Chapter 10

Center Of Mass

C.O.M OF DISCRETE PARTICLES AND SYMMETRIC BODIES

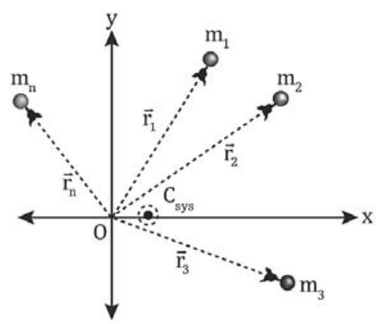
Centre of Mass of Discrete Particles

The center of mass of an object or discrete particles is a singular point where it's possible to envision the entire mass as being concentrated.

For a system comprising n point masses, denoted as m_1, m_2, m_3, ..., m_n, with their respective position vectors provided by $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \ldots \vec{r}_n$, The position vector of the center of mass of the system is expressed as follows, with "respectively" indicating the correspondence with the mentioned point masses:

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 \ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 \ldots\ldots + m_n} = \frac{\sum_1^n m_i\vec{r}_i}{\sum_1^n m_i}$$

Hence, the center of mass represents the weighted mean of all the masses.



Along the x, y, and z axes, we have the following:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 \ldots\ldots + m_nx_n}{m_1 + m_2 + m_3 \ldots.. + m_n} = \frac{\sum_1^n m_ix_i}{\sum_1^n m_i}$$

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3 \ldots\ldots + m_ny_n}{m_1 + m_2 + m_3 \ldots.. + m_n} = \frac{\sum_1^n m_iy_i}{\sum_1^n m_i}$$

$$z_{com} = \frac{m_1z_1 + m_2z_2 + m_3z_3 \ldots.. + m_nz_n}{m_1 + m_2 + m_3 \ldots.. + m_n} = \frac{\sum_1^n m_iz_i}{\sum_1^n m_i}$$

Ex. The position vectors of three particles of masses, $m_1 = 1$ kg, $m_2 = 2$ kg, and, $m_3 = 3$ kg, are $\vec{r}_1 = (\hat{i} + 4\hat{j} + \hat{k})$m, $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})$m, and, $\vec{r}_3 = (2\hat{i} - \hat{j} - 2\hat{k})$m respectively. Find the position vector of their centre of mass.

Sol. We know that the centre of mass for a discrete particle system is given by the following;

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 \ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 \ldots\ldots + m_n} = \frac{\sum_1^n m_i\vec{r}_i}{\sum_1^n m_i}$$

$$\vec{r}_{com} = \frac{[1\times(\hat{i}+4\hat{j}+\hat{k})]+[2\times(\hat{i}+\hat{j}+\hat{k})]+[3\times(2\hat{i}-\hat{j}-2\hat{k})]}{1+2+3}$$

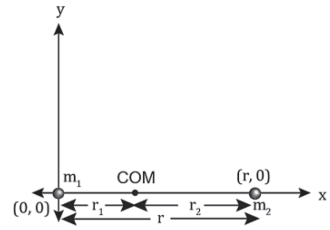
$$\vec{r}_{com} = \frac{9\hat{i}+3\hat{j}-3\hat{k}}{6} = \frac{3\hat{i}+\hat{j}-\hat{k}}{2}\text{m}$$

Centre of mass of two particles

Let's examine two particles with masses m_1 and m_2, positioned at a separation distance r.

Let us choose the origin at particle m_1.

The center of mass is located at a distance r_1 from m_1 and r_2 from m_2, as depicted in the illustration.



$$r_1 = \frac{m_1\times 0+m_2\times r}{m_1+m_2} = \frac{m_2 r}{m_1+m_2} \quad \text{... (1)}$$

And,

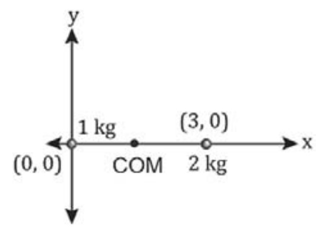
$$r_2 = r - r_1 = r - \frac{m_2 r}{m_1+m_2} = \frac{m_1 r}{m_1+m_2} \quad \text{... (2)}$$

Dividing equation (1) by equation (2),

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

The center of mass divides the line connecting the two particles in a proportion inverse to their masses.

Ex. Two particles of masses, 1 kg and 2 kg, are located at x = 0 and x = 3. Find the position of their centre of mass.



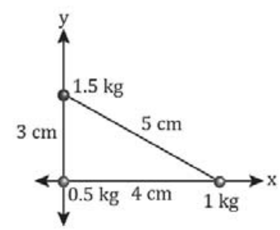
Sol. For a two-particle system, the centre of mass is given as follows:

$$x_{com} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{(1\times 0)+(2\times 3)}{1+2} = 2$$

Also $y_{com} = 0$

Therefore, the coordinates of the centre of mass are (2, 0)

Ex. Three particles of masses, 0.5 kg, 1 kg, and 1.5 kg, are placed at the three corners of a right-angled triangle of sides 3 cm, 4 cm, and 5 cm as shown in the figure. Locate the centre of mass of the system.

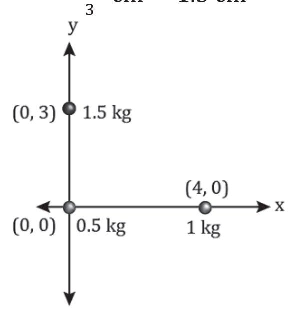


Sol. Let the coordinates of the centre of mass be (x_{com}, y_{com}). Then, for a three-particle system, we have the following:

$$x_{com} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3} = \frac{0.5(0)+1(4)+1.5(0)}{0.5+1+1.5} = \frac{4}{3}\text{ cm}$$

$$y_{com} = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3} = \frac{0.5(0)+1(0)+1.5(3)}{0.5+1+1.5}$$

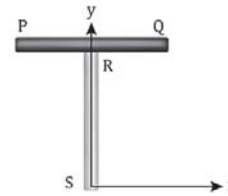
$$= \frac{4.5}{3}\text{ cm} = 1.5\text{ cm}$$



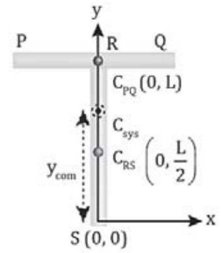
Centre of Mass of Symmetric Bodies

- Integration is employed to determine the center of mass of continuous bodies.
- The center of mass may exist beyond the physical boundaries of the body.
- If an object exhibits uniform mass distribution and possesses symmetrical geometry, then the center of mass is situated at the center of symmetry.

Ex. Two identical uniform rods, PQ and RS, each of length L, are joined to form a T-shaped frame as shown in the figure. Locate the centre of mass of the frame.



Sol. We know that the centre of mass of a uniform symmetric body lies at the centre of symmetry. Thus, the centre of mass of PQ and RS lies as shown in the figure.

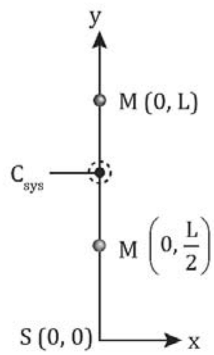


This is equivalent to a system of two particles that are located at the centre of masses of the two rods, respectively

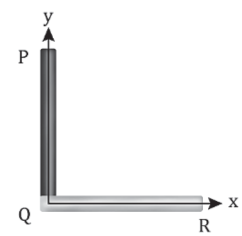
Now, let the centre of mass of the two particle system be located at (0, y_{com}).

$$y_{com} = \frac{(M\times\frac{L}{2})+(M\times L)}{M+M} = \frac{3L}{4}$$

Thus the center of mass of the system is located at $(0, \frac{3L}{4})$



Ex. Two identical uniform rods, PQ and QR, each of length L, are joined to form a L-shaped frame as shown in figure. Locate the centre of mass of the frame.



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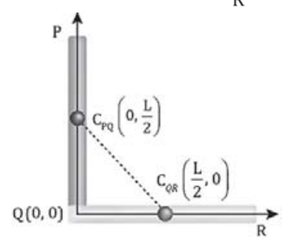

This is equivalent to a two particle system, masses of which are located at the centre of masses of the two rods, respectively. Now, let the centre of mass of this two particle system be located at (x_{com}, y_{com}).

$$x_{com} = \frac{(M\times 0)+(M\times\frac{L}{2})}{M+M} = \frac{L}{4}$$

$$y_{com} = \frac{(M\times\frac{L}{2})+(M\times 0)}{M+M} = \frac{L}{4}$$

Coordinate of the center of mass $= (x_{com}, y_{com}) = (\frac{L}{4}, \frac{L}{4})$