CONICAL PENDULUM

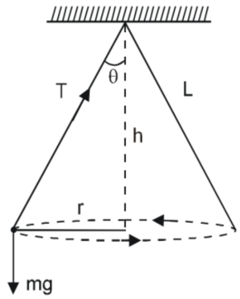
Ex. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r. Find

(a) The angular speed of the particle and

(b) The tension in the string. (Such a system is called a conical pendulum)

Sol. The situation is shown in figure. The angle θ made by the string with the vertical is given b.

$$\sin\theta = r/L, \cos\theta = h/L = \frac{\sqrt{L^2-r^2}}{L} \quad \text{... (1)}$$



The forces on the particle are

(a) The tension T along the string and

(b) The weight mg vertically downward.

The particle is moving in a circle with a constant speed v. Thus , the radial acceleration towards the centre has magnitude v^2/r. Resolving the forces along the radial direction and applying Newton's second law,

$$T\sin\theta = m(v^2/r) \quad \text{... (2)}$$

As there is no acceleration in vertical direction, we have from Newton's law

$$T\cos\theta = mg \quad \text{... (3)}$$

Dividing (2) by (3),

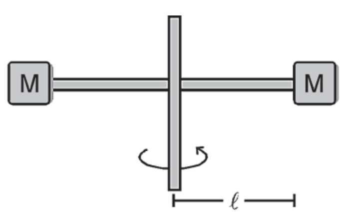
$$\tan\theta = \frac{v^2}{rg} \text{ or,}$$

$$v = \sqrt{rg\tan\theta}$$

$$\omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{L\cos\theta}} = \sqrt{\frac{g}{(L^2-r^2)^{\frac{1}{2}}}}$$

And from (3), $T = \frac{mg}{\cos\theta} = \frac{mgL}{(L^2-r^2)^{\frac{1}{2}}}$

Ex. Two blocks each of mass M are connected to the ends of a light frame as shown in figure. The frame is rotated about the vertical line of symmetry. The rod breaks if the tension in it exceeds T_0. Find the maximum frequency with which the frame may be rotated without breaking the rod.



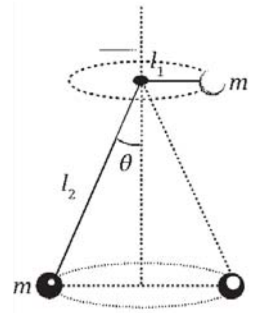
Sol. Consider one of the blocks . If the frequency of revolution is f, the angular velocity is $\omega = 2\pi f$ The acceleration towards the centre is $\omega^2 l = 4\pi^2 f^2 l$ The only horizontal force on the block is the tension of the rod. At the point of breaking , this force is T_0 . So from Newton's law,

$$T_0 = M.4\pi^2 f^2 1$$

$$f = \frac{1}{2\pi}\left[\frac{T_0}{M\ell}\right]^{1/2}$$

Ex. Two particles, each of mass m, are attached to the two ends of a light string of length l, which passes through a hole at the center of a table. One particle describes a circle on the table with angular velocity ω_1 and the other describes a circle as a conical pendulum with angular velocity ω_2 below the table, as shown in the figure. If l_1 and l_2 are the lengths of the portion of the string above and below the table, then which of the following options is correct?

(A) $\frac{l_1}{l_2} = \frac{\omega_2^2}{\omega_1^2}$ (B) $\frac{l_1}{l_2} = \frac{\omega_1^2}{\omega_2^2}$ (C) $\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{l}{g}$ (D) $\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} > \frac{l}{g}$



Sol. Given,

Mass of each particle = m

Angular velocity of horizontally rotating particle = ω_1

Angular velocity of conical pendulum particle = ω_2

Length of portion of string above the table = l_1 = Radius of horizontal circle on table

Length of portion of string below the table = l_2

Let θ be the angle that the conical pendulum makes with the vertical.

Let the tension in the string be T.

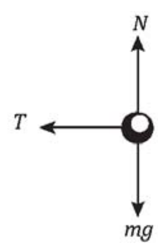
Consider the particle above the table.

Balancing forces in the vertical direction, we get,

$$\Sigma \vec{F}_y = \vec{0}$$
$$N - mg = 0$$
$$N = mg \quad \text{... (1)}$$

Balancing forces in the radial direction, we get

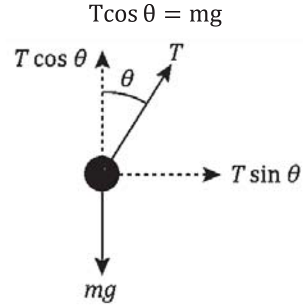
$$\Sigma \vec{F}_c = m\vec{a}_c$$
$$T = m(\omega_1)^2 l_1 \quad \text{... (2)}$$



Consider the particle below the table.

Balancing forces in the vertical direction, we get

$$\Sigma \vec{F}_y = \vec{0}$$
$$T\cos\theta - mg = 0$$
$$T\cos\theta = mg \quad \text{... (3)}$$



Balancing forces in the radial direction, we get,

$$\Sigma \vec{F}_c = m\vec{a}_c$$

$$T\sin\theta = m(\omega_2)^2 r$$

$$T \times \frac{r}{l_2} = m(\omega_2)^2 r (\because \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}})$$

$$T = m(\omega_2)^2 l_2 \qquad \dots (4)$$

From equations (2) and (4), we get,

$$m(\omega_1)^2 l_1 = m(\omega_2)^2 l_2$$

$$\frac{\omega_1^2}{\omega_2^2} = \frac{l_2}{l_1}$$

Thus, option (A) is the correct answer.