CENTRIFUGAL FORCE AND CONCEPT OF APPARENT WEIGHT

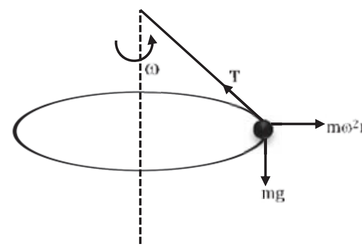
Centrifugal Force:

When a body undergoes rotation within a circular path and the centripetal force diminishes, the body will deviate from the circular trajectory. Observer A, who does not partake in the circular motion, perceives the body as veering off tangentially at the release point. However, for observer B, who rotates along with the released body, it seems as though an external force propelled the body outward along the radius away from the center. This apparent force acting outward is known as centrifugal force.

Its magnitude is equal to that of the centripetal Force $= \frac{mv^2}{r} = m\omega^2 r$

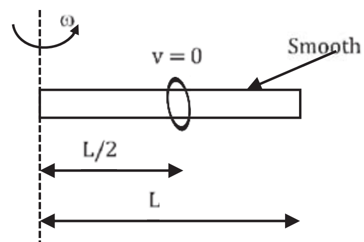
Direction of centrifugal force, it is always directed radially outward.

Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame. FBD of ball w.r.t. non-inertial frame rotating with the ball.



Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyses the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

Ex. A ring which can slide along the rod are kept at mid-point of a smooth rod of length L. The rod is rotated with constant angular velocity ω about vertical axis passing through its one end. Ring is released from midpoint. Find the velocity of the ring when it just leave the rod.



Sol. Centrifugal force

$$m\omega^2 x = ma$$

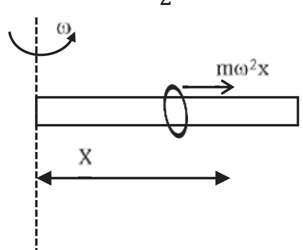
$$\omega^2 x = \frac{vdv}{dx}$$

$$\int_{\frac{L}{2}}^{L} \omega^2 x dx = \int_0^v vdv \text{ (integrate both side.)}$$

$$\omega^2\left(\frac{x^2}{2}\right)_{\frac{L}{2}}^{L} = \left(\frac{v^2}{2}\right)_0^v$$

$$\omega^2\left(\frac{L^2}{2} - \frac{L^2}{8}\right) = \frac{v^2}{2}$$

$$v = \frac{\sqrt{3}}{2}\omega L.$$



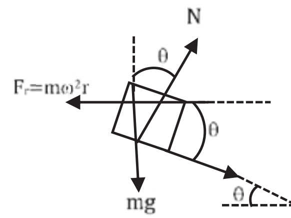
Velocity at time of leaving the rod

$$v' = \sqrt{(\omega L)^2 + \left(\frac{\sqrt{3}}{2}\omega L\right)^2} = \frac{\sqrt{7}}{2}\omega L$$

Ex. A small block is placed on a rough triangular shaped wedge which is revolving as shown such that the block undergoes circular path of radius R. The coefficient of friction between the block and the wedge is μ. Find the range of angular speed w so that the block does not slip with respect to the wedge.

Sol. If we select the frame of reference as the point at which the small block is placed. We take centrifugal force $m\omega^2 r$ outward.

1. Maximum angular speed (ω_{max})



More isω, more is centrifugal force to balance which more horizontal force is required radially inwards. The friction force acts down the incline as shown so that its horizontal component helps horizontal component of Normal force to balance centrifugal force.

$(F_{ne})_y = 0 \Rightarrow N\cos\theta - f\sin\theta - mg = 0$

Under limiing condition i.e. when the block is about to slip, $f = \mu N$

$$N(\cos\theta - \mu\sin\theta) = mg$$

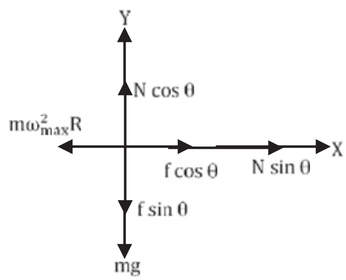
$$N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$(F_{net})_x = 0 \Rightarrow N\sin\theta + (\mu N)\cos\theta - m\omega_{max}^2 R = 0$$

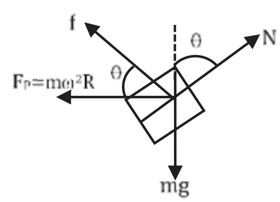
$$N(\sin\theta + \mu\cos\theta) = m\omega_{max}^2 R$$

$$mg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) = m\omega_{max}^2 R$$

$$\omega_{max} = \sqrt{\frac{g}{R}\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$



2. For minimum angular speed (ω_{max})



Lesser is w, lesser is centrifugal force thus friction now acts up the incline so as to provide horizontal component along centrifugal force to balance horizontal component of Normal force.

$$(F_{ne})_y = 0 \Rightarrow N\cos\theta + f\sin\theta - mg = 0$$

$$N(\cos\theta + \mu\sin\theta) = mg$$

$$N = \frac{mg}{\cos\theta + u\sin\theta}$$

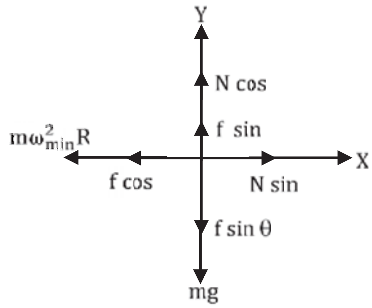
$$(\ Fnet\ )_x = 0 \Rightarrow N\sin\theta - f\cos\theta - m\omega_{min}^2 R = 0$$

$$N(\sin q - \mu\cos q) = mw_{min}^2\ R$$

$$mg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right) = m\omega_{min}^2 R$$

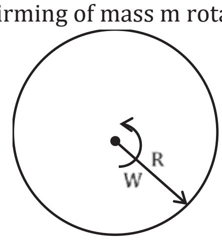
$$\omega_{min} = \sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{R(\cos\theta + \mu\sin\theta)}}$$

$$\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{R(\cos\theta + \mu\sin\theta)}} \le \omega \le \sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{R(\cos\theta - \mu\sin\theta)}}$$

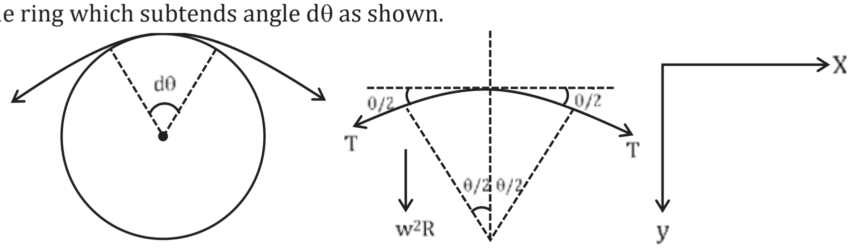


Tension In Rotating Ring:

Diagram shown top view of a unfirming of mass m rotating in horizontal plane.



Here tension inside the ring is an internal force for the ring so to find it, we take a very small part of the ring which subtends angle dθ as shown.



We take y-axis towards the centre and x-axis in tangential direction and origin as the mid-point of the element

$$(F_{nat}) = (dm)\omega^2 R$$

$$2T\sin\left(\frac{d\theta}{2}\right) = (dm)\omega^2 R \qquad \text{... (1)}$$

$$\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2} \ (\because d\theta \text{ is very small})$$

Also dm = (mass per unit length of ring) × length of element

$$dm = \left(\frac{M}{2\pi R}\right)(Rd\theta) = \left(\frac{M}{2\pi}\right)d\theta$$

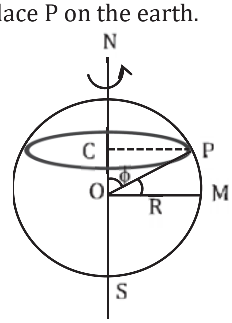
Putting these values in equation (i), we get

$$2\ T = \left(\frac{d\theta}{2}\right) = \left(\frac{M}{2\pi}\right)d\theta \times \omega^2 R$$

$$T = \frac{M\omega^2 R}{2\pi}$$

Effect Of Earths Rotation On Apparent Weight:

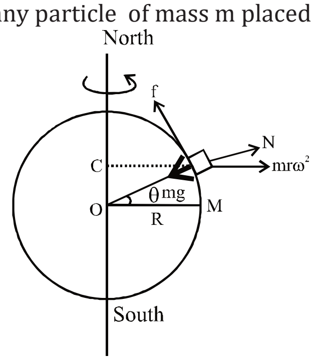
The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation. Every point on the moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place P on the earth.



Draw a perpendicular PC from P to the axis SN. The places P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place P. We have

$$CP = OP\cos\theta \text{ or, } r = R\cos\theta$$

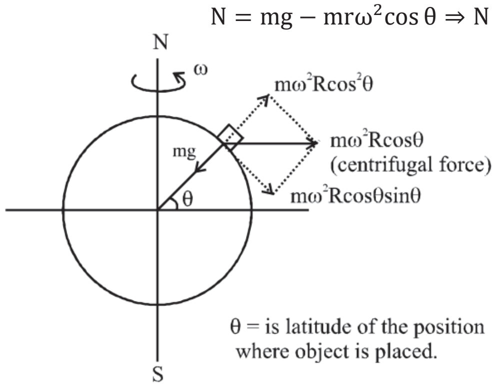
Here R is the radius of the earth and is colatitude angle. If we work from the frame of reference of the earth, we shall have to assume the existence of pseudo force. In particular, a centrifugal force mw^2r has to be assumed on any particle of mass m placed at P.

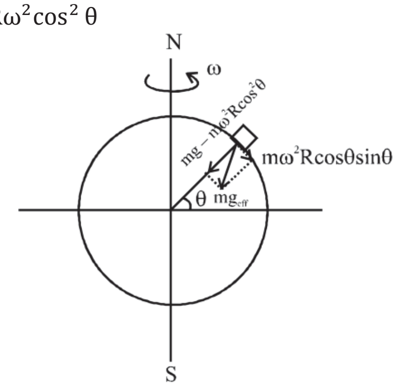


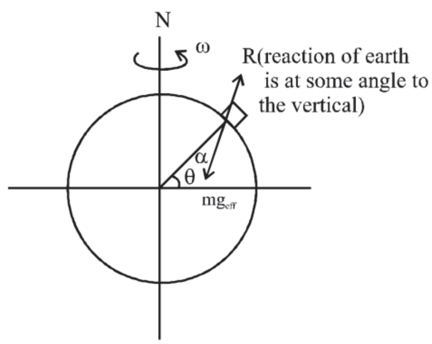
If we consider a block of mass m at point P then this block is atrest with respect to earth. If resolve the forces along and perpendicular the centre of earth then

$$N + mr\omega^2\cos\theta = mg$$

$$N = mg - mr\omega^2\cos\theta \Rightarrow N = mg - mR\omega^2\cos^2\theta$$







Note: At equator ($\theta = 0$) Wapp. is minimum and at pole ($\theta = \pi/2$) Wapp.. is maximum . This apparent weight is not along normal but at some angle   w.r.t. it. At all point except poles and equator (= 0 at poles and equator)

Ex. A body weighs 98N on a spring balance at the north pole. What will be the reading on the same scale if it is shifted to the equator?

$$\text{Use } g = GM/R^2 = 9.8 \text{ ms}^2 \text{ and } R_{earth} = 6400 \text{ km}$$

At poles , the apparent weight is same as the true weight.

Thus, $98N = mg = m\,(9.8 \text{ m/s}^2)$

At the equator, the apparent weight is

$$mg' = mg - m\omega^2 R$$

The radius of the earth is 6400 km and the angular speed is

$$\omega = \frac{2\pi \text{rad}}{24 \times 60 \times 60 \text{s}} = 7.27 \times 10^{-6} \text{rad/s}$$

$$mg' 98 \text{ N} - (10\text{kg}(7.27 \times 10^{-5} \text{ s}^{-1})^2(6400 \text{ km})$$

$$97.66 \text{NAns}$$