KINEMATICS OF CIRCULAR MOTION

Non-Uniform Circular Motion (NUCM)

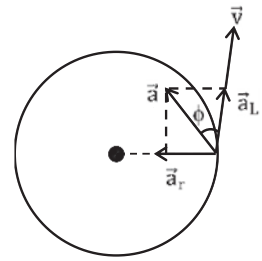
In non-uniform circular motion, both the direction and magnitude of the velocity of a particle vary.

$$\text{Now, } \vec{v} = v(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -v(\sin\theta\hat{i} + \cos\theta\hat{j})\frac{d\theta}{dt} + (-\sin\theta\hat{i} + \cos\theta\hat{j})\frac{dv}{dt}$$

we can write $\vec{a} = \vec{a}_r + \vec{a}_t$

$$\vec{a}_r = v\omega(-\hat{r}) \text{ i.e., radial acceleration and } \vec{a}_t = (\frac{dv}{dt})\hat{v}$$



With a unit vector equal to , it also lies in the tangential direction and is referred to as tangential acceleration. Consequently, in this scenario, acceleration exhibits two components: one along the velocity, namely tangential acceleration, and another perpendicular to the velocity, denoted as radial acceleration, as illustrated.

If net acceleration $(\vec{a})$ makes angel ϕ with tangential direction, we may write.

$$\tan\phi \frac{a_r}{a_t}$$

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2}$$

Some point to remember:

$$a_t = \frac{dv}{dt} = \frac{rd\omega}{dt} = r\alpha \Rightarrow |\vec{a}| = \sqrt{(\omega^2 r)^2 + (r^\alpha)^2}$$

Students need not be confused with $|\frac{d\vec{v}}{dt}|$ and $\frac{d|\vec{v}|}{dt}\cdot\vec{v}$ Contains both magnitude and direction; thus, $\frac{d\vec{v}}{dt}$ means $\vec{a}_{net}\,|\frac{d\vec{v}}{dt}| = |\vec{a}_{net}|$

Also $|\vec{v}|$Refers to the change in the magnitude of velocity solely due to tangential acceleration.

$$|\frac{dv}{dt}| = a_t$$

Radial And Tangential Acceleration

In circular motion, there exist three forms of acceleration: tangential acceleration and centripetal acceleration Total acceleration.

Tangential acceleration

Component of acceleration directed along tangent of circle is called tangential acceleration. It is responsible for changing the speed of the particle. It is defined as,

$$a_i = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt} = \text{ Rate of change of speed.}$$

$$a = \alpha_r$$

Centripetal acceleration

It is responsible for change in direction of velocity. In circular motion, there is always a centripetal acceleration.

Centripetal acceleration is always variable because it changes in direction.

Centripetal acceleration is also called radial acceleration or normal acceleration.

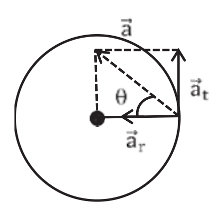
Total acceleration

Total acceleration is vector sum of centripetal acceleration and tangential acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_r + \vec{a}_t$$

$$a = \sqrt{a_t^2 + a_r^2}$$

$$\tan\theta = \frac{a_r}{a_t}$$



Ex. A particle is revoluting a circular of radius 0.2 m with angular velocity $\omega = 20t^{\circ}\text{rad/s}$ where t is in seconds. Find its acceleration at $t = 0.5\text{sec}$.

Sol. $a_r = \omega^2 r = (20t^2)^2(0.2) = 80t^2 \text{m/s}^2$

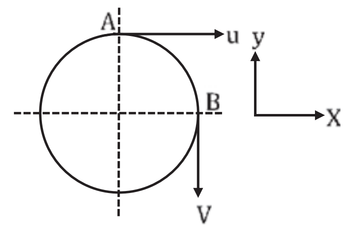
$$t = 0.5\text{sec}; a_r = 80(0.5)^4 = 5\text{m/s}^2$$
$$\alpha = \frac{d\omega}{dt} = 40t\text{rad/s}^2$$
$$a_t = r\alpha = 8t$$
$$t = 0.5\text{sec}; a_t = 8(0.5) = 4 \text{ m/s}^2$$
$$|\vec{a}| = \sqrt{a_t^2 + a_r^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{41} \text{ m/s}^2$$
$$a = 6.4 \text{ m/s}^2$$

Ex. A particle is moving in a circular path of radius R with speed u, when it begins to speed up at a constant rate. After that when it completes and fourth revolution, change in its velocity vector has magnitude 2u. At that moment, find

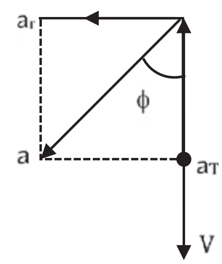
1. Its radial acceleration
2. Angle it makes with its velocity vector.

Sol. 1. Suppose initially it was at point A with speed u and now it is at point B as shown with speed v.

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_{iu}$$
$$= (-\hat{j}) - (u\hat{i})$$
$$|\Delta\vec{v}| = \sqrt{v^2 + u^2}$$
$$\sqrt{v^2 + u^2} = 2u$$
$$v^2 = 3u^2$$
$$a_r = \frac{v^2}{R} \Rightarrow a_r = \frac{3u^2}{R}$$



2.
$$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$
$$(\frac{v}{R})^2 = (\frac{u}{R})^2 + 2\alpha(\frac{\pi}{2})$$
$$\alpha = \frac{v^2 - u^2}{\pi R^2} = \frac{3u^2 - u^2}{\pi R^2} = \frac{2u^2}{\pi R^2}$$
$$a_t = R\alpha = \frac{2u^2}{\pi R}$$
$$\tan\phi = \frac{a_r}{a} \Rightarrow \phi = \tan^{-1}(\frac{3\pi}{2})$$



3. Average Angular Acceleration:

Let w_1 and w_2 be the instantaneous angular speed at times t_1 and t_2 respectively, then average angular acceleration αav is defined as

$$\vec{\alpha}_w = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{dt}$$

4. Instantaneous Angular Acceleration:

It is the limit of average angular acceleration as Δt approaches zero i.e.

$$\vec{\alpha} \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{dt} = \frac{d\vec{\omega}}{dt}$$

Since $\vec{\omega} = \frac{d\vec{\theta}}{dt}, \therefore \vec{\alpha} = \frac{d\vec{\omega}}{dt} \therefore = \frac{d^2\vec{\theta}}{dt^2}$

Also $\vec{\alpha} = \frac{\omega d\vec{\omega}}{d\theta}$

Note Both average and instantaneous angular acceleration are axial vector with dimension. $[T^{-2}]$ and unit rad/s^2.

If a = 0, circular motion is said to be uniform.