POTENTIAL ENERGY

Potential Energy

Potential energy is the energy a system holds due to its shape, size, or arrangement.

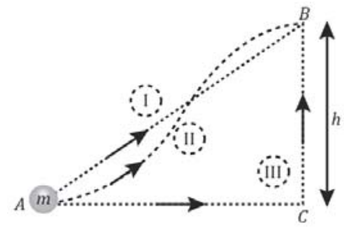
Ex. Gravitational potential energy stemming from altitude, spring potential energy, and so forth.

Conservative Force

If the work performed by a force remains unaffected by the path taken by the object, the force is deemed conservative.

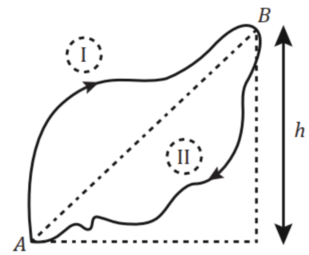
Ex. Gravitational force

Contemplate a mass m being raised against gravity from point A to point B along three distinct routes, as depicted in the provided diagram.



The work done by gravity on the body along all three paths will be identical since the work under gravitational force is determined by the variation in height rather than the specific path taken. $(W_g)_I = (W_g)_{II} = (W_g)_{III} = -mgh$

In general form, a force is conservative if the work it does around any closed path is zero. $(W_g)_I + (W_g)_{II} = 0$



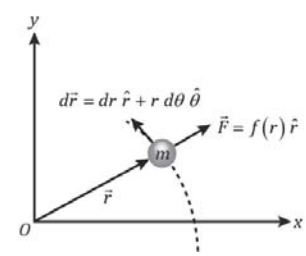
Non-Conservative Force

If the work performed by a force is contingent on the path taken by the object, the force is characterized as a non-conservative force.

1. Conservative forces are described as state or point functions, indicating their reliance solely on the initial and final states.
2. Non-conservative forces are defined as path functions, signifying their dependence on the traveled path.

Central Forces

Forces whose magnitude varies with the position vector $\vec{r}$ and direction is also along the position vector $\vec{r}$ are known as central forces. $\vec{F} = f(r)\hat{r}$



Ex. Electrostatic force, $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Gravitational force, $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$

Spring force, $\vec{F} = -kx\hat{x}$

Let's examine a particle of mass m moving from point A with a position vector. $\vec{r}_A$ to a point be with position vector $\vec{r}_B$ under a central force. $\vec{F} = f(r)\hat{r}$

Work done by the force in moving the particle is given as follows.

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B [(f(r)\hat{r})] \cdot (dr\hat{r} + rd\theta\hat{\theta})$$

$$W_{AB} = \int_A^B f(r)dr$$

Note: While all central force exhibit conservation behavior, the opposite statement might not hold.

Difference between Conservative and Non-Conservative Forces

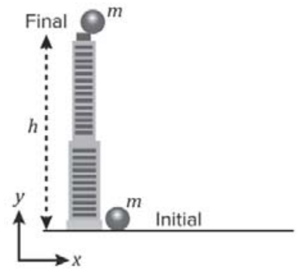
	Conservation Force		Non-conservation Force
1.	Work done is independent of the path.	1.	Work done depends on the path.
2.	Work done along a closed path is zero.	2.	Work done along a closed path is not zero.
3.	It is generally central in nature.	3.	Force velocity-depends and retarding in future.
4.	Total energy remains constant.	4.	Energy is dissipated as heat energy.
5.	Work done is completely recoverable.	5.	Work done is not completely recoverable.

Change in Potential Energy

As the configuration of a system changes, conservative forces exert work on it, which is subsequently stored as potential energy within the system. The alteration in potential energy equals the negative of the work accomplished by the internal conservative force as the system transitions from its initial to final configuration.

$$\Delta U = U_f - U_i = -(W_{conservative})_{i\to f} = -\int_i^f \vec{F}\cdot d\vec{r}$$

Gravitational Potential Energy



The alteration in gravitational potential energy is equal to the negative of the work executed by the internal conservative force as the system transitions from its initial to final configuration.

Let's consider a mass m initially situated at ground level. It is then lifted to the summit of a building with a height h in opposition to the force of gravity.

$U_i =$ Initial potential energy $U_f =$ Final potential energy

Work done by gravity is given as follows:

$W_g = m\vec{g}\cdot\vec{h} = -mgh$ (As the force and displacement are in opposite direction)

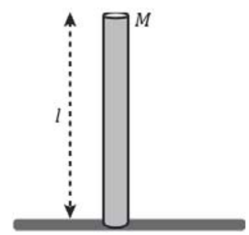
Change in potential energy is as follows.

$$U_f - U_i = -W_g = -(-mgh) = mgh$$

Taking the potential energy at ground level as zero, we get

$$U_f = mgh$$

Note: Ground level is commonly chosen as the reference line for potential energy, serving as the datum point.



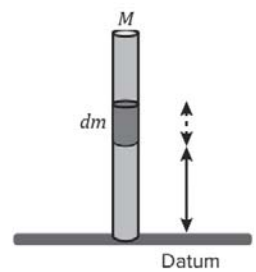
Ex. Calculate the potential energy of a uniform vertical rod of mass M and length I.

Sol. Given

Mass of the rod =M

Length of the rod = l

Consider an elements of length dx and mass dm at a distance x form the bottom as show in the given figure.



Since the density of the rod is constant,

Mass of the elements, $dm = \frac{M}{l}dx$

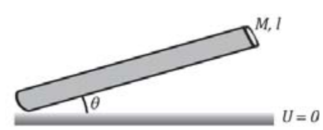
$$dU = (dm)gx = \frac{M}{l}gxdx$$

$$\int_0^U dU = \int_0^l \frac{M}{l}gxdx = \frac{Mg}{l}\int_0^l xdx$$

$$U = \frac{Mg}{l}\left[\frac{x^2}{2}\right]_0^l = \frac{Mgl}{2}$$

This is actually the potential energy of a point mass (M) kept at a distance of $\frac{1}{2}$ from the datum, which is also the location of the center of the rod.

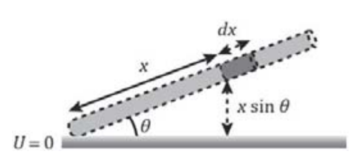
Ex. A uniform rod of mass M and length l is held in position shown in the figure. Find the potential energy of the rod.



Sol. Given, Mass of the rod = M

Length of the rod = l

Consider an elements of length dx and mass dm at a distance x from the bottom end as shown in the given figure. Since the density of the rod is constant.



Mass of the elements, $dm = \frac{M}{l}dx$

$$dU = (dm)(g)(x\sin\theta) = \frac{M}{l}gx\sin\theta dx$$

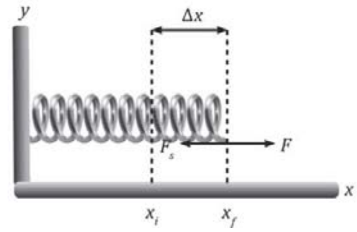
$$\int_0^U dU = \int_0^l \frac{M}{l}gx\sin\theta dx = \frac{Mg\sin\theta}{l}\int_0^l xdx$$

$$U = \frac{Mg\sin\theta}{l}\left[\frac{x^2}{2}\right]_0^l = \frac{Mgl\sin\theta}{2}$$

This is actually the potential energy of a point mass (M) kept at a distance of $\frac{l\sin\theta}{2}$ from the datum, which is also the location of the centre of the rod.

Note: For a uniform body (density constant), gravitational potential energy is, $U = mgh_{centre}$

Spring Potential Energy



The variation in spring potential energy is the opposite of the work performed by the conservative spring force as the system transitions from its initial to final arrangement.

Consider a spring which is stretched from an initial position x_i to the final position x_f.

Work done by the spring fore is a follows.

$$W_{speing} = -\frac{1}{2}k(x_f^2 - x_i^2)$$

Potential energy is as follows.

$$U_f - U_i = -W_{sping} = \frac{1}{2}k(x_f^2 - x_i^2)$$

If, $x_i = 0$ (natural length), and $x_f = x$,

$$U_f - U_i = -W_{sping} = \frac{1}{2}k(x^2 - 0^2) = \frac{1}{2}kx^2$$

Consider the natural length of the spring as datum we get $U(l_0) = 0$

So the potential energy becomes. $U = \frac{1}{2}kx^2$