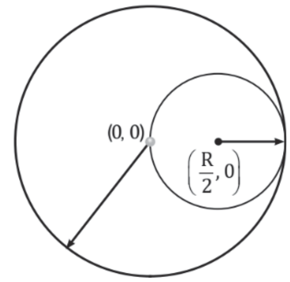
## C.O.M OF SYSTEMS WITH PART EXTRACTIONS AND DISPLACEMENT OF C.O.M

**Ex.** From a uniform circular disc of mass M and radius R, a small disc of radius $\frac{R}{2}$ is removed such that both have a common point of tangency on the x-axis. Find the position of centre of mass of the remaining portion from the centre of the original disc.



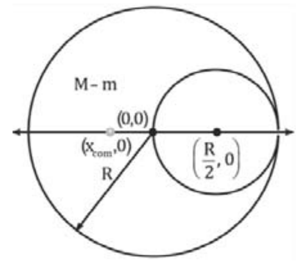

**Sol.** Given,

Mass of the original disc = M

Radius of the original disc = R

Radius of the smaller disc that is removed = $\frac{R}{2}$

Since both the discs are uniform and circular, the centre of mass of both the discs will lie on their respective centres.




Thus, Centre of mass of original disc = (0, 0)

Centre of mass of smaller disc = $\left(\frac{R}{2}, 0\right)$

Let be the area density of the disc.

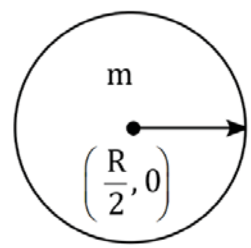
$$\sigma = \frac{M}{\pi R^2}$$

Mass of the smaller disc is as follows:

$$m = \sigma \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{\pi R^2} \times \pi \times \frac{R^2}{4}$$

$$m = \frac{M}{4}$$

Now, since the remaining figure is also symmetrical about the x-axis, the centre of mass of the new figure will lie on the x-axis.




Let the coordinates of the new centre of mass be ($x_{com}$, 0).

Since some mass is removed, we have to take the removed mass as negative in the formula.

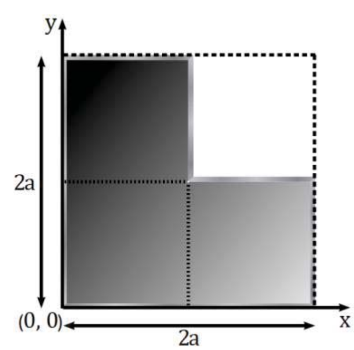
Let $x_1$ and $x_2$ be the x-coordinates of the centre of masses of the original and removed part respectively.

$$x_{com} = \frac{Mx_1 + (-m)x_2}{M + (-m)} = \frac{(M \times 0) - \left(\frac{M}{4} \times \frac{R}{2}\right)}{M - \frac{M}{4}}$$

$$x_{com} = -\frac{R}{6}$$

Therefore, the center of mass of the new figure is, $\left(-\frac{R}{6}, 0\right)$.

**Ex.** Find the position of centre of mass of the section with uniform mass distribution as shown in the figure.

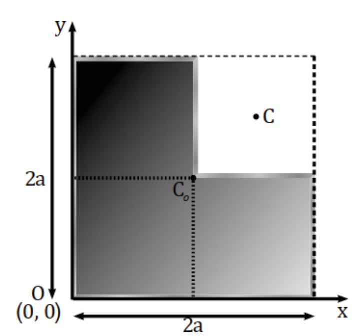



**Sol.** In this figure, a square of side a has been removed from the larger square of side 2a. Since both the squares are uniform, their centre of masses lie on their respective geometric centres as shown in the figure.

$$C_o = (a, a)$$

$$C = \left(\frac{3a}{2}, \frac{3a}{2}\right)$$

Let M be the mass of the original larger square and m be the mass of the smaller square.




Let be the area density of the section.

$$\sigma = \frac{M}{(2a)^2} = \frac{M}{4a^2}$$

Mass of the smaller disc is as follows.

$$m = \sigma \times a^2 = \frac{M}{4a^2} \times a^2 = \frac{M}{4}$$

$$m = \frac{M}{4}$$

Let ($x_{com}$, $y_{com}$) be the centre of mass of the given figure.

Since some mass is removed, we have to take the removed mass as negative in the formula. Let ($x_1$, $y_1$) and ($x_2$, $y_2$) be the coordinates of the centre of masses of the original and removed part respectively.

$$x_{cam} = \frac{Mx_1+(-m)x_2}{M+(-m)} = \frac{(M\times a)-(\frac{M}{4}\times\frac{3a}{2})}{M-\frac{M}{4}}$$

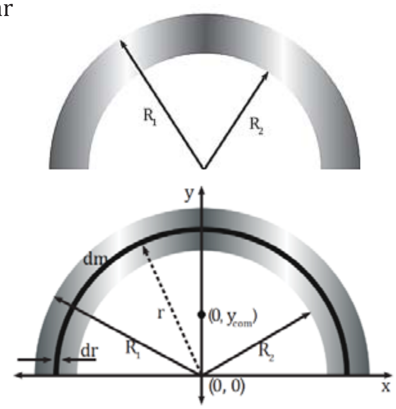
$$x_{cam} = \frac{5a}{6}$$

$$y_{com} = \frac{My_1+(-m)y_2}{M+(-m)} = \frac{(M\times a)-(\frac{M}{4}\times\frac{3a}{2})}{M-\frac{M}{4}}$$

$$y_{com} = \frac{5a}{6}$$

Therefore the coordinate of the center of mass of the given figure is $(\frac{5a}{6},\frac{5a}{6})$

**Ex.** Find the centre of mass of a uniform plate having semicircular outer and inner boundaries of radius $R_1$ and $R_2$, respectively.




**Sol.** Given,

Outer radius = $R_1$

Inner radius = $R_2$

Consider an elemental plate of mass dm and thickness dr at a radius r as shown in the figure.

Let be the area density of plate.

$$\sigma = \frac{M}{\frac{\pi}{2}(R_1^2-R_2^2)}$$

Mass of the smaller plate is,

$$dm = \sigma \times \pi r dr = \frac{2M}{\pi(R_1^2-R_2^2)} \times \pi r dr$$

$$dm = \frac{2Mr\times dr}{R_1^2-R_2^2}$$

Since the plate is symmetrical about the y-axis, the centre of mass of the plate lies on the y-axis. Let the centre of mass be (0, $y_{com}$).

$y_{com} = \frac{1}{M}\int y dm$; where, y = Centre of mass of the elemental plate $= \frac{2r}{\pi}$

$$y_{cam} = \frac{1}{M}\int_{R_2}^{R_1}\frac{2r}{\pi}\times\frac{2Mr\times dr}{R_1^2-R_2^2} = \frac{4}{\pi(R_1^2-R_2^2)}\times\int_{R_2}^{R_1} r^2 dr$$

$$y_{cam} = \frac{4}{\pi(R_1^2-R_2^2)}\times\left[\frac{r^3}{3}\right]_{R_2}^{R_1} \Rightarrow y_{cam} = \frac{4(R_1^3-R_1^3)}{3\pi(R_1^2-R_2^2)}$$

Therefore, the centre of mass of the given plate is, $\left(0,\frac{4(R_1^3-R_1^3)}{3\pi(R_1^2-R_2^2)}\right)$.

**Displacement of Centre of Mass**

In a system containing n particles with masses $m_1$, $m_2$ … $m_n$ and their respective position vectors $\vec{r}_1, \vec{r}_2, \ldots\ldots, \vec{r}_n$ the position vector of the center of mass is given as follows for each respective particle.

$$\vec{r}_{com} = \frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3+\cdots....+m_n\vec{r}_n}{m_1+m_2+m_3+\cdots...+m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} \quad \ldots (1)$$

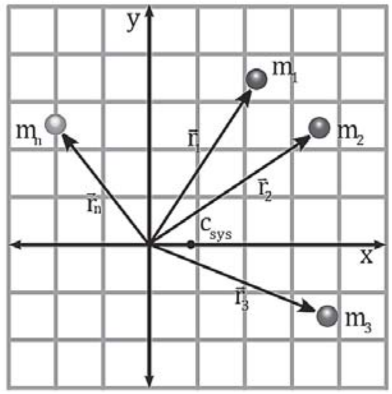
If the system is in motion, consider the following $\vec{v}_{com}$ and $\vec{a}_{com}$ represent the velocity and acceleration of the system's center of mass, respectively.

Differentiating equation (1) with respect to time t, we get,

$$\frac{d}{dt}(\vec{r}_{com}) = \frac{m_1\frac{d\vec{r}_1}{dt}+m_2\frac{d\vec{r}_2}{dt}+m_3\frac{d\vec{r}_3}{dt}+\cdots....+m_n\frac{d\vec{r}_n}{dt}}{m_1+m_2+m_3+\cdots....+m_n} = \frac{\sum_1^n m_i\frac{d\vec{r}_i}{dt}}{\sum_1^n m_i}$$

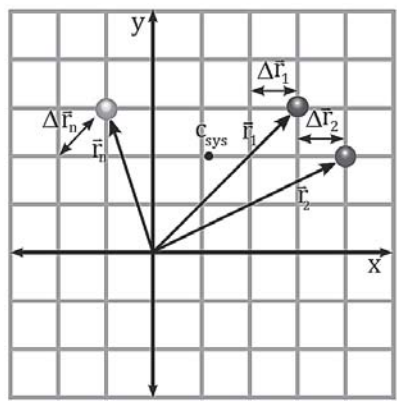
$$\vec{v}_{com} = \frac{m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3+\cdots...+m_n\vec{v}_n}{m_1+m_2+m_3+\cdots...+m_n} = \frac{\sum_1^n m_i\vec{v}_i}{\sum_1^n m_i}$$

$$\vec{a}_{com} = \frac{\sum_1^n m_i\vec{a}_i}{\sum_1^n m_i}$$




In a system consisting of n particles with masses $m_1, m_1, . . , m_n$ displaced by $\Delta\vec{r}_1, \Delta\vec{r}_2, . . , \Delta\vec{r}_n$ respectively, the change in position vector of the centre of mass is given as follows:

$$\Delta\vec{r}_{com} = \frac{m_1\Delta\vec{r}_1+m_2\Delta\vec{r}_2+m_3\Delta\vec{r}_3+\cdots...+m_n\Delta\vec{r}_n}{m_1+m_2+m_3+\cdots...+m_n} = \frac{\sum_{i=1}^{n} m_i\Delta\vec{r}_i}{\sum_{i=1}^{n} m_i}$$




**Ex.** A stationary pulley carries a rope whose one end supports a ladder hanging with a man and the other end supports a counterweight of mass M. A man of mass m climbs up a distance $l_0$ with respect to the ladder and then stops. Neglecting the mass of the rope and the friction in the pulley axle, find the displacement (l) of the centre of mass of this system.


**Sol.** Given, Mass of the man = m

Mass of the counterweight = M

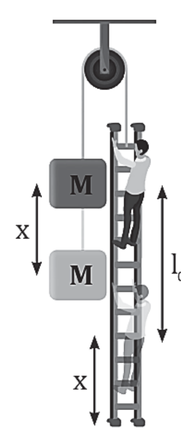
For the system to be in equilibrium, Mass of the ladder = Mass of the counterweight – Mass of the man = M – m

Distance climbed by the man with respect to the ladder = $l_0$

Let l be the displacement of the centre of mass of the system.

As the man climbs up the ladder, the ladder also moves down and the counterweight moves up.

Let the ladder move x distance down, resulting in the counterweight moving x distance upwards. The change in the position of centre of mass is given as follows:




$$\Delta\vec{r}_{com} = \frac{m_1\Delta\vec{r}_1+m_2\Delta\vec{r}_2+m_3\Delta\vec{r}_3}{m_1+m_2+m_3}$$

$m_1 =$ Mass of the counterweight $= M$

$m_2 =$ Mass of the ladder $= M - m$

$m_3 =$ Mass of the man $= m$

$$\Delta\vec{r}_{com} = \frac{M(x)+(M-m)(-x)+m(l_0-x)}{M+(M-m)+m}\hat{j}$$

$$\Delta\vec{r}_{com} = \frac{ml_0}{2M}\hat{j}$$