DYNAMICS OF CIRCULAR MOTION & APPLICATIONS OF CENTRIPETAL FORCE

If there is no force acting on a body it will move in a straight line (with constant speed). Hence if an object moving in a circular path or any curved path, there must be some force acting on the body.
If speed of body is constant, the net force acting on the body is along the inside normal to the path of the body and it is called centripetal force.

$$\text{Cntripetal Force} = ma_c = \frac{mv^2}{r} = m\omega^2 r$$

However if speed of the body varies then, in addition to above centripetal force which acts along inside normal, there is also a force acting along the tangent of the path of the body which is called tangential force.

$$\text{Tangential Force } (F_t) = Ma_t = \frac{Mdv}{dt} = M\alpha r;$$

(Where is the angular acceleration?)

Note

Remember $\frac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal which is responsible for circular motion. This force may be friction, normal, tension, spring force, gravitation force or a combination of them.
So to solve any problems in uniform circular motion we identify all the forces acting along the normal (towards center), calculate their resultant and equate it to $\frac{mv^2}{r}$
If circular motion is non-uniform then in addition to above step we also identify all the circular path, calculate their resultant and equate it to $\frac{mdv}{dt}$ or $\frac{md|\vec{v}|}{dt}$.

Example of Centripetal Force

Combination of forces which provide centripetal acceleration necessary for the revolution of a particle is called centripetal force or radial force for example

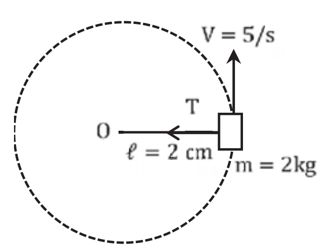
1. For object tied to a string and is revolving on a smooth horizontal surface, tension is centripetal force.
2. To revolve satellite around the earth, gravitational force provides centripetal. So gravitational force is centripetal force on it.
3. For motion of electron around nucleus, the electrostatic force on electron is centripetal force on it.
4. For an object placed on a rough rotating table, friction on the object due to table is centripetal force.

Problem solving

1. Identify the plane of circular motion
2. locate the centre of rotation and calculate the radius.
3. Make F.B.D.
4. The net force along radial direction is mass times the radial acceleration i.e., $m(\frac{v^2}{R})$ or $m(\omega^2 R)$

   Centripetal force $\left(\frac{mv^2}{R}\right)$ is no seperate force like tension, weight, spring force, normal reaction, friction etc. In fact anyone of these centripetal force.

Circular Motion In Horizontal Plane:

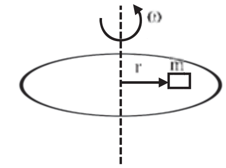


Ex. A block of mass 2kg is tied to a string of length 2m, the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed 5 m/s. Find the tension in the string.

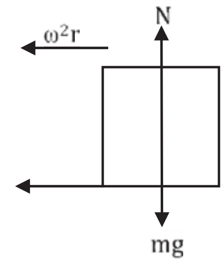
Sol. Here centripetal force is provided by tension.

$$T = \frac{mv^2}{r} = \frac{2 \times 5^2}{2} = 25 \text{ N}$$

Ex. A rough horizontal table can rotate about its axes as shown. A small block is placed at r = 20 cm from its axis. The coefficient of friction between them is 0.5. Find the maximum angular speed that can be given to the block table system so that the block does not slip on the table.

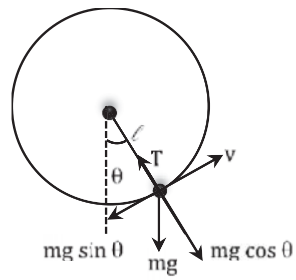


Sol. The force acting on the block are as shown below. N and mg are in vertical direction, so the only force that can provide necessary centripetal acceleration in horizontal plane is friction. It makes the block to revolve with the table without slipping.



$$(F_{net})_y = 0 \Rightarrow N = mg$$
$$(F_{net})_x = ma$$
$$f = m\omega^2 r$$
$$f \le \mu N$$
$$m\omega^2 r \le \mu mg \Rightarrow \omega \le \sqrt{\frac{\mu g}{r}}$$
$$\omega_{max} = \sqrt{\frac{\mu g}{r}} = \sqrt{\frac{0.5 \times 10}{0.2}}$$
$$\omega_{m\geqslant x} = 5 rad/s^2$$

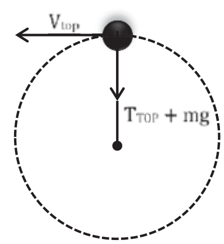
Motion in a vertical circle



Let us consider the motion of a point mass tied to a string of length l and whirled in a vertical circle. If at any time the body is at angular position q, as shown in the figure, the forces acting on it are tension T in the string along the radius towards the center and the weight of the body mg acting vertically down wards.

Applying Newton's law along radial direction

$$T - mg\cos\theta = m \cdot a_c = \frac{mv^2}{\ell}$$
$$T = \frac{mv^2}{\ell} + mg\cos\theta \quad \text{... (1)}$$



The point mass will complete the circle only and only if tension is never zero (except momentarily, if at all) if tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile.

From equation ... (1), it is evident that tension decreases with increase in q because cos q is a decreasing function and v decreases with height. Hence tension is minimum at the top most point. i.e. $T_{min} = T_{topmost}$ $T > 0$ at all points. $\Rightarrow T_{min} > 0$

However if tension is momentarily zero at highest point the body would still be able to complete the circle.

Hence condition for completing the circle (or looping the loop) is

$$T_{min} \ge 0 \text{ or } T_{top} \ge 0$$
$$T_{top} + mg = \frac{mv_{top}^2}{\ell} \quad \text{... (2)}$$

Equation...(2) could also be obtained by putting q = p in equation ..(1).

For looping the loop, $T_{top} \ge 0$

$$\frac{mv_{top}^2}{\ell} \ge mg \Rightarrow v_{top} \ge \sqrt{g\ell} \quad \text{... (3)}$$

Condition for looping the loop is $v_{mpp} \ge \sqrt{g\ell}$.

If speed at the lowest point is u, then from conservation of mechanical energy between lowest point and top most point.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_{top}^2 + mg \cdot 2\ell$$

Using Equation (3) for v_{top} we get $u \ge \sqrt{5g\ell}$

i.e., for looping the loop, velocity at lowest point must be $\ge \sqrt{5g\ell}$