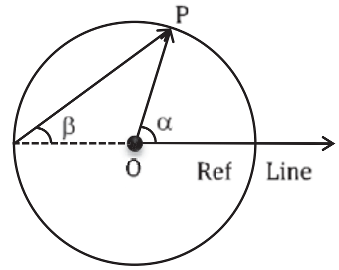
RELATIVE ANGULAR VELOCITY

Relative Angular Velocity



Just as velocities are always relative, angular velocity is also inherently relative; there is no concept of absolute angular velocity. Angular velocity is defined in relation to an origin, the point from which the position vector of the moving particle is drawn.

Examine a particle P in motion along a circular path, as depicted in the provided figure. In this scenario, the angular velocity of particle P with respect to both 'O' and 'A' will vary.

Angular velocity of a particle P w.r.t. O.

$$\omega_{PO} = \frac{d\alpha}{dt}$$

Angular velocity of a particle P w.r.t. A,

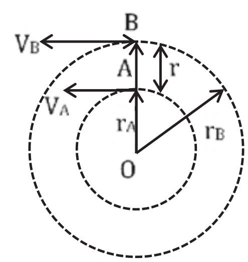
$$\omega_{PA} = \frac{d\beta}{dt}$$

Definition:

The angular velocity of a particle 'A' relative to another moving particle 'B' is the speed at which the position vector of 'A' rotates concerning 'B' at a given moment (or simply put, the angular velocity of A with the origin fixed at B). Mathematically, the angular velocity of A with respect to B, denoted as W_{AB}, is defined as:

$$\omega_{AB} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicualr to line}}{\text{separation between A and B}} = \frac{(V_{AB})_\perp}{r_{AB}}$$

Key Point



If two particles are moving on separate concentric circles with distinct velocities, the angular velocity of B as perceived by A will be contingent upon their positions and velocities. Let's consider the scenario when A and B are closest to each other, moving in the same direction as illustrated in the figure. In this situation,

$$(V_{AB})_\perp = V_B - V_A$$

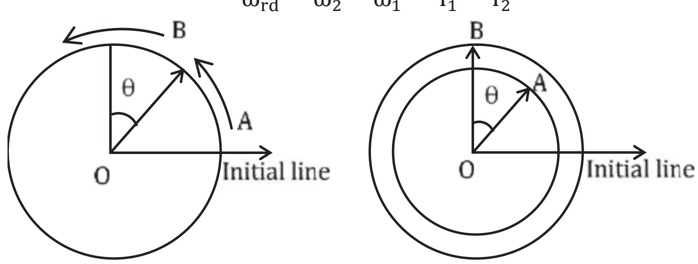
Separation between A andB is $r_{BA} = r_B - r_A$

$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}} = \frac{V_B - V_A}{r_{AB}}$$

If two particles are moving on the same circle or different coplanar concentric circles in the same direction, each with distinct uniform angular speeds, WA and WB respectively, the rate of change of the angle between $\vec{OA}$ and $\vec{OB}$ is $\frac{d\theta}{dt} = \omega_B - \omega_A$

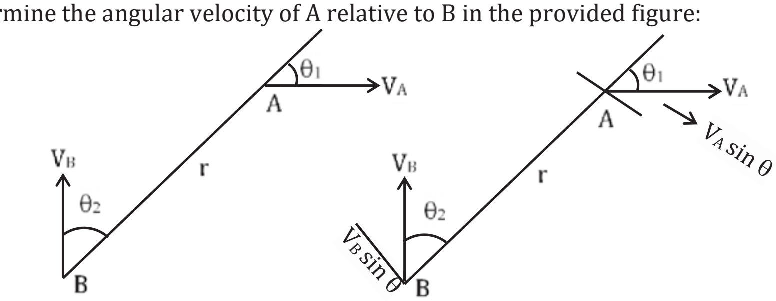
Therefore, the duration for one to complete a single revolution around point O with respect to the other.

$$T = \frac{2\pi}{\omega_{rd}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2}$$



$\omega_B - \omega_A$ It is the rate of change of the angle between $\vec{OA}$ and $\vec{OB}$ this does not represent the angular velocity of B with respect to A, which is the rate at which the line AB rotates.

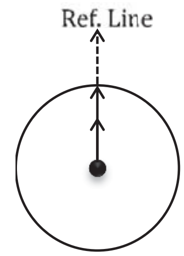
Ex. Determine the angular velocity of A relative to B in the provided figure:



Sol. Angular velocity of A with respect to B;

$$\omega_{AB} = \frac{(V_{AB})_{\perp}}{r_{AB}}$$
$$(V_{AB})_{\wedge} = V_A \sin\theta_1 + V_B \sin\theta_2$$
$$r_{AB} = r$$
$$\omega_{AB} = \frac{V_A \sin\theta_1 + V_B \sin\theta_2}{r}$$

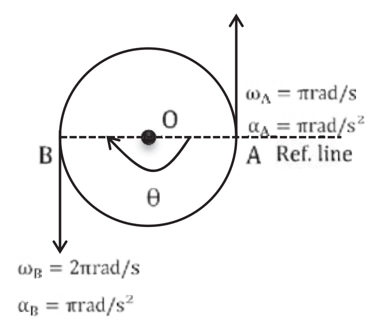
Ex. Determine the time interval for the minute hand and second hand of a clock to meet.



$$\omega_{min} = \frac{2\pi}{60}\text{rad/min}; \omega_{sec} = \frac{2\pi}{1}\text{rad/min.}$$
$$\theta_{sec} - \theta_{min} = 2\pi \text{ (for second and minute hand to meet again)}$$
$$(\omega_{sec} - \omega_{min})t = 2\pi$$
$$2\pi(1 - 1/60)t = 2\pi \Rightarrow t = \frac{60}{59}\text{min}$$

Ex. Two particles, A and B, are in motion on a circle. Initially, Particle A and B are positioned diagonally opposite to each other. Particle A moves with an angular velocity of π rad/sec and an angular acceleration of $\pi/2$ rad/sec^2. Particle B moves with a constant angular velocity of 2π rad/sec. Determine the time at which both particles A and B will collide.

Sol. Suppose angle between OA and OB = θ
then, rate of change of θ,



$$\dot{\theta} = \omega_B - \omega_A = 2\pi - \pi = \pi\text{rad/sec}$$
$$\ddot{\theta} = \alpha_B - \alpha_A = -\frac{\pi}{2}\text{rad/sec}^2$$

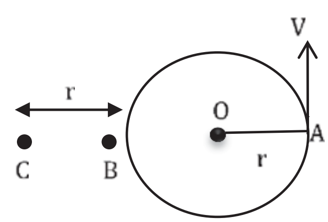
If angular displacement is Δθ,

$$\Delta\theta = \dot{\theta}t + \frac{1}{2}\ddot{\theta}t^2$$

For A and B to colide angular displacement Δθ = π

$$\pi = \pi t + \frac{1}{2}\left(\frac{-\pi}{2}\right)t^2 \Rightarrow t^2 - 4t + 4 = 0 \Rightarrow t = 2\text{sec}$$

Ex. A particle moves with a constant speed in a circle, as illustrated. Determine the angular velocity of particle A with respect to the fixed points B and C, given that the angular velocity with respect to O is denoted as ω.



Sol. Angular velocity of A with respect to O is ;

$$\omega_{AO} = \frac{(V_{AO})_{\perp}}{r_{AO}} = \frac{v}{r} = \omega$$
$$\omega_{AB} = \frac{(V_{AB})_{\perp}}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2}$$
$$\omega_{AC} = \frac{(V_{AC})_{\perp}}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$$