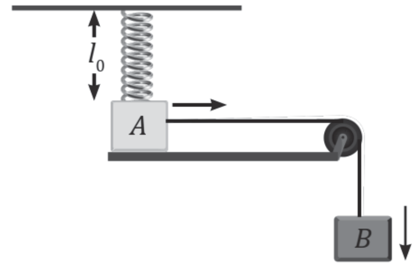
POWER
Illustrations Of Power

Ex. The figure shows two blocks A and B, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which the block A can slide is smooth. The block A is attached to a spring of spring constant 40 Nm^{-1} whose other end is fixed to a support 40 cm above the horizontal surface. Initially, the spring is vertical and un-stretched when the system is released to move. Find the velocity of the block A at the instant it loses contact with the surface below it. (Take g = 10 ms^{-2})



Sol. Mass of the block A and B = 320g = 0.32 Kg = $\frac{8}{25}$ Kg

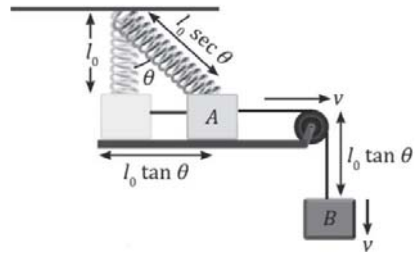
Spring constant (k) = $40Nm^{-1}$

Natural length of the spring = (l_0) = 40 cm = 0.4 m = $\frac{2}{5}$ m

Let the velocity of the block A be v.

Total length of spring after extension (l) = $l_0 \sec\theta$

Extension of spring (x) = Total length after extension – Natural length



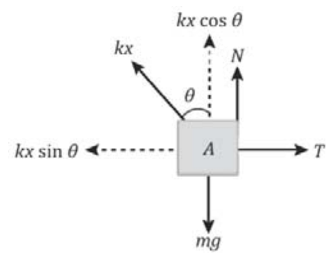
$$x = l_0\sec\theta - l_0$$

On balancing the force in the vertical direction, we get,

$$kx\cos\theta + N = mg$$
$$kl_0(\sec\theta - 1)\cos\theta + N = mg$$
$$kl_0(1 - \cos\theta) = mg$$

($\because$ N = 0, when the block loses contact with the surface)

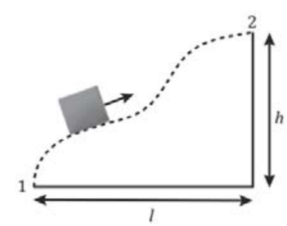
$$40 \times 0.4 \times (1 - \cos\theta) = 0.32 \times 10$$
$$\cos\theta = \frac{4}{5} \Rightarrow \theta = 37°$$



On taking (A + B + spring) as a system and applying work-energy theorem, we get,

$$W_{net} = \Delta KE$$
$$W_g + W_N + W_T + W_{spring} = \Delta KE = (KE_{final})_A + (KE_{final})_B - (KE_{initial})_A - (KE_{initial})_B$$
$$mgl_0\tan\theta + 0 + 0 - \frac{1}{2}k(l_0\sec\theta - l_0)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \frac{1}{2}m \times 0^2 - \frac{1}{2}m \times 0^2$$
$$\left(\frac{8}{25} \times 10 \times \frac{2}{5} \times \frac{3}{4}\right) - \left(\frac{1}{2} \times 40 \times \frac{4}{25} \times \frac{1}{16}\right) = \frac{8v^2}{25}$$
$$v = \sqrt{\frac{19}{8}}ms^{-1}$$

Ex. A body of mass m was slowly hauled up the hill by a force F which, at each point, was directed tangential to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base is l, and the coefficient of friction is k.



Sol. Given,

Force = F

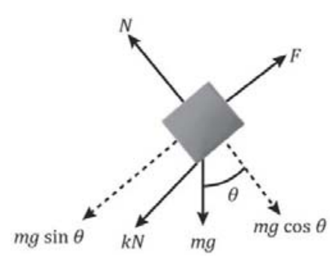
Mass of the body = m

Height of the hill = h

Coefficient of friction = k

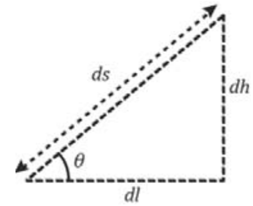
Given that the applied force was always tangential to the trajectory. Also, we know that the velocity at any point on any curved surface is along the tangent to the surface at that point. So, the external force is always parallel to the velocity at every point.

We know that the friction force acts opposite to the direction of motion. Thus, at every point, the friction force will be antiparallel to the displacement.



$$\vec{F}_{ext} \| d\vec{v} \Rightarrow \vec{F}_{ext} \| d\vec{s}$$
$$\vec{f}_k \| -d\vec{s}$$

The normal reaction is always perpendicular to the displacement. Hence, the work done by the normal reaction is zero. Applying work-energy theorem, we get



$$W_g + W_N + W_f + W_{ext} = \Delta KE$$

However, Final velocity $=$ Initial velocity $\Rightarrow \triangle\ KE = 0$

$$W_g + 0 + W_f + W_{ext} = 0$$

$$W_{ext} = -W_g - W_f \quad \dots (1)$$

Work done by the friction is as follows:

$$dW_f = \vec{f}_s \cdot d\vec{s} = f_s ds\cos 180^\circ = -f_s ds = -kNds = -k(mg\cos\theta)ds$$

$$\Sigma dW_f = -kmg\Sigma ds\cos\theta = -kmg\Sigma dl$$

$$W_f = -kmgl \quad \dots (2)$$

Work done by the gravity is as follows:

$$W_g = -mgh \quad \dots (3)$$

Putting the values from equation (2) and equation (3) in equation (1), we get,

$$W_{ext} = -(-mgh) - (-kmgl)$$

$$W_{ext} = mgh + kmgl$$

Power

Average power

Average power is defined as the average rate of work done or energy converted per unit time. It is computed using the following mathematical expression.

$$\text{Average power} = \frac{\text{Total work}}{\text{Total time}}$$

$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous power

As the time interval Δt approaches zero, the force reaches its maximum average power value, known as instantaneous power.

$$P_{ins} = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \frac{d}{dt}\vec{F} \cdot d\vec{r} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$P_{ins} = \vec{F} \cdot \vec{v}$$

Unit of power is watt (W):

When a body performs work equivalent to one joule in one second, it is termed as having a power of one watt.

Ex. A man weighing 60 kg climbs up a staircase carrying a 20 kg load on his back. The staircase has 20 steps and each step has a height of 20 cm. If he takes 10 seconds to climb, calculate the power. (Take $g = 9.8\ ms^{-2}$).

Sol. Given, Total mass (m) $=$ Mass of man $+$ Mass of load $= 60 + 20 = 80$ kg

Height of each step $= 20$ cm

Number of steps $= 20$

Total vertical displacement, $h = 20 \times 20 = 400\ cm = 4\ m$

Time taken $(t) = 10\ s$

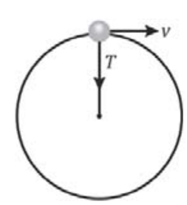
Now, work done by the man against the gravity, $mgh = 80 \times 9.8 \times 4 = 3136\ J$

Average power of man is as follows: Average power $= \frac{\text{Total work}}{\text{Total time}} = \frac{3136}{10} = 313.6\ W$

Ex. A block moves in a uniform circular motion because a cord tied to the block is anchored at the centre of a circle. What is the power delivered by the tension force exerted by the cord?

Sol. We know that in a circular motion, the centripetal force always acts towards the centre. Here, the force due to tension is acting as the centripetal force. So, the direction of the tension force is along the radius, i.e., towards the centre.

Also, at any point of time, in a circular motion, the velocity of the particle is always tangential. So, the tension force and velocity is perpendicular to each other. Therefore, the power delivered by tension force is as follows:



$$P = \vec{T} \cdot \vec{v} = Tv\cos 90^\circ = 0$$

Ex. An engine pumps up 100 kg of water through a height of 10 m in 5 s. Given that the efficiency of the engine is 60 % and g = 10 ms^{-2}. What is the power of the engine? (Neglect the speed of water being drawn by the pump)

Sol. Given,

Mass of the water pumped (m) = 100 Kg

Height to which water is pumped (h) = 10 m

Time taken to pump (t) = 5 s

Efficiency of the engine = 60%

$$\text{Power required to pump } = \frac{\text{Work done against gravity}}{\text{Time taken}}$$

$$P_{req} = \frac{mgh}{t} = \frac{100\times10\times10}{5} = 2000\text{ W}$$

However, efficiency of the engine is 60%.

$$0.6 \times P_{engine} = P_{req}$$

$$P_{engine} = \frac{P_{req}}{0.6} = \frac{2000}{0.6} = 3333\text{ W}$$

Ex. A body of mass m is thrown at an angle α to the horizontal with initial velocity v_0. Find the mean power developed by the gravity over the whole time of motion of the body, and the instantaneous power of gravity as a function of time.

Sol. Given, Mass = m

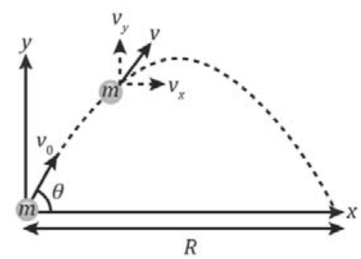
Angle of the projection = α

Initial velocity = v_0

For average power,

The displacement of the body is in horizontal direction but the gravity force is in the vertical direction. Hence, both are perpendicular to each other. So, the total work done in this case is zero.

So, the average power is zero.



For instantaneous power,

At any instant of time, the velocity and acceleration along x-axis and y-axis is given as follows.

x-axis	y-axis
$u_x = v_0\cos\alpha$	$u_y = v_0\sin\alpha$
$a_x = 0$	$a_y = -g$
$v_x = u_x = v_0\cos\alpha$	$v_y = v_0\sin\alpha - gt$

Velocity at any point of time is given as follows:

$$v = v_0\cos\alpha\hat{i} + (v_0\sin\alpha - gt)\hat{j}$$

Now, instantaneous power is as follows:

$$P_{ins} = \vec{F}\cdot\vec{v} = (-mg\hat{j})\cdot[v_0\cos\alpha\hat{i} + (v_0\sin\alpha - gt)\hat{j}]$$

$$P_{ins} = -mg(v_0\sin\alpha - gt)$$

Ex. A force F acting on a body depends on its displacement S as $F \propto S^{\frac{1}{3}}$ how will the power delivered by the force depend on displacement?

Sol. Given,

$$F \propto S^{\frac{1}{3}}$$

$$F = kS^{\frac{1}{3}} = mv\frac{dv}{dS}$$

where, k = Propotionality constant

$$mvdv = kS^{\frac{1}{3}}dS$$

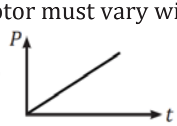
$$m\int_0^v vdv = k\int_0^S S^{\frac{1}{3}}dS$$

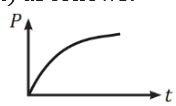
$$m[\frac{v^2}{2}]_0^v = \frac{3k}{4}[S^{\frac{4}{3}}]_0^S$$

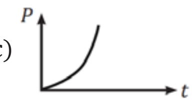
$$mv^2 = \frac{3kS^{\frac{4}{3}}}{2}$$

$$v = \sqrt{\frac{3k}{2m}} S^{\frac{2}{3}}$$
$$v \propto S^{\frac{2}{3}}$$
$$P = \vec{F} \cdot \vec{v}$$
$$P \propto (S^{\frac{1}{3}} \times S^{\frac{2}{3}})$$
$$P \propto S$$

Ex. A motor drives a body along a straight line with a constant force. The power (P) developed by the motor must vary with time (t) as follows:

(a) 

(b) 

(c) 

(d) 

Sol. Given that the force is constant.
$F = \text{ constant } = k$
By Newton's second law of motion, $ma = k$
$$a = \frac{k}{m} = \text{ constant}$$
Since the acceleration is constant we can apply the equation of motion, velocity at any time (t) is given as follows. $v = u + at = 0 + \frac{k}{m}t$
Therefore power is as follows. $P = \vec{F} \cdot \vec{v} = k \times (\frac{k}{m})t$
$$P \propto t$$
Thus the power (P) developed by the motor must vary linearly with time (t) starting from the origin. Hence option (A) is the correct answer.