RADIUS OF CURVATURE AND HORIZONTAL CIRCULAR TURNINGS

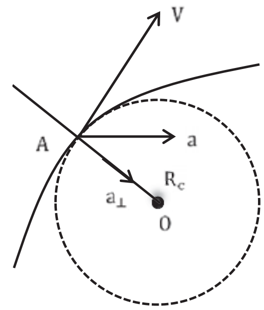
Radius of Curvature:

If a body is moving in any curvilinear path, then at different locations, the curvature would be different, thus the radius would be different.

For general curvilinear motion, when the particle crosses a point A, it is satisfying condition of moving on an imaginary circle, At this instant if

$a_\perp = \frac{v^2}{R_c}$ (where R_c is radius of curvature at this instant)

$R_c = \frac{v^2}{a_\perp} \Rightarrow R_c = \frac{(\text{Speed})^2}{\text{comp. of acceleration perpendicular to velocity}}$



Ex. An object is projected with speed 50 m/s at an angle 53° with the horizontal from ground. Find radius of its trajectory

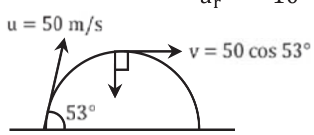
1. At the instant it is at highest point

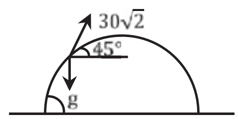
2. At t = 1 sec. after projection.

Sol. 1. At any instant acceleration of the projectile is 'g' downward. At the highest point velocity has magnitude = 50 cos 53° = 30 m/s and is in horizontal direction. Thus acceleration perpendicular to velocity is 'g' itself.

$$a_r = \frac{v^2}{R} = g$$

$$R = \frac{v^2}{a_r} = \frac{(30)^2}{10} = 90 \text{ m}$$





2. At t = 1sec $V_x = 50 \cos 53° = 30$m/s

$$V_y = 50\sin 53° - g(1) = 30\text{m/s}$$

$$V = \sqrt{V_x^2 + V_y^2} = 30\sqrt{2} \text{ m/s}$$

i.e. $\vec{v}$ is at angle 45° with the horizontal

$a_\perp =$ componant of g perpendicular to velocity $= \frac{g}{\sqrt{2}}$

$$\frac{(30\sqrt{2})^2}{R} = \frac{10}{\sqrt{2}}$$

$$R = 180\sqrt{2}\text{m}$$

Note If a particle follows a path described by y = f(x), then the radius of curvature at any point (x, y) on the trajectory is determined by...

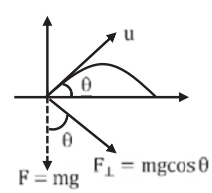
$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$

Ex. A particle of mass m is projected with speed u at an angle q with the horizontal. Find the radius of curvature of the path traced out by the particle at the point of projection and also at the highest point of trajectory.

Sol. At point of projection $R = \frac{mvv^2}{F_\perp} = \frac{mu^2}{mg\cos\theta}$

$$R = \frac{u^2}{g\cos\theta}$$

At highest point $a_\perp = g, v = u\cos\theta: R = \frac{v^2}{a_\perp} = \frac{u^2\cos^2\theta}{g}$



Ex. A particle moves along the plane trajectory y(x) with constant speed v. Find the radius of curvature of the trajectory at the point x = 0 if the trajectory has the form of a parabola $y = ax^2$ where 'a' is a positive constant.

Sol. If the equation of the trajectory of a particle is given we can find the radius of trajectory of the instantaneous circle by using the formula.

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$y = ax^2 \Rightarrow \frac{dy}{dx} = 2ax = 0(\text{ at } x = 0) \text{ and } \frac{d^2y}{dx^2} = 2a$$

Now radius of trajectory is given by

$$R = \frac{[1 + 0]^{3/2}}{2a} = \frac{1}{2a}$$

OR

This problem can also be solved by using the formula : $R = \frac{v^2}{a_\perp}. y = ax^2$,

differentiate with respect to time $\frac{dy}{dt} = 2ax\frac{dx}{dt}$

at $x = 0, v_y = \frac{dy}{dt} = 0$ hence $v_x = v$

Since v_x is constant, $a_x = 0$

Now, differentiate (1) with respect to time $\frac{d^2y}{dt^2} = 2ax\frac{d^2x}{dt^2} + 2a(\frac{dx}{dt})^2$

at $x = 0, v_x = v$

net acceleration, $a_y = a_y = 2av^2(\text{ since } a_x = 0)$

this acceleration is perpendicular to velocity (v_x)

Hence it is equal to centripetal acceleration

$$R = \frac{v^2}{a} = \frac{v^2}{2av^2} = \frac{1}{2a}$$

Circular Turning On Roads:

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. By friction only
2. By banking of roads only.
3. By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

$$f = \frac{mv^2}{r}$$

Further, limiting value of f is μN

$$f_L = \mu N = \mu mg (N = mg)$$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$

$$\frac{mv^2}{r} \le \mu mg \text{ or } \mu \ge \frac{v^2}{rg} \text{ or } v \le \sqrt{\mu rg}$$

Here, two situations may arise. If m and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$

Ex. A bend in a level road has a radius of 100 m. Calculate the maximum speed which a car turning this bend may have without skidding. Given : $\mu = 0.8$.

Sol. $V_{max} = \sqrt{\mu rg} = \sqrt{0.8 \times 100 \times 10} = \sqrt{800} = 28 \text{ m/s}$