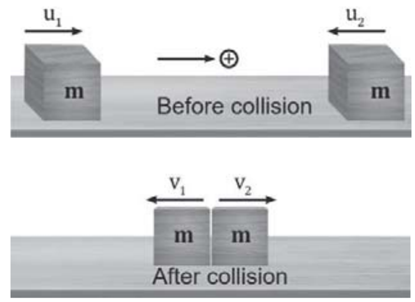
HEAD-ON COLLISION

Special cases of Collision

Case 1



Two particles of equal mass collide directly with each other. The collision is assumed to be perfectly elastic.

Thus, $e = 1$ and $m_1 = m_2 = m$

Consider their velocities before the collision as u_1 and u_2, and after the collision as v_1 and v_2, as illustrated in the diagram. Take the positive direction to be towards the right.

By applying the principle of momentum conservation, we obtain the following:

$$mu_1 - mu_2 = -mv_1 + mv_2$$

$$v_2 - v_1 = u_1 - u_2 \quad \text{... (1)}$$

Coefficient of restitution is given as follows:

$$e = \frac{v_{sep}}{v_{app}} = \frac{v_2-(-v_1)}{u_1-(-u_2)} = 1$$

$$\frac{v_2+v_1}{u_1+u_2} = 1$$

$$v_2 + v_1 = u_1 + u_2 \quad \text{... (2)}$$

Adding equations and we get the following:

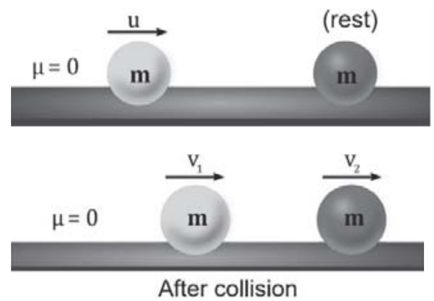
$$2v_2 = 2u_1$$

$$v_2 = u_1$$

Substituting in equation (1), we get the following:

$$v_1 = u_2$$

Hence, in a head-on unrestricted elastic collision, particles of equal masses interchange their velocities.



Ex. On a frictionless surface, a ball of mass m moving with a speed u makes a head-on collision with an identical ball at rest. The kinetic energies of the balls after the collision is three-fourths of the initial kinetic energy. Find the coefficient of restitution (e).

Sol. Let the velocities of the balls be v_1 and v_2 after the collision as shown in the figure.

Given condition,

$$(KE_f)_{sys} = \frac{3}{4}(KE_i)_{sys}$$

By using the principle of conservation of momentum, we get the following:

$$mu + 0 = mv_1 + mv_2$$

$$v_1 + v_2 = u \quad \text{... (1)}$$

Coefficient of restitution is given as follows:

$$e = \frac{v_{sep}}{v_{app}} = \frac{v_2 - v_1}{u}$$

$$v_2 - v_1 = eu \quad \text{... (2)}$$

By adding equation (1) and equation (2), we get the following:

$$v_2 = \frac{(1+e)u}{2}$$

By subtracting equation (2) from equation (1), we get the following:

$$v_1 = \frac{(1-e)u}{2}$$

From the given condition of kinetic energy, we have the following:

$$\left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right) = \frac{3}{4}\left(\frac{1}{2}mu^2 + 0\right)$$

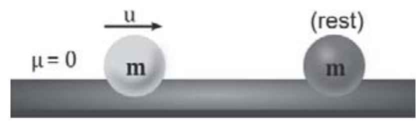
$$v_1^2 + v_2^2 = \frac{3u^2}{4}$$

By putting value of v_1 and v_2 in the above equation, we get the following:

$$\frac{(1-e)^2u^2}{4}+\frac{(1+e)^2u^2}{4}=\frac{3u^2}{4}$$
$$(e^2-2e+1)+(e^2+2e+1)=3$$
$$2e^2+2=3$$
$$e^2=\frac{1}{2}$$

Since e cannot be negative, $e=\frac{1}{\sqrt{2}}$

Alternative way



We know that the loss of kinetic energy during collision of masses m_1 and m_2 and is given by,

$$(KE)_{loss}=\frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_{app})^2(1-e^2) \quad \text{... (1)}$$

Here, $m_1=m_2$ and, $u_{app}=u$

Also, $(KE_f)_{sys}=\frac{3}{4}(KE_i)_{sys}$

$$(KE)_{loss}=(KE_i)_{sys}-\frac{3}{4}(KE_i)_{sys}=\frac{1}{4}(KE_i)_{sys} \quad \text{... (2)}$$

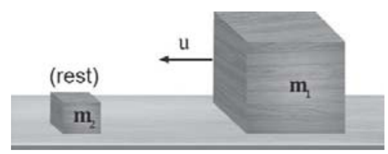
From equation (1) and (2),

$$\frac{1}{2}\frac{(m)(m)}{(2m)}u^2(1-e^2)=\frac{1}{4}(\frac{1}{2}mu^2)$$
$$\frac{1}{2}(\frac{m}{2})u^2(1-e^2)=\frac{1}{4}(\frac{1}{2}mu^2)$$
$$1-e^2=\frac{1}{2}$$

Since e cannot be negative, we get the following:

$$e=\frac{1}{\sqrt{2}}$$

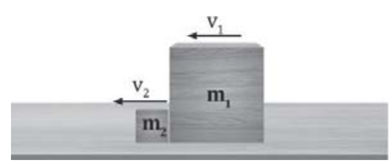
Case 2



Let mass m_1 be very heavy as compared to mass m_2 ($m_1 >> m_2$).

This means

$$m_1+m_2\approx m_1 \text{ or, } \frac{m_2}{m_1}\to 0$$



Let m_2 initially be stationary, while m_1 moves to the left with velocity u. consider the positive direction to be towards the left. Let v_1 and v_2 denote the final velocities of masses m_1 and m_2, respectively.

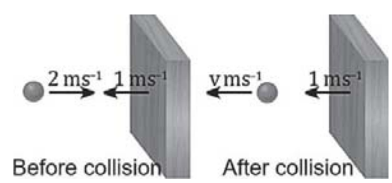
By applying the law of momentum conservation,

$$m_1 u + 0 = m_1 v_1 + m_2 v_2$$

As m_2 is very small, $u = v_1$

This demonstrates that in a head-on collision, when one body is infinitely heavier than the other, it maintains its velocity after the collision.

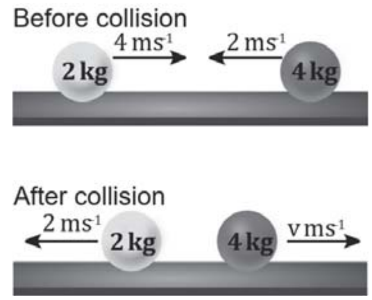
Ex. A ball is moving with a speed of 2 ms^{-1} towards a heavy wall that is moving towards the ball with a speed of 1 ms^{-1} as shown in the figure. Assuming collision to be elastic, find the speed of the ball immediately after the collision.



Sol. Consider the right direction as positive. As the collision is elastic, we get the following:

$$e=\frac{v_{sep}}{v_{app}}=1$$
$$\frac{v-1}{2-(-1)}=1$$
$$v=4ms^{-1}$$

Ex. Two balls of masses 2 kg and 4 kg are moving towards each other with velocities 4 ms^{-1} and 2 ms^{-1}, respectively, on a frictionless surface. After colliding, the 2 kg ball returns back with velocity 2 ms^{-1}. Find the following:



(a) Velocity of 4 kg ball after collision
(b) Coefficient of restitution (e)
(c) Impulse of deformation (J_d) on the 2 kg ball
(d) Impulse of reformation (J_r) on the 2 kg ball
(e) Maximum potential energy of deformation.

Sol. **(a)** Velocity of 4 kg ball after collision Using the principle of conservation of momentum, we get the following:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(2)(4) - (4)(2) = -(2)(2) + (4)(v)$$
$$8 - 8 = -4 + 4v$$
$$v = 1m^{-1}$$

(b) Coefficient of restitution (e)

$$e = \frac{v_{sep}}{v_{app}}$$
$$e = \frac{1-(-2)}{4-(-2)} = \frac{1}{2}$$

(c) Impulse of deformation (J_d) on the 2 kg ball
At the instant of collision, the velocities of two balls will be the same. Let this velocity be v. By using principle of conservation of momentum for both balls together as a system, we get the following:

$$\vec{p}_{initial} = \vec{p}_{max\ deformation}$$
$$(2)(4) - (4)(2) = (2 + 4)v$$
$$v = 0$$

Impulse of deformation on the 2Kg ball is as follows:

$$\vec{J}_d = \vec{p}_{max\ deformation} - \vec{p}_{initial}$$
$$= (2 \times 0) - (2 \times 4) = -8Ns$$

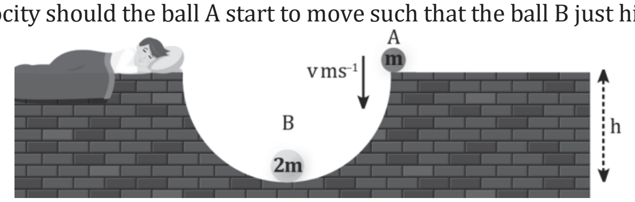
(d) Impulse of reformation (J_r) on the 2Kg ball Coefficient of restitution (e) is given in terms of impulse as follows:

$$e = \frac{J_{ref}}{J_{def}}$$
$$\frac{1}{2} = \frac{J_{ref}}{-8}$$
$$J_{ref} = -4Ns$$

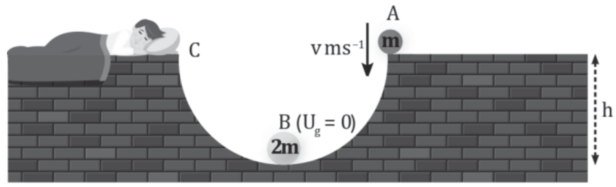
(e) Maximum potential energy of deformation we know that,

$$(KE)_i - (KE)_{max\ deformation} = \text{Maximum PE of deformation}$$
$$[\frac{1}{2}(2)(4)^2 + \frac{1}{2}(4)(2)^2] - \frac{1}{2} \times (2 + 4) \times 0^2 = U_{max}$$
$$U_{max} = 16 + 8 = 24J$$

Ex. The track shown in the figure is frictionless. The ball B of mass 2m is lying at rest and the ball A of mass m is pushed along the track with some speed. The collision between A and B is perfectly elastic. With what velocity should the ball A start to move such that the ball B just hits the sleeping man?

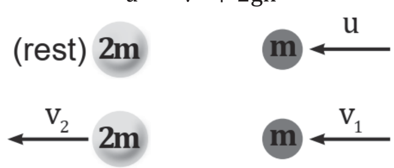


Sol. Let the ball A of mass m be pushed along the track with initial speed v. The ball B can have zero velocity when it touches the head. Let the datum for potential energy be at the initial position of ball B.



Using the principle of conservation of mechanical energy for the ball of mass m at points A and B, we get the following:

$$K_A + U_A = K_B + U_B$$
$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mu^2 + 0$$
$$u^2 = v^2 + 2gh \qquad \ldots (1)$$



Let the velocities of mass m and 2m, after collision be v_1 and v_2 respectively..

Coefficient of restitution for elastic collision is as follows:

$$e = \frac{v_2 - v_1}{u} = 1$$
$$v_2 - v_1 = u. \qquad \ldots (2)$$

By using the principle of conservation of momentum, we get the following:

$$mu + 0 = mv_1 + 2mv_2$$
$$v_1 + 2v_2 = u \qquad \ldots (3)$$

By adding equation (2) and equation (3), we get the following:

$$v_2 = \frac{2u}{3}$$

By using principle of conservation of mechanical energy for ball of mass 2m at points B and C, we get the following:

$$K_B + U_B = K_C + U_C$$
$$\frac{1}{2}(2m)\left(\frac{2u}{3}\right)^2 + 0 = 0 + (2m)gh$$
$$\frac{4u^2}{9} = 2gh$$
$$u^2 = \frac{9gh}{2}$$

By putting value of u^2 in equation (1), we get the following:

$$\frac{9gh}{2} = v^2 + 2gh$$
$$v^2 = \frac{5gh}{2}$$
$$v = \sqrt{\frac{5gh}{2}}$$

This is the minimum initial velocity required for the ball of mass m so that ball B should just hit the man.