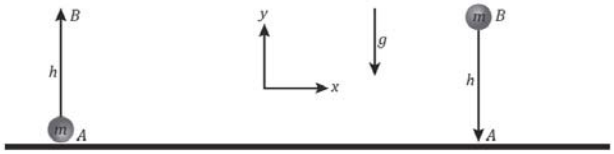
WORK DONE BY STANDARD FORCES

Work Done by Gravity

The gravitational force acting on a particle is equivalent to the particle's weight.

$$\vec{F}_g = m\vec{g}$$



Let's examine two scenarios: one where the particle travels from point A to point B, and the other where it moves from point B to point A.

Let h be the distance between points A and B.

The work done by gravity when moving the particle from point A to point B is expressed as follows:

$$W_{AB} = m\vec{g} \cdot \vec{h} = mgh\cos 180^\circ = -mgh$$

The work done by gravity when moving the particle from point B to point A is described as follows:

$$W_{BA} = m\vec{g} \cdot \vec{h} = mgh\cos 0^\circ = mgh$$

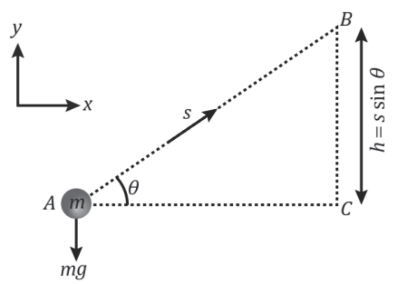
Conservative Nature of Gravity Force

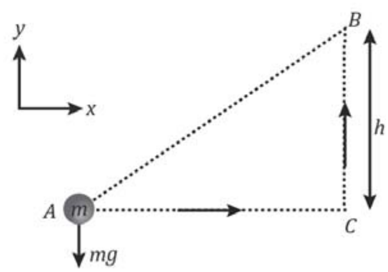
Let's consider a particle transitioning from point A to point B, as depicted in the diagram.

Work done by gravity is as follows:

$$W = \vec{F}_g \cdot \vec{s} = mgs\cos(90 + \theta) = -mg(s\sin\theta)$$

$$W = -mgh$$





Let's now focus on the same particle, but this time it moves from point A to point C to point B, as illustrated in the diagram.

Work done by gravity is as follows:

$$W = W_{AC} + W_{CB} = mg(AC)\cos 90^\circ + (-mgh)$$

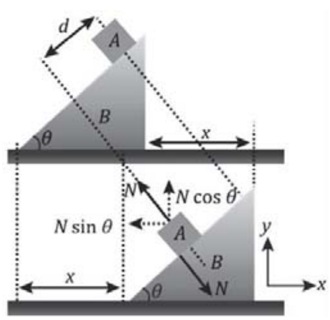
$$W = -mgh$$

Gravity is a conservative force. Therefore, the work performed by gravity does not rely on the path taken during motion. The work executed by a conservative force is solely contingent on the displacement.

Work Done by Normal Reaction

Let's examine a block positioned on a wedge, as depicted in the diagram.

As the block moves a distance d along the wedge, the wedge moves a distance x along the ground, as illustrated in the figure.



For block A:

The Free Body Diagram (FBD) yields the normal reaction at any given moment, described as follows:

$$\vec{N} = -N\sin\theta\hat{i} + N\cos\theta\hat{j}$$

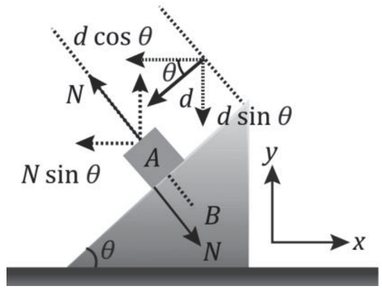
For displacement, $\vec{d} = (x - d\cos\theta)\hat{i} - d\sin\theta\hat{j}$

Next, the work performed by the normal force on block A can be expressed as follows:

$$(W_N)_A = \vec{N}\cdot\vec{d} = (-N\sin\theta\hat{i} + N\cos\theta\hat{j})\cdot((x - d\cos\theta)\hat{i} - d\sin\theta\hat{j})$$

$$(W_N)_A = -N\sin\theta(x - d\cos\theta) - Nd\sin\theta\cos\theta$$

$$(W_N)_A = -Nx\sin\theta \quad \text{... (1)}$$



For wedge B:

The Free Body Diagram (FBD) provides the normal reaction at any given moment, as follows:

$$\vec{N} = N\sin\theta\hat{i} - N\cos\theta\hat{j}$$

Displacement of B is as follows: $\vec{d} = x\hat{i}$

Now, the work accomplished by the normal force on B is as follows:

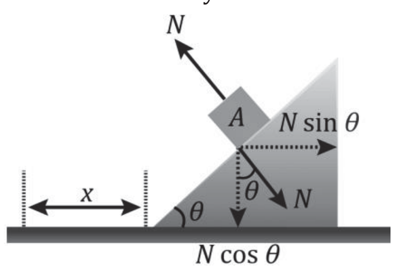
$$(W_N)_B = \vec{N}\cdot\vec{d} = (N\sin\theta\hat{i} - N\cos\theta\hat{j})\cdot(x\hat{i})$$

$$(W_N)_B = Nx\sin\theta \quad \text{... (2)}$$

Total work done by the normal reaction on the system is as follows:

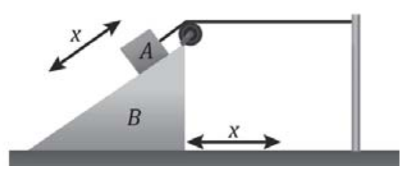
$$W_N = (W_N)_A + (W_N)_B = -NX\sin\theta + NX\sin\theta = 0$$

Thus, the work done by normal reaction in a system is zero.



Work Done by Tension Force

Contemplate a block denoted as A positioned atop a wedge identified as B, joined by means of a string, as illustrated in the accompanying diagram. Designate x to signify the displacement of wedge B and T to denote the tension within the string.



Work done on block A

The tension exerted on block A can be expressed as follows:

$$\vec{T}_A = T\cos\theta\hat{i} + T\sin\theta\hat{j}$$

Displacement of block A is as follows

$$\vec{x}_A = (x - x\cos\theta)\hat{i} - x\sin\theta\hat{j}$$

Work done on block A is as follows:

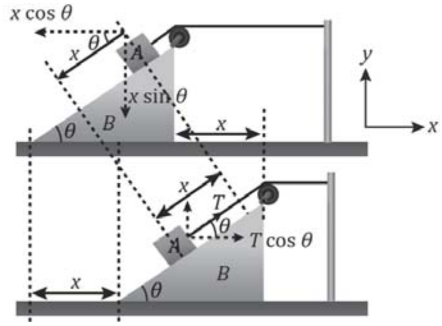
$$(W_T)_A = \vec{T}_A\cdot\vec{x}_A$$

$$\{T\cos\theta\hat{i} + T\sin\theta\hat{j}\}\cdot\{(x - x\cos\theta)\hat{i} - x\sin\theta\hat{j}$$

$$(W_T)_A = T\cos\theta(x - x\cos\theta) - T\sin\theta(x\sin\theta)$$

$$(W_T)_A = Tx\cos\theta - Tx\cos^2\theta - Tx\sin^2\theta$$

$$(W_T)_A = Tx\cos\theta - Tx(\cos^2\theta + \sin^2\theta)$$
$$(W_T)_A = Tx(\cos\theta - 1) \quad \text{... (1)}$$

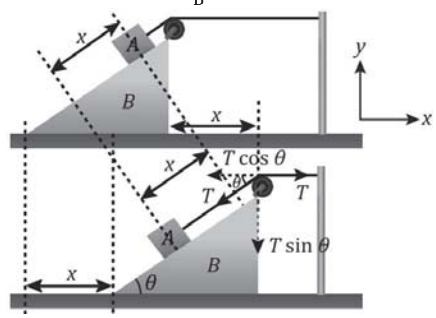


Work done on wedge B

The tension force acting on wedge B is described as follows:

$$\vec{T}_B = (T - T\cos\theta)\hat{i} - T\sin\theta\hat{j}$$

Displacement of wedge B is as follows: $\vec{x}_B = x\hat{i}$



Work done on wedge B is as follows:

$$(W_T)_B = \vec{T}_B \cdot \vec{x}_B = \{(T - T\cos\theta)\hat{i} - T\sin\theta\hat{j}\} \cdot x\hat{i}$$
$$(W_T)_B = Tx(1 - \cos\theta) \quad \text{... (2)}$$

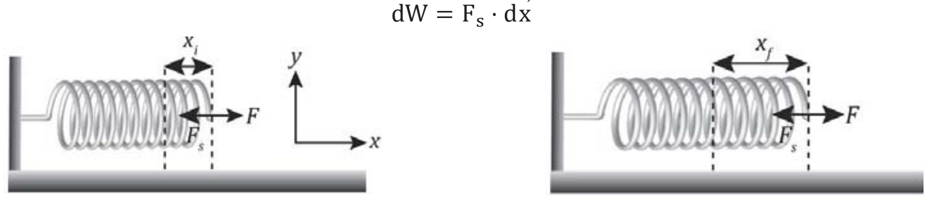
Total work done on the system by tension force is as follows

$$W_T = (W_T)_\Delta + (W_T)_R = Tx(\cos\theta - 1) + Tx(1 - \cos\theta) = 0$$

Work Done by Spring

Imagine a spring with an initial length denoted as l_0, which has been elongated from its starting point x_i to its ending point x_f as depicted in the diagram. For any infinitesimally tiny displacement dx, the minimal work performed by the spring force is expressed as follows:

$$dW = \vec{F}_s \cdot d\vec{x}$$



Where, $\vec{F}_s =$ Spring force

By Hooke's law, $\vec{F}_s = -(kx)\hat{x}$

$$dW = -kxdx$$
$$W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} (-kx)dx$$
$$W = -k\int_{x_i}^{x_f} xdx = -\frac{k[x^2]_{x_i}^{x_f}}{2} = -\frac{1}{2}k(x_f^2 - x_i^2)$$

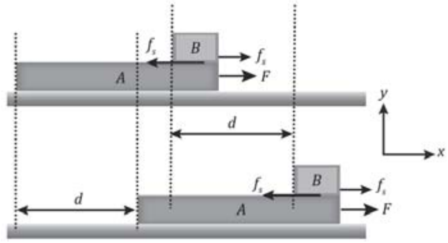
Note: Hooke's law asserts that in an ideal spring (one without mass), the restorative force generated by compression or elongation (deformation) varies directly in proportion to the extent of the deformation. $|\vec{F}_s| = kx$

Where, k is the spring constant/stiffness.

Work Done By Friction Force

Work Done By Static Friction

Static friction functions to prevent objects from sliding along their point of contact on a surface. Let's examine two blocks, labeled A and B, which are drawn towards each other. Despite the pulling force applied, static friction between them prevents any relative motion. Over time, suppose the displacement of both blocks becomes d.



Work done by static friction on B is as follows.

$$(W_{fs})_B = f_s d\cos 0^\circ = f_s d$$

Work done by statics friction on A is as follows.

$$(W_{fs})_A = f_s d\cos 180^\circ = -f_s d$$

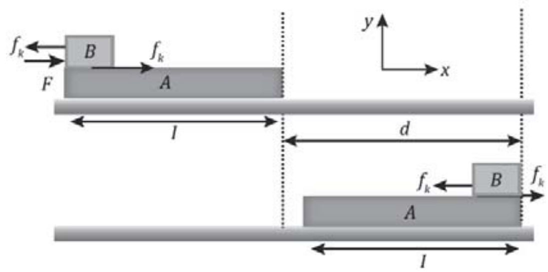
Total work done by statics friction on the system is as follows.

$$W_{fs} = (W_{fs})_B + (W_{fs})_A = f_s d - f_s d = 0$$

Work done by kinetic friction

Kinetic friction, also referred to as dynamic or sliding friction, arises when two objects are in relative motion with each other. Let's consider two blocks, labeled A and B, which have been drawn towards each other. However, because of kinetic friction, there is some degree of relative motion between them. Assume the size of block B is insignificant compared to the displacement l.

At a specific moment later, suppose block A has undergone a displacement of d, while block B has experienced a relative displacement of l, as depicted in the illustration. (l is much smaller than d)



Work done by kinetic friction on B is as follows.

$$(W_{f_k})_B = f_k(d + l)\cos 180^\circ = -f_k(d + l)$$

Work done by kinetic friction on A is as follows.

$$(W_{f_k})_A = f_k d\cos 0^\circ = f_k d$$

Total work done by kinetic friction on the system is as follows.

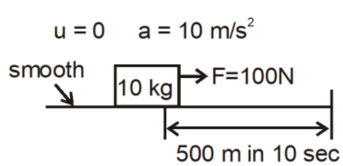
$$W_{f_k} = (W_{f_k})_B + (W_{f_k})_A = -f_k(d + l) + f_k d = -f_k l$$

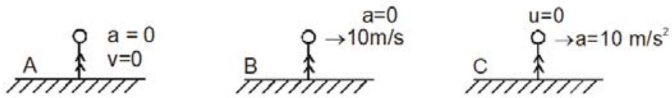
Note: Kinetic friction consistently performs negative work on the system. In essence, it invariably converts mechanical energy into unusable non-mechanical forms, such as heat, thereby dissipating energy.

Work Done by Pseudo Force

The work done by a pseudo force is zero since it arises solely due to the perspective of the observer.

Ex. Find out work done by the force F in 10 seconds as observed by A, B & C.





Sol. $(W_F)_{\text{on block w.r.t A}} = 100 \times 500 \text{ J} = 50{,}000 \text{ J}$
$(W_F)_{\text{on block w.r.t}} = 100[500 - 10 \times 10] = 40{,}000 \text{ J}$
$(W_F)_{\text{on block w.r.tc}} = 100[500 - 500] = 0$