# Chapter 9

# Work Power Energy



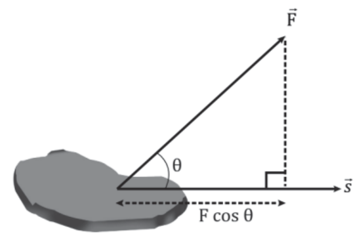
**WORK DONE**

**Work**

**Work Done by a Constant Force**

The work performed by a constant force equals the dot product of force and displacement.

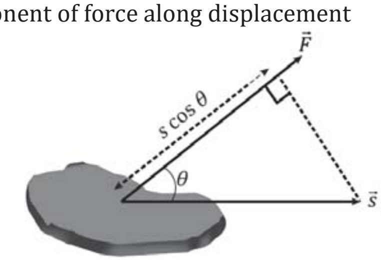
$$W = \vec{F} \cdot \vec{S} = FS\cos\theta$$




$$W = (F\cos\theta)s = (F_{\parallel})s$$

Where

$F_{\parallel}$ Component of force along displacement




$$W = F(s\cos\theta) = F(s_{\parallel})$$

Where

$s_{\parallel}$ = Component of displacement along force

Special cases

Case I:

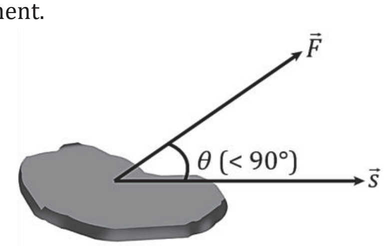
When $0 \leq \theta < 90°$, $\cos\theta$ id positive.

$$W = F s\cos\theta$$

However, F, s, and cos θ are positive.

W = positive

The work done by a force is positive when the applied force has a component aligned with the direction of displacement.



**Case II:**

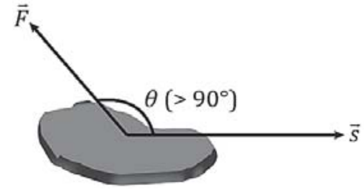
When $90^\circ < \theta \leq 180^\circ$, $\cos\theta$ is negative.

$$W = F s\cos\theta$$

However, $\cos\theta$ is negative.

$W =$ negative

The work done by a force is negative when the applied force has a component opposite to the direction of displacement.

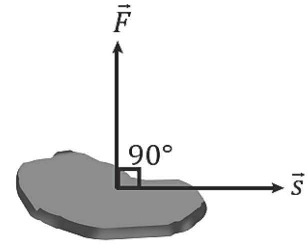



**Case III:**

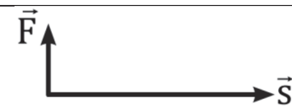
When $\theta = 90^\circ$, $\cos\theta$ is zero.

$$W = F s\cos\theta = 0$$

The work done by a force is zero when the body is displaced in a direction perpendicular to the force.

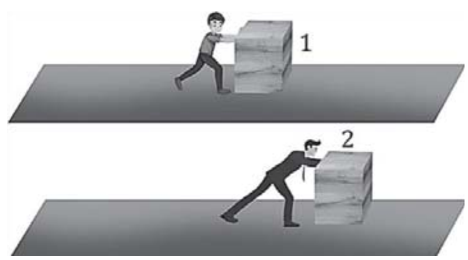



**Conclusion**

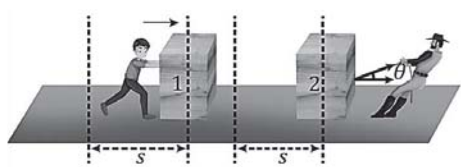
| Angle between applied force and displacement ($\theta$) | Orientation of F and s | Work done |
|---|---|---|
| $0 \leq \theta < 90^\circ$, $\cos\theta$ is positive. |  | Positive |
| $90^\circ < \theta \leq 180^\circ$, $\cos\theta$ is negative. |  | Negative |
| $\theta = 90^\circ$, $\cos\theta$ is zero. |  | Zero |

**Ex.** Consider two identical blocks starting from the same initial position and pushed with an equal force, but one of the blocks undergoes greater displacement. Which block experiences greater work done?

**Sol.** The final position of the blocks are shown in the figure. In this case, work done on the second block is more since its displacement is more and force and displacement are in the same direction for both the blocks.




**Ex.** Consider two identical blocks; one being pushed and other being pulled with the same force, but acting at an angle $\theta$ with the horizontal as shown in the figure. Displacement of both blocks is same. On which block is the work done more?




**Sol.** The two blocks have displacement s as given in the question.

For the first block,

As force and displacement are along the same line,

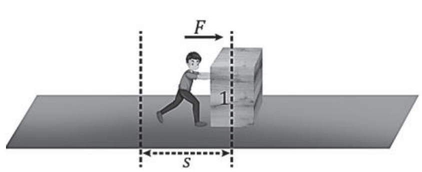
Work done, $W_1 = F s\cos 0^\circ = Fs$ ... (1)

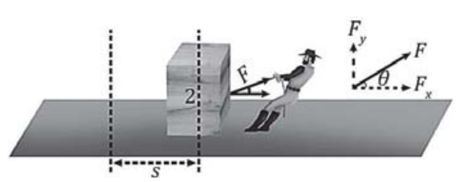
For the second block,

Work done, $W_2 = F s\cos\theta = (F\cos\theta)s = F_x S$ ... (2)

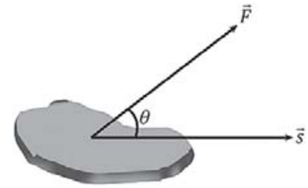
Comparing equation (1) and (2) We get

$$W_1 > W_2$$



**Work Done in Component Form**

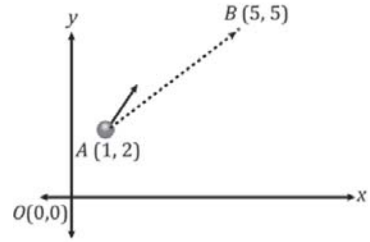



Let a force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ act on a particle, due to which the displacement of the particle is $\vec{s} = s_x\hat{i} + s_y\hat{j} + s_z\hat{k}$.

Work done is given by the following:

$$W = \vec{F} \cdot \vec{s}$$
$$W = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (s_x\hat{i} + s_y\hat{j} + s_z\hat{k})$$
$$W = F_x s_x + F_y s_y + F_z s_z$$

**Ex.** A particle is displaced from point A (1, 2) to B (5, 5) by applying a constant force, $\vec{F} = (2\hat{i} + 3\hat{j})$N Find the work done by the force to move the particle from point A to point B.




**Sol.** Given,

Force, $\vec{F} = 2\hat{i} + 3\hat{j}$

Initial position, $\vec{r}_1 = \hat{i} + 2\hat{j}$

Final position, $\vec{r}_2 = 5\hat{i} + 5\hat{j}$

Displacement vector $\vec{s} = \vec{r}_2 - \vec{r}_1 = 4\hat{i} + 3\hat{j}$ work done by the force is given by the following:

$$W = \vec{F} \cdot \vec{s}$$
$$W = (2\hat{i} + 3\hat{j}) \cdot (4\hat{i} + 3\hat{j})$$
$$W = 17\text{J}$$

**Ex.** A block of mass 10 kg is kept on a smooth horizontal surface. If a force of 10 N is applied on the block for 2 s, what is the work done by that force?


**Sol.** Given, Mass of block (m) = 10 kg

Magnitude of force (F) = 10 N

Time interval (t) = 2 s

Let the displacement of the block in 2 seconds be s. Let a be the acceleration of the block. Applying Newton's second law of motion, we get,

$$F = ma$$
$$a = \frac{10}{10} = 1\text{ms}^{-2}$$

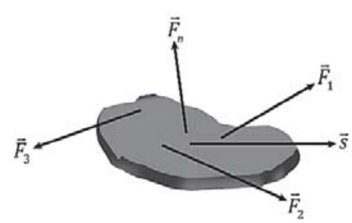
Using second equation of motion, we get,

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + (\frac{1}{2} \times 1 \times 2^2) = 2\text{ m}$$

Work done by the force is given by the following:

$$W = Fs\cos\theta = 10 \times 2 \times \cos 0°$$
$$W = 20\text{ J}$$

**Work Done by Multiple Forces**




The cumulative work performed on a particle equals the sum of the work done by all the constant forces acting upon it.

Let multiple forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots \dots . . \vec{F}_n$ act upon a particle such that the particle undergoes a total displacement of $\vec{S}$ as shown in the figure.

The work done is given by the following:

$$W = \vec{F}_1 \cdot \vec{s} + \vec{F}_2 \cdot \vec{s} + \cdots \dots \dots + \vec{F}_n \cdot \vec{s}$$

As the displacement of particle is constant,

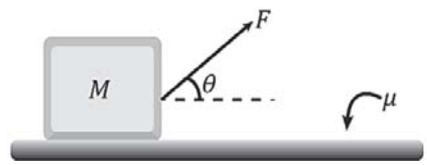
$$W = (\vec{F}_1 + \vec{F}_2 + \cdots \ldots + \vec{F}_n) \cdot \vec{s}$$

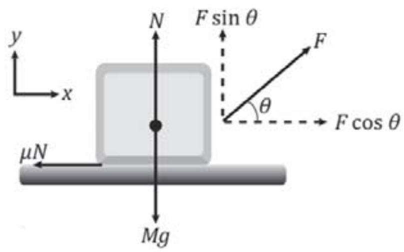
$$W = (\vec{F}_{net}) \cdot \vec{s}$$

**Salient features of work**

1. The force's source or agent is responsible for performing the work.
2. Work done is a scalar value.
3. Displacement is measured from the point where the force is applied.
4. The joule is the SI unit of work.
5. When multiple constant forces are at play, the work done by one force for a given displacement remains unaffected by the presence of other forces.
6. A force is independent of the reference frame, whereas its displacement varies with the reference frame, making the work done by a force frame-dependent.

**Ex.** A block of mass M is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. The friction coefficient between the block and the surface is μ. If the block travels at a uniform velocity, find the work done by this applied force during a displacement d of the block.




**Sol.** Given,

Mass of the block = M

Friction coefficient between the block and surface = μ

Displacement of the block = d

Let the work done by the force F be W.

Since the block is moving with a uniform velocity, so the acceleration is zero. By Newton's second law of motion,

$$\vec{F}_{net} = m\vec{a} = 0$$

Balancing force in the horizontal direction,

$$\Sigma\vec{F}_x = 0$$

$$F\cos\theta = \mu N \quad \ldots (1)$$

Balancing force in the vertical direction,

$$\Sigma\vec{F}_y = 0$$

$$N + F\sin\theta = Mg \quad \ldots (2)$$

Substituting the value of N from equation (2) in equation (1),

$$F\cos\theta = \mu\{Mg - F\sin\theta\}$$

$$F\cos\theta = \mu Mg - \mu F\sin\theta$$

$$F\{\cos\theta + \mu\sin\theta\} = \mu Mg$$

$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

Work done by the force is given by the following;

$$W = Fd\cos\theta = \frac{\mu Mg}{\cos\theta + \mu\sin\theta} \times d\cos\theta$$

$$W = \frac{\mu Mgd\cos\theta}{\cos\theta + \mu\sin\theta}$$

**Work Done by a Variable Force**

The cumulative work done equals the summation of infinitesimal work done over infinitesimal displacements.

$$W = \int dW$$

The work done by a force in transporting a particle from point A to point B is expressed as follows:

$$W = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B F\cos\theta dr$$

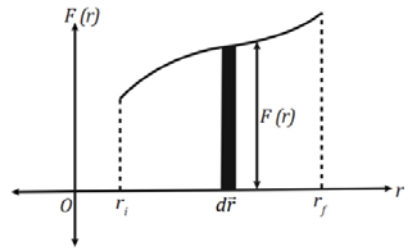
**Note:** When a force's magnitude or direction varies with time or position, it is termed as a variable force.

**Work Done by a Variable Force in Rectangular Form**

Let a force, $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ result in the displacement of the particle, $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
Consider A and B as the starting and ending positions, respectively, of the particle.
Work done by the force is given by,

$$dW = \vec{F}\cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$
$$W = \int dW$$
$$W = \int_A^B (F_x dx + F_y dy + F_z dz)$$
$$W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

**Area Under the Force vs Displacement Graph**




The area under the graph of force versus displacement provides the measure of work done by that force over the given displacement.

$$W = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

The total work done is the accumulation of infinitesimal work over infinitesimal displacements along the track.

**Ex.** A force $\vec{F} = (0.5x + 10)\hat{i}$ acts on a particle. Here, $\vec{F}$ is in newton and x is in metre. Calculate the work done by the force during the displacement of the particle from x = 0 m to x = 2 m

**Sol.** Given,

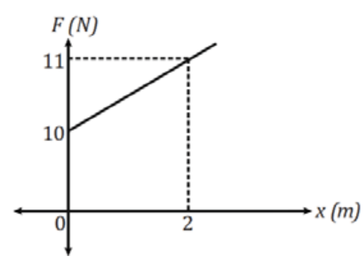
Force, $\vec{F} = (0.5x + 10)\hat{i}$

Infinitesimal work done by the force is given by the following

$$dW = \vec{F}\cdot d\vec{x} = (0.5x + 10)\hat{i}.(dx)\hat{i} = 0.5xdx + 10dx$$

Total work done by the force is given by the following:

$$W = \int dW = \int_0^2 (0.5xdx + 10dx)$$
$$W = \frac{0.5}{2}[x^2]_0^2 + 10[x]_0^2$$
$$W = 21J$$



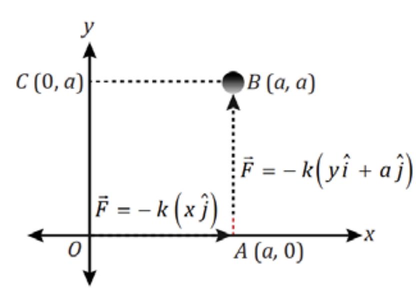

Alternate way

$$F = 0.5x + 10N$$
$$F(0) = 10N$$
$$F(2) = 11N$$

The force vs displacement graph is as shown in the figure.
Area under the force vs displacement curve gives the work done.

$$W = \frac{1}{2}\times\{(10-0) + (11-0)\}\times(2-0)$$
$$W = 21J$$

**Ex.** A force, $\vec{F} = -k(y\hat{i} + x\hat{j})$, acts on a particle moving in the x-y plane, where, k is a positive constant. The particle starts from the origin and is taken along the positive x-axis to the point (a, 0), and then perpendicularly to the point (a, a). Calculate the total work done on the particle by the force F.




**Sol.** Given,

Force, $\vec{F} = -k(y\hat{i} + x\hat{j})$

Total work done = Work done in taking the particle from O to A + Work done in taking the particle from A to B.

$$W = W_{OA} + W_{AB}$$

Case 1: Particle moving from O to A

The displacement along OA is on the x-axis. So, the y-coordinate is 0.

So, force, $\vec{F} = -k(x\hat{j})$

Also, $dy = 0$,

Displacement, $d\vec{r} = dx\hat{i}$

Work done, $W_{OA} = \int \vec{F} \cdot d\vec{r} = \int (-kx\hat{j}) \cdot dx\hat{i} = 0$

Case 2: Particle moving from A to B

Here, $dx = 0$ and, $x = a$

So, force, $\vec{F} = -k(y\hat{i} + a\hat{j})$

Displacement, $d\vec{r} = dy\hat{j}$

Work done is given by the following:

$$dW_{AB} = \vec{F} \cdot d\vec{r} = (-ky\hat{i} - ka\hat{j}) \cdot dy\hat{j}$$

$$dW_{AB} = -kady$$

$$W_{AB} = \int dW_{AB} = \int_0^a -kady = -ka^2$$

$$W_{total} = W_{OA} + W_{AB} = -ka^2$$