IMPULSE

The momentum alteration experienced by a system resulting from the application of a force (F) over an infinitesimal period is termed impulse (J), which is imparted to the system by the force.

$$\vec{J} = \int \vec{F}dt = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

The impulse exerted on an object within a specific time interval corresponds to the area beneath the force-versus-time (F-t) graph within that interval.

Area $= \vec{J} = \int \vec{F}dt = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

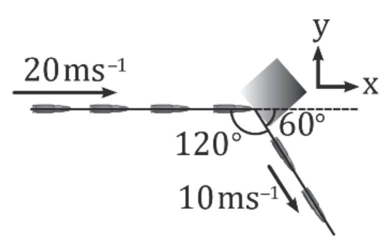
Note: If a force is capable of significantly altering momentum even within an infinitesimally small duration, it qualifies as an impulsive force.
Ex: Normal reaction, friction force, tension force, etc.

Ex. A ball of mass m travelling with velocity $2\hat{i} + 3\hat{j}$ receives an impulse $-3m\hat{i}$. What is the velocity of the ball immediately afterwards?

Sol. Given,
Mass of the ball $= m$
Initial velocity, $u = 2\hat{i} + 3\hat{j}$
Impulse, $\vec{J} = -3m\hat{i}$
Let $\vec{v}$ be the final velocity of the ball Here,
Initial momentum, $\vec{p}_i = m(2\hat{i} + 3\hat{j})$
Final momentum, $\vec{p}_f = m\vec{v}$
We know that the impulse imparted on a system by a force is equal to the change in momentum of the system.

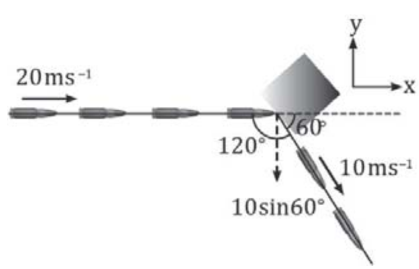
$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
$$-3m\hat{i} = m\vec{V} - m(2\hat{i} + 3\hat{j})$$
$$-3\hat{i} + (2\hat{i} + 3\hat{j}) = \vec{V}$$
$$\vec{V} = -\hat{i} + 3\hat{j}$$

Ex. A bullet of mass 10^{-3} kg strikes an obstacle and moves 60° to its original direction. If its speed also changes from 20 ms^{-1} to 10 ms^{-1}, find the magnitude of impulse acting on the bullet.



Sol. Given,
Mass of the bullet, $m = 10^{-3}$ kg
Initial speed, $v_i = 20\ ms^{-1}$
Final speed, $v_f = 10\ ms^{-1}$

$$\vec{v}_i = 20\hat{i}ms^{-1}$$
$$\vec{v}_f = 10\cos 60°(\hat{i}) + 10\sin 60°(-\hat{j}) = (5\hat{i} - 5\sqrt{3}\hat{j})ms^{-1}$$
$$\Delta\vec{v} = \vec{v}_f - \vec{v}_f = \{(5\hat{i} - 5\sqrt{3}\hat{j}) - 20\hat{i}\}$$
$$\Delta\vec{v} = (-15\hat{i} - 5\sqrt{3}\hat{j})ms^{-1}$$



We know that the impulse imparted by a force on a system is equal to the change in momentum of that system.

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

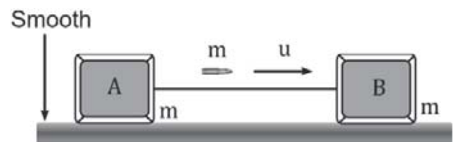
$$\vec{J} = m\vec{v}_f - m\vec{v}_i = m\Delta\vec{v}$$

$$\vec{J} = m(-15\hat{i} - 5\sqrt{3}\hat{j}) = 10^{-3}(-15\hat{i} - 5\sqrt{3}\hat{j})$$

$$|\vec{J}| = 10^{-3}\sqrt{(-15)^2 + (-5\sqrt{3})^2} = 10^{-3}\sqrt{300} = 10^{-2}\sqrt{3}\text{Ns}$$

$$|\vec{J}| = \sqrt{3} \times 10^{-2}\text{Ns}$$

Ex. Two identical blocks A and B of mass m that are connected by a massless string are placed on a frictionless horizontal plane. A bullet having the same mass and moving with speed u strikes block B as shown in the figure. If the bullet gets embedded into the block B, then find the following:



(a) The final velocity of blocks A, B, and the bullet

(b) The impulse on block A due to the tension in the string

(c) The impulse on the bullet due to the normal force

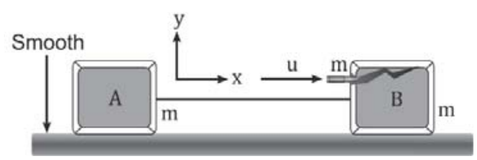
. **(d)** The impulse on block B due to the normal force

Sol. Given,

Mass of each block = Mass of the bullet = m

Speed of the bullet = u

(a) Final velocity of blocks A, B, and the bullet Consider the two blocks and the bullet together as a system.



Let $\vec{v}$ be the final velocity of the system. Since the net external force acting on the system along the x-direction is zero, the momentum of the system is conserved along the x-direction.

$$\sum(\vec{F}_{ext})_X = \vec{0} \Rightarrow (\vec{p}_i)_X = (\vec{p}_f)_X$$

$$0 + 0 + mu = (m + m + m)V$$

$$V = \frac{u}{3}$$

(b) Impulse on block A due to the tension in the string

Let $\vec{J}_T$ be the impulse acting on block A due to the tension force. Then

$$\vec{J}_T = m(\vec{V}_f - \vec{V}_i) = m(\frac{\vec{u}}{3} - \vec{0})$$

$$\vec{J}_T = \frac{m\vec{u}}{3}$$

$$|\vec{J}_T| = \frac{mu}{3}$$

(c) Impulse on the bullet due to the normal force

Let $\vec{J}_B$ be the impulse acting on the bullet due to the normal force from block B. Then,

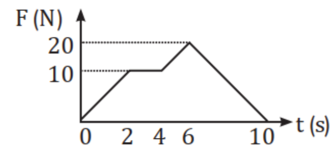
$$\vec{J}_B = m(\vec{V}_f - \vec{V}_i) = m(\frac{\vec{u}}{3} - \vec{u})$$

$$\vec{J}_B = -\frac{2m\vec{u}}{3}$$

$$|\vec{J}_B| = \frac{2mu}{3} \text{ (along negative x-axis)}$$

(d) Impulse on block B due to the normal force By Newton's third law of motion, the impulse on block B will be negative of the impulse on the bullet.

Ex. A particle of mass 2 kg is initially at rest. A force starts acting on the particle in one direction whose magnitude changes with time. The force vs time graph is shown in the figure. Find the speed of the particle at the end of 10 s.



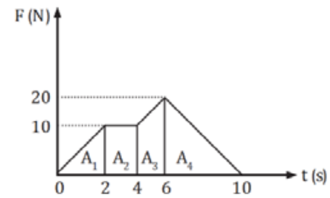
Sol. Given,

Mass of the particle, m = 2 kg

Initial speed of particle, $v_i = 0 \ ms^{-1}$

Let v_f be the final speed of the particle after 10 s.

We know that the area under the force vs time graph gives us impulse.



Thus,

$$J = A_1 + A_2 + A_3 + A_4$$

$$J = (\tfrac{1}{2} \times 2 \times 10) + (2 \times 10) + (\tfrac{1}{2} \times (10 + 20) \times 2) + (\tfrac{1}{2} \times 4 \times 20)$$

$$J = 100 Ns$$

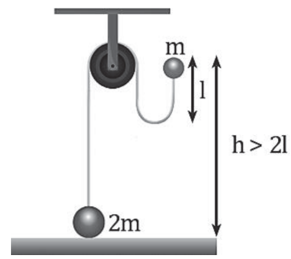
However, impulse imparted is equal to change in the momentum.

$$J = m\Delta v = m(v_f - 0)$$

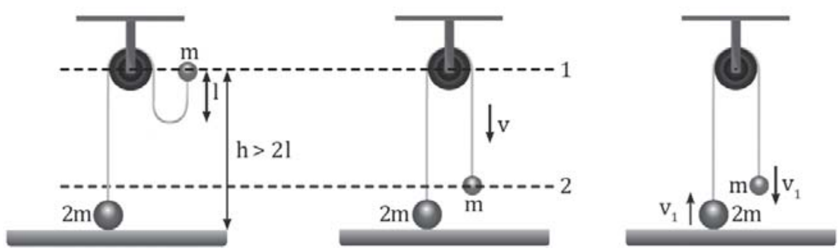
$$100 = 2v_f$$

$$v_f = 50 ms^{-1}$$

Ex. In the figure shown, the heavy ball of mass 2m rests on the horizontal surface and the lighter ball of mass m is dropped from a height h > 2l. At the instant, the string gets taut. What is the upward velocity of the heavy ball?



Sol. When the lighter ball of mass m is dropped from the initial position, it falls freely for length 2l with the final velocity being v. At this instant, the string gets taut and the velocity of the system becomes v_1 as shown in the figure.



Choosing point 2 as the datum and applying principle of conservation of mechanical energy between points 1 and 2, we get the following:

$$K_1 + U_1 = K_2 + U_2$$

$$0 + mg(2l) = \tfrac{1}{2}mv^2 + 0$$

$$v = 2\sqrt{gl} \quad \text{... (1)}$$

As soon as the lighter ball reaches the point 2, both the balls experience an impulse J_T due to the tension force.

For the heavier ball, we get the following:

$$J_T = 2m(v_1 - 0)$$

$$J_T = 2mv_1 \quad \text{... (2)}$$

For the lighter ball, we get the following:

$$J_T = m(-v_1 - (-v))$$

$$J_T = m(v - v_1) \quad \text{... (3)}$$

By comparing equations (2) and (3), we get the following:

$$2mv_1 = m(v - v_1)$$

$$v_1 = \frac{v}{3}$$

Putting the value of from equation (1), we get the following:

$$v_1 = \frac{2\sqrt{gl}}{3}$$

Impulsive force :

An impulsive force, characterized by its higher magnitude and brief duration, is known for its ability to swiftly alter the momentum of an object within a short span of time. It's important to note that the distinction between impulsive and non-impulsive forces is relative and lacks a precise boundary.

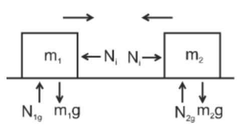
Note: Typically, forces involved in collisions exhibit impulsive characteristics. Due to their brief application period, there is minimal displacement of particles, resulting in limited motion.

1. Gravitational force and spring force consistently lack impulsive properties.
2. The nature of normal, tension, and friction forces varies depending on the situation.
3. An impulsive force can only be counteracted by another impulsive force.

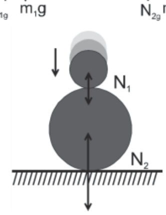
1. Impulsive Normal :

During a collision, the normal forces acting on the collision surface are invariably impulsive.

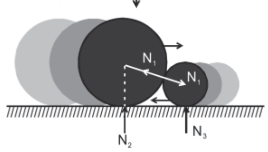
Ex:



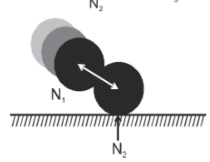
N_i = Impulsive; N_g = Non-impulsive



Both normal are Impulsive



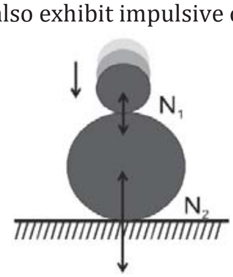
N_1,N_3 = Impulsive; N_2 = non-impulsive



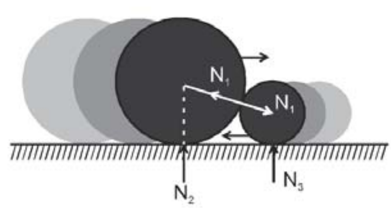
Both normal are Impulsive

2. Impulsive Friction:

If the normal force exerted between the two objects is impulsive, then the friction force between them will also exhibit impulsive characteristics.



Friction at both surfaces is impulsive



Friction due to N_2 is non-impulsive and due to N_3 And N_1 are impulsive

3. Impulsive Tensions:

When a string experiences a sudden jerk, equal and opposite tensions arise simultaneously at each end. As a result, equal and opposite impulses are exerted on the bodies connected to the string in the direction of the string. This scenario presents two distinct cases that need to be examined.

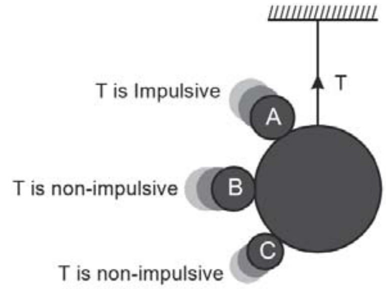
(a) One end of the string is fixed:

The impulse exerted at the fixed end of the string does not alter the momentum of the fixed object located there. Conversely, the object connected to the free end experiences a momentum change aligned with the direction of the string. Additionally, momentum remains unaltered in a direction perpendicular to the string, where impulsive forces are absent.

(b) Both ends of the string attached to movable objects:

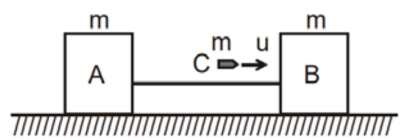
In this scenario, equal and opposite impulses affect the two objects, resulting in equivalent alterations in momentum. Consequently, the total momentum of the system remains constant, even though each object experiences a change in momentum aligned with the string's direction. However, perpendicular to the string, no impulse is applied, thus leaving the momentum of each particle in this direction unaffected.

For a rod, tension consistently exhibits impulsive characteristics, whereas for a spring, tension consistently lacks impulsive properties.



All normal are impulsive but tension T is impulsive only for the ball A

Ex. Two identical blocks, labeled A and B, are connected by a massless string and situated on a frictionless horizontal surface. A bullet with the same mass, traveling at speed u, collides with block B from behind, as illustrated. If the bullet becomes embedded in block B, determine:



(a) The velocity of A,B,C after collision. **(b)** Impulse on A due to tension in the string

(c) Impulse on C due to normal force of collision. **(d)** Impulse on B due to normal force of collision.

Sol. **(a)** By Conservation of linear momentum $v = \frac{u}{3}$ **(b)** $\int T dt = \frac{mu}{3}$

(c) $\int N dt = m(\frac{u}{3} - u) = \frac{-2mu}{3}$ **(d)** $\int (N - T)dt = \int N dt - \int T dt = \frac{mu}{3}$

$\int N dt = \frac{2mu}{3}$