Chapter 11

Collision

Introduction to Collision

A collision occurs when an impulsive force acts briefly between two or more bodies, causing a change in their momenta.

Salient features of collision

The influence of non-impulsive forces such as gravity is not considered.
Impulsive forces comprise friction, tension, and normal reaction.
Non-impulsive forces encompass gravity, spring force, and electrostatic force.
Particles may be either in physical contact or not.
The time interval being considered is brief owing to the impulsive nature of the forces.

Coefficient of restitution (e)

It represents the ratio of reformation impulses (J_r) and deformation (J_d) of anybody experiencing a collision.

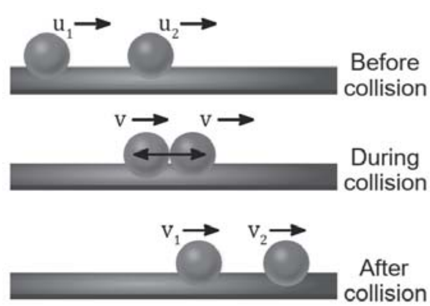
Mathematically, it can be expressed as follows:

$$e = \frac{J_r}{J_d} = \frac{\int F_r dt}{\int F_d dt}$$

The coefficient of restitution indicates the extent of deformation experienced by the bodies as a result of the collision.

Its value can range from a maximum of 1 to a minimum of 0.

Consider two balls moving in the same direction with velocities u_1 and u_2 as depicted in the figure. These balls undergo deformation and regain their shapes upon collision. At a specific moment when maximum deformation occurs, both balls attain the same velocity v. Subsequently, upon separation, the balls move with velocities v_1 and v_2.



According to Newton's law of collision, the coefficient of restitution (e) can be expressed as follows:

$$e = \frac{J_r}{J_d} = \frac{v_{sep}}{v_{app}}$$

Here $v_{Sep} = v_2 - v_1$

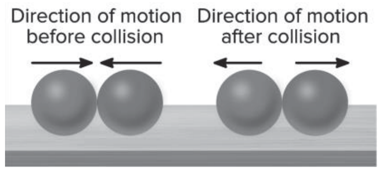
And $v_{app} = u_1 - u_2$

Thus $e = \frac{v_2 - v_1}{u_1 - u_2}$

Classification of Collision

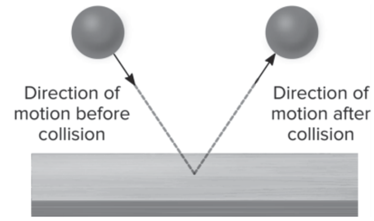
1. **Head-on collision**

In head-on collisions, the particles' velocities are either in the same direction or in opposite directions before and after the collision.



2. **Oblique collision**

In oblique collisions, the particles' velocities are along different paths before and after the collision.



On the basis of kinetic energy conservation

1. **Elastic collision**

In elastic collisions, particles fully recover their shape after the collision. This represents an ideal scenario that might not always be achievable in reality. Since the particles maintain the same shape before and after the collision, the coefficient of restitution is 1 for elastic collisions.

$$J_{ref} = J_{def}$$
$$v_{sep} = v_{app}$$

In elastic collision, as $v_{sep} = v_{app}$ the same. The system's kinetic energy remains constant before and after the collision.

2. **Inelastic collision**

In inelastic collisions, the bodies will deform and cannot fully recover their original shape. Since the shape of particles before and after the collision differs, the coefficient of restitution ranges between 0 and 1 for inelastic collisions. In such collisions, there is a loss of kinetic energy, resulting in the final kinetic energy of the system being less than the initial kinetic energy. Mathematically,

$$J_{ref} < J_{def}$$
$$v_{sep} < v_{app}$$

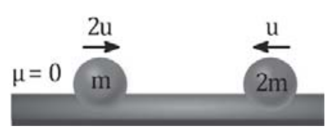
3. **Perfectly inelastic collision**

In perfectly inelastic collisions, the separation velocity along the collision line is 0. Consequently, the coefficient of restitution is also 0 in perfectly inelastic collisions, indicating maximum loss of kinetic energy. The bodies remain permanently attached and never separate in perfectly inelastic collisions. Mathematically,

$$J_{ref} = 0$$
$$v_{sep} = 0$$

Note: When friction is present on the surfaces of colliding bodies, kinetic energy is invariably dissipated during the collision, as friction acts as an impulsive force.

Ex. Two particles of masses m and 2m, moving in opposite directions with velocity 2u and u, respectively, on a frictionless surface collide elastically. Find the following:

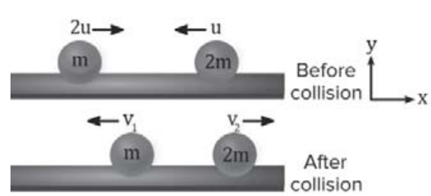


(a) Their velocities after the collision.

(b) The fraction of kinetic energy lost by the colliding particles.

Sol. **(a)** **Velocities after the collision**

Let the axis system and velocities of the balls before and after the collision be directed as shown in figure.



Here

The external force that is acting on the system of the two particles at time of the collision is zero. Using the principle of conservation of linear momentum for a system of two particles, Momentum of system before collision = Momentum of system after collision

$$m(2u) - 2m(u) = -m(v_1) + 2m(v_2)$$

$$2v_2 - v_1 = 0$$

$$v_1 = 2v_2 \quad \text{... (1)}$$

As the collision is elastic

$$e = \frac{v_{sep}}{v_{app}} = 1$$

$$1 = \frac{v_2+v_1}{2u-(-u)}$$

$$v_2 + v_1 = 3u \quad \text{... (2)}$$

By putting the value of v_1 from equation (1) in equation (2),we get the following:

$$v_2 + 2v_2 = 3u$$

$$v_2 = u \text{ and } v_1 = 2u$$

(b) Loss of kinetic energy

Kinetic energy of the system before the collision,

$$(KE)_i = \frac{1}{2}m(2u)^2 + \frac{1}{2}(2m)u^2$$

Kinetic energy after the collision

$$(KE)_f = \frac{1}{2}m(2u)^2 + \frac{1}{2}(2u)u^2$$

$$(KE)_{loss} = (KE)_i - (KE)_f$$

$$= \frac{1}{2}m(2u)^2 + \frac{1}{2}(2u)u^2 - [\frac{1}{2}m(2u)^2 + \frac{1}{2}(2u)u^2] = 0$$

Note: In the case of an inelastic collision, the reduction in kinetic energy is as follows:

$$(KE)_{loss} = \frac{1}{2}(\frac{m_1m_2}{m_1+m_2})(u_{app})^2(1-e^2)$$