulse component acting along the common tangent direction. The linear momentum of each particle remains constant in this direction.
- The total impulse acting on the system is zero throughout the collision.
- The total momentum of both particles remains constant before and after the collision in every direction.
- The coefficient of restitution can be utilized in the direction of the line of impact.

Ex. A ball of mass m hits a floor with a speed u making an angle of incidence α with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.



Sol. Let the ball reflect at an angle of reflection β with the vertical with speed v. We know that the linear momentum of the particles remains unchanged along a line perpendicular to the LOI. Thus, for the constant mass, the velocity components along this direction are equal.

$$u \sin \alpha = v \sin \beta \qquad \dots (1)$$

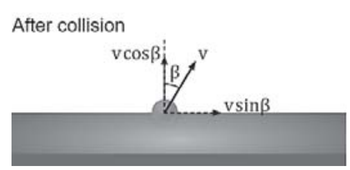


Coefficient of restitution along the line of impact is as follows:

$$e = \frac{v_{sep}}{v_{app}}\Big|_{LOI} \Rightarrow e = \frac{v\cos\beta}{u\cos\alpha}$$

$$u\cos\alpha = \frac{v\cos\beta}{e} \qquad \dots (2)$$

By dividing equation (1) by equation (2) we get the following:



$$\tan\beta = \frac{\tan\alpha}{e}$$

Or,

$$\beta = \tan^{-1}\left(\frac{\tan\alpha}{e}\right)$$

Final velocity, $$v = \sqrt{(v\sin\beta)^2 + (v\cos\beta)^2}$$

Or,

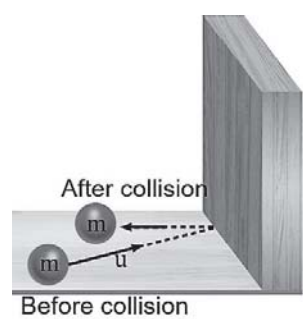
$$v = \sqrt{(u\sin\alpha)^2 + (eu\cos\alpha)^2}$$

Ex. A smooth sphere of mass m is moving on a horizontal plane with a velocity, $u = 3\hat{i} + \hat{j}\,ms^{-1}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$ If the coefficient of restitution between the spheres and the wall is 0.5, find the following:

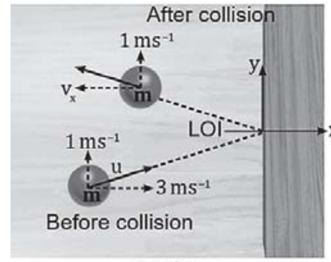
(a) Velocity of the sphere after the impact

(b) Loss in kinetic energy caused by the impact

(c) Impulse, $\vec{J}$, that acts on the sphere



Sol. (a) **Velocity of the sphere after the impact**



Top view

Let the velocity of the sphere in negative x-direction after the collision be v_x as shown in the figure. In the tangential direction (y-axis), the velocity will remain the same. Coefficient of restitution along the line of impact is as follows:

$$e = \frac{v_{sep}}{v_{app}}\Big|_{\text{along LOI}} = \frac{1}{2}$$

$$e = \frac{v_x}{3} = \frac{1}{2} \Rightarrow v_x = \frac{3}{2}$$

$$\vec{v} = (-\frac{3}{2}\hat{i} + \hat{j})m^{-1}$$

(b) **Loss in kinetic energy caused by the impact**

We have, initial and final velocities of the sphere,

$$\vec{u} = 3\hat{i} + \hat{j}$$

$$\vec{v} = \frac{-3}{2}\hat{i} + \hat{j}$$

Thus, initial kinetic energy is as follows:

$$(KE)_i = \frac{1}{2}(m)(3^2 + 1^2) = (5m)J$$

Final kinetic energy is as follows:

$$(KE)_f = \frac{1}{2}(m)(\frac{9}{4} + 1^2) = (\frac{13m}{8})J$$

Loss in kinetic energy is as follows:

$$(KE)_{loss} = (KE)_i - (KE)_f = 5m - \frac{13m}{8} = \frac{27m}{8}\ J$$

(c) **Impulse acting on the sphere**

$$\vec{p}_i = m\vec{u} = m(3\hat{i} + \hat{j})$$

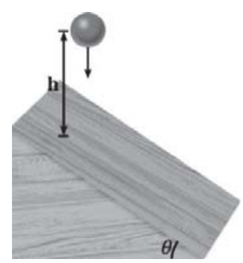
$$\vec{p}_f = m\vec{v} = m(\frac{-3}{2}\hat{i} + \hat{j})$$

Thus, impulse acting on sphere is as follows:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

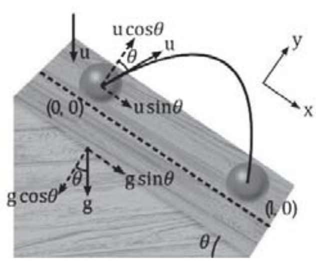
$$\vec{J} = m[\frac{-3}{2}\hat{i} + \hat{j} - 3\hat{i} - \hat{j}]$$

$$\vec{J} = -\frac{9m}{2}\hat{i}Ns$$



Ex. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a perfectly elastic collision. Where will it hit the plane again? Find the value of length between positions of two collisions along the inclined surface.

Sol. Let the ball strike the inclined plane with velocity u. Let the ball strike again at l distance. We know that it will acquire velocity $u = \sqrt{2gh}$ when it falls from height h.



As the collision is elastic and the surface is stationary, it will bounce back with the same velocity u in the direction as shown in the figure. Let the incline direction be x-axis and let us resolve the components of the velocity and acceleration due to gravity along x and y axes.

This is a projectile motion along an inclined plane. When the ball strikes again on the surface at length l, the displacement along the y-axis is zero.

Thus,

$$y = u_y t + \frac{1}{2}a_y t^2 = 0$$

By substituting the value of velocity and acceleration along y-axis, we get the following:

$$t = \frac{2u_y}{|a_y|} = \frac{2(u\cos\theta)}{g\cos\theta} = \frac{2u}{g}$$

Range of projectile, $$x = u_x t + \frac{1}{2}a_x t^2$$

$$I = (u\sin\theta)(\frac{2u}{g}) + \frac{1}{2}(g\sin\theta)(\frac{4u^2}{g^2})$$

As, $$u = \sqrt{2gh}$$

$$l = (\sin\theta)\left(\frac{2}{g}\right) \times 2gh + \frac{1}{2}(g\sin\theta)\left(\frac{4\times 2gh}{g^2}\right)$$

$$l = 8h\sin\theta$$