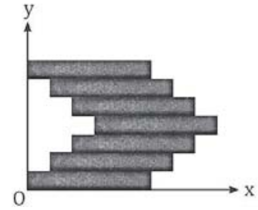
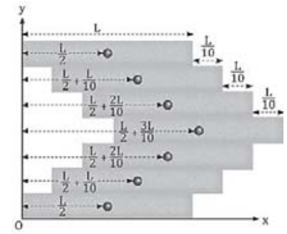
C.O.M OF CONTINUOUS BODIES



Ex. Seven bricks, each of length L, are arranged as shown in the figure. Each brick is displaced with respect to the one in contact by $\frac{L}{10}$Find the x-coordinate of the centre of mass relative to the origin shown.



Sol. Given, Length of each brick = L Since each brick is symmetrical, the centre of mass of each brick will lie on the corresponding geometric centers, i.e., $\frac{L}{2}$ distance from the end of the brick.

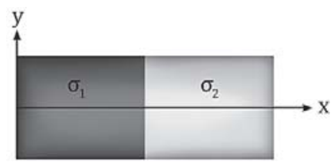
Thus, the centre of masses of the bricks lies as shown in the figure.

Now, the x-coordinate of the centre of mass of this system is given as follows:

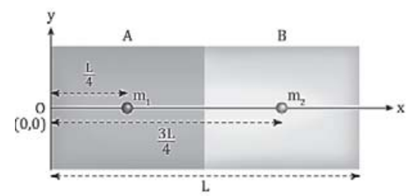
$$X_{com} = \frac{\sum_{i=1}^{n} m_i X_i}{\sum_{i=1}^{n} m_i}$$

$$X_{com} = \frac{(m+m)\times\frac{L}{2}+(m+m)\times(\frac{L}{2}+\frac{L}{10})+(m+m)\times(\frac{L}{2}+\frac{2L}{10})+m\times(\frac{L}{2}+\frac{3L}{10})}{7m}$$

$$X_{com} = \frac{L+L+L+\frac{L}{2}+\frac{L}{5}+\frac{2L}{5}+\frac{3L}{10}}{7} = \frac{44L}{70}$$



Ex. Half of the rectangular plate shown in figure is made of a material of area density σ_1 and the other half of area density σ_2. The length of the plate is L. Locate the centre of mass of the plate.



Sol. Given,

Length of the whole plate = L

So, the length of each part is $\frac{L}{2}$

Let m_1 and m_2 be the mass of each plate and A_1 and A_2 be the areas.

$$m_1 = \sigma_1 A_1 = \sigma_1 \times \frac{L^2}{4}$$

$$m_2 = \sigma_2 A_2 = \sigma_2 \times \frac{L^2}{4}$$

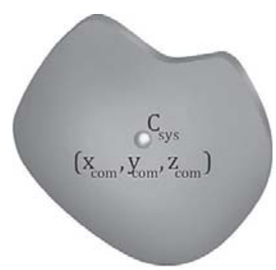
Now, the centre of mass of each plate lies at its respective geometric centres as shown in the figure. Therefore, the centre of mass of this system is given as follows:

$$X_{com} = \frac{\sum_{i=1}^{2} m_i x_i}{\sum_{i=1}^{2} m_i}$$

$$X_{com} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2} = \frac{(\sigma_1\times\frac{L^2}{4})\times(\frac{L}{4})+(\sigma_2\times\frac{L^2}{4})\times(\frac{3L}{4})}{(\sigma_1\times\frac{L^2}{4})+(\sigma_2\times\frac{L^2}{4})}$$

$$X_{com} = (\frac{\sigma_1+3\sigma_2}{\sigma_1+\sigma_2}) \times (\frac{L}{4})$$

Centre of Mass of Continuous Bodies



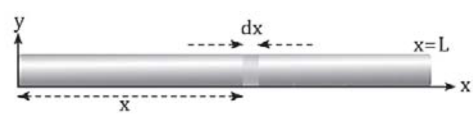
Let's contemplate a body with a continuous distribution of matter; in such a case, the center of mass of the body is determined as follows:

$$x_{com} = \frac{\int x\, dm}{\int dm}$$

$$y_{com} = \frac{\int y dm}{\int dm}$$

$$z_{com} = \frac{\int z dm}{\int dm}$$

Center of Mass of a uniform rod



Let's examine a rod of mass M and length L positioned along the x-axis, with one end located at the origin. To determine the center of mass of this rod, we'll examine a small element of length dx and mass dm at a distance X from the origin along the x-axis, as illustrated in the diagram.

Now, by unitary method, we get,

$$dm = \frac{M}{L}dx$$

The centre of mass is given as follows:

$$x_{com} = \frac{\int x dm}{\int dm}$$

$$X_{com} = \frac{\int_0^L \frac{M}{L} X dx}{M} = \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L$$

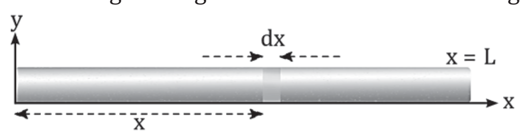
$$X_{com} = \frac{L}{2}$$

Ex. A rod of length L is placed along the x-axis between x = 0 and x = L. The linear density (λ) of the rod varies with the distance x from the origin as, $\lambda = kx$. Here, k is a positive constant. Find the position of the centre of mass of this rod.

Sol. Given, Linear density of rod, $\lambda = kx$

Length of the rod = L

For finding the centre of mass of this rod, let us consider a small element of length dx and mass dm at a distance x from the origin along the x-axis as shown in the figure.



Now, $dm = \lambda\, dx = kx\, dx$

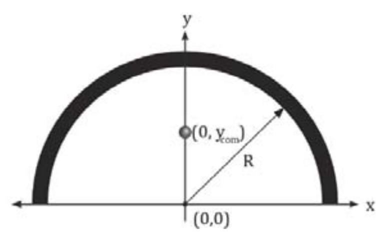
The centre of mass is given as follows:

$$x_{com} = \frac{\int x dm}{\int dm}$$

$$x_{com} = \frac{\int_0^L x(kxdx)}{\int_0^L kxdx} = \frac{k\left[\frac{x^3}{3}\right]_0^L}{k\left[\frac{x^2}{2}\right]_0^L}$$

$$x_{com} = \frac{2L}{3}$$

Centre of Mass of a Semi-circular Ring



Let's contemplate a uniformly distributed semi-circular ring with radius R and mass M as depicted in the figure.

Because the ring exhibits symmetry about the y-axis, the center of mass will be situated along the y-axis.

Let the coordinates of the centre of mass be $(0, y_{com})$.

Let's examine a tiny element with a length $dl = Rd\theta$ mass dm, which spans an angle $d\theta$ at the center, positioned at an angle θ with respect to the positive x-axis, as illustrated in the figure.

Now, mass of the element is as follows:

$$dm = \frac{M}{\pi R} \times (Rd\theta) = \frac{M}{\pi}d\theta$$

And, $y = R\sin\theta$,

The centre of mass is given as follows:

$$y_{com} = \frac{\int y dm}{\int dm}$$

$$y_{com} = \frac{\int_0^\pi (R\sin\theta)\left(\frac{M}{\pi}\right)d\theta}{\int_0^\pi \left(\frac{M}{\pi}\right)d\theta} = \frac{-\frac{MR}{\pi}[\cos\theta]_0^\pi}{M}$$

$$y_{com} = \frac{2R}{\pi}$$

Therefore, the coordinates of centre of mass are, $(0, y_{com}) = \left(0, \frac{2R}{\pi}\right)$.